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Interference Alignment for Cognitive Radio Communications and Networks: A Survey

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Abstract: Interference alignment (IA) is an innovative wireless transmission strategy that has shown to be a promising technique for achieving optimal capacity scaling of a multiuser interference channel at asymptotically high-signal-to-noise ratio (SNR). Transmitters exploit the availability of multiple signaling dimensions in order to align their mutual interference at the receivers. Most of the research has focused on developing algorithms for determining alignment solutions as well as proving interference alignment's theoretical ability to achieve the maximum degrees of freedom in a wireless network. Cognitive radio, on the other hand, is a technique used to improve the utilization of the radio spectrum by opportunistically sensing and accessing unused licensed frequency spectrum, without causing harmful interference to the licensed users. With the increased deployment of wireless services, the possibility of detecting unused frequency spectrum becomes diminished. Thus, the concept of introducing interference alignment in cognitive radio has become a very attractive proposition. This paper provides a survey of the implementation of IA in cognitive radio under the main research paradigms, along with a summary and analysis of results under each system model.

Keywords: opportunistic interference alignment (OIA); spatial water-filling (SWF); space-time water-filling (ST-WF); maximum eigenmode beamforming (MEB)

1. Introduction

The increased deployment of wireless services has on the one hand, consistently led to greater scarcity of the licensed frequency spectrum [1] and on the other hand however, has resulted in underutilization of frequency spectrum. This is largely due to the current fixed spectrum access (FSA) policy, which is based on a static allocation of the spectrum resources. This gives licensed users exclusive rights to the spectrum and results in several portions of the licensed spectrum being underutilized [2–4]. This justifies demand for a more flexible access policy called dynamic spectrum access (DSA) because it ensures better utilization of the spectrum resources. The spectrum resources are still allocated to the licensed/primary users (PUs) but its usage is not exclusively granted. Unlicensed/secondary users (SUs) with cognition capability [5] are able to access the spectrum resources opportunistically, as long as the PUs remain idle. These SUs or cognitive radios (CRs) can also concurrently share the spectrum with the PUs, as long as the PUs transmissions are adequately protected. The radio spectrum can therefore be reused in an opportunistic manner or shared all the time, to improve spectral efficiency [6].

This cognition capability of CRs can be divided into three main components namely spectrum sensing, spectrum analysis and spectrum access decision. Spectrum sensing is by far the most important component in the establishment of CRs as it provides the awareness of the overall spectrum utilization as well as which bands to sense for gaps in the spectrum (also called spectrum holes), when to sense for those gaps and for how long to sense in real-time [7]. There are two techniques usually involved in spectrum sensing (SS), namely direct spectrum sensing and indirect spectrum sensing, the former being used to detect the receiver (Rx) of the PU, while the latter is used to detect the transmit (Tx) of the PU. Even though it is more effective for SS to be employed directly, it is the more challenging of the two techniques because the Rx does not usually transmit when it works [7,8]. Therefore, most of the existing SS schemes employ indirect SS techniques. Such schemes are generally classified to matched filter detection, energy detection, feature detection, and interference temperature measurement.

The reality however, is that SUs are almost unable to operate concurrently with the PUs without causing harmful interference to the PU system, thus a very crucial task in the design of CR networks is about how best the SUs can avoid interfering with the PUs in their vicinity [9–11]. This becomes particularly challenging nowadays due to data communication, where the PU is seldom idle, such that the availability of spectrum holes becomes diminished [12].

In attempting to solve to this issue, the emphasis of research has moved in the direction of a innovative interference management strategy called interference alignment (IA) [13–15]. It is a radical cooperative interference management strategy that has recently emerged through rigorous analysis of interference channels and networks. This approach exploits the availability of multiple signaling dimensions either from multiple antennas, time slots or frequency blocks as the case may be. The transmitters linearly encode their signals over these multiple signaling dimensions, such that the resulting interference signal observed at each receiver lies in a lower dimensional subspace and is orthogonal to the one spanned by the signal of interest at each receiver while the other subspace is reserved for interference free communication. This helps to achieve the interference channels maximum multiplexing gain, otherwise known as degrees of freedom (DoF) [16,17]. DoF is an essential measurement used for capacity approximation. DoF is the amount of signaling dimensions, respective dimension corresponding to an interference-free additive white Gaussian noise (AWGN) channel with signal-to-noise ratio (SNR) that increases proportionally with the overall transmit power [18].

The study of DoF initiated in ref. [19] found surprisingly high DoF, as SNR approached infinity. The results of ref. [19] in which users are able to transmit at a data rate equal to one-half of their capacity in an ideal interference-free channel motivated the work of refs. [17,20], which presented the IA scheme in its linear form for the two user X-channel as a general principle. It was particularly interesting to see that the potential DoF that could be created largely depended on the alignment techniques involved. This implies that the signal space across the entire network nodes could potentially have numerous spatial dimensions as the overall number of transmit and receive antennas. However, achieving ideal signal alignment is a very complex task [19] of which techniques such as message sharing in the manner of CR, beamforming, zero forcing and successive decoding may be combined in many different ways across users, data streams and antennas to establish inner bounds on the DoF [19]. This overview will focus on the technique of message sharing.

The earliest work to explore the increase in DoF with message sharing in the manner of CR was done in refs. [20,21] for both the multiple-input multiple-output (MIMO) interference and X-channels. This is because the CR network can be seen as an interference channel when the SUs coexist with the PUs and the SU transmission is subjected to the PU receiver threshold as well as the cross interference between the SUs themselves. Thus the IA strategy tends to as expected fit in with CR systems mechanisms of managing interference between the PU and SU [10]. This work explored the impact of distributing a user's information with other the user's transmitter or receiver in that manner of singularity. In terms of performance, the DoF for the MIMO interference channel (IC) remained the same as without any form of cognition at either transmitters or receivers. Indeed, even this was a good enough result for further research bearing in mind that with all nodes having equal number of antennas, any increment on the number of antennas would yield higher data rates. However, the IC was shown to achieve higher DoF if both users have some form of cognition at the same time i.e., they either both make use of cognitive receivers, or they both make use cognitive transmitters or one has a cognitive transmitter although the other has a cognitive receiver.

A number of other practical IA algorithms have been developed in the manner of message sharing such as refs. [22,23], but these have primarily been developed for the single-tier K-user IC where each transmitter has an intended receiver, and the remaining transmitters are considered as interferers for that receiver. Despite these studies focusing on single-tier systems, they have provided a significant research platform that has been translated into mainstream two-tier CR networks. From all the work done to further exploit this research subject, it has emerged that broadly speaking there are two main system models or paradigms in the design and implementation of IA in two-tier CR networks [24].

The first paradigm considers a CR network having a number of SU pairs and a single PU-Tx and PU-Rx pair, where the PU-Tx is assumed far from its Rx and the SU-Rx's are considered not to be influenced by the interference from the PU-Tx (which is somewhat similar to the direct spectrum sensing scenario [7,8]). The main goal of this IA approach is to choose appropriate transmit precoding matrices and receiver interference suppression subspaces for the SUs to ensure that individual SU-Rx can decode their own signals while maintaining an allowable interference level to the PU within the specified threshold by minimizing the interference leaked into the received signal subspace. These matrices impose an upper limit on the interference temperature, short of the constraint on the amount of SUs. This optimization design is solved iteratively until the algorithm converges monotonically, where transmitters and receivers characterise the precoders and receiver subspaces by turns.

The second paradigm considers the same CR network as the first paradigm, but the SUs are in closer proximity to the PU-Tx (in the manner of indirect spectrum sensing [8]). However, this paradigm proposes that under power-limitation, a PU that maximizes its own rate by using appropriate algorithms (usually water-filling algorithms) on its MIMO channel singular values might leave some of them unused, i.e., no transmission takes place along the corresponding spatial directions. These unused directions may be opportunistically utilized by one of the SU-Tx by designing an IA technique for the SUs, in which a linear pre-coder perfectly aligns the interference generated by the SU-Tx with such unused spatial directions, thereby enabling the SUs to share the licensed spectrum with zero interference to the PU transmission [24]. An optimization scheme can also be designed to maximize the transmission rates of the SUs.

In this paper, our main objective is to provide a systematic overview on IA in CR communications and networking. We review the key system models/paradigms involved in the implementation of IA in CR networks and explain how these are crossly related. In particular, for each paradigm, we review various strategies for calculating alignment results and wide-ranging interference management techniques. It should be noted that these two paradigms are merely a representation of the direction of research on this subject because majority of the work done fall into either of the two. However, there are quite a number of other equally significant methods of implementing IA in CR that do not fall under the scope of either paradigm. Thus, this overview will include discussions on some of these other research endeavours, some practical implementations of this network model as well as future research challenges that remain in accomplishing IA in practical CR networks.

The remainder of this paper is organized as follows: Section 2 gives a brief introduction of IA principles, techniques and applications. Section 3 focuses on a CR network having a number of SU pairs and a single PU-Tx and PU-Rx pair, where the PU-Tx is assumed far from its Rx and the SU-Rx's are considered not to be influenced by the interference from the PU-Tx. Section 4 considers the same CR network as the first paradigm in Section 3, but the SUs are in closer proximity to the PU-Tx in the manner of indirect spectrum sensing. Comparison of research literature and analysis, computational complexity of different methods, observations and recommendations are presented in Section 5. Section 6 presents open research challenges and lastly, Section 7 provides conclusions.

Notation

The definitions of the acronyms that will be frequently used in this paper are summarized in Table 1 for ease of reference.

Table 1. Summary of Important Acronyms.

Acronym	Definition	Acronym	Definition
AS	antenna selection	OFDM	orthogonal frequency-division multiplexing
BER	bit error rate	OIA	opportunistic interference alignment
BS	base stations	OSO	opportunistic spatial orthogonalisation
CP	cyclic-prefix	OTD	optimal transceiver design
CR	cognitive radios	PA	power allocation
CSI	channel state information	PU	primary users
CSS	cooperative spectrum sensing	Rx	receiver
DoF	degrees of freedom	SA	space-alignment
DOIA	distributed OIA	SD	spatial directions
DPC	dirty paper coding	SIA	selective IA
DSA	dynamic spectrum access	SIC	successive IC
EE	energy efficiency	SIMO	single-input multiple-output
FAP	femtocell access points	SINR	signal-to-interference-plus-noise ratio
FBS	femtocell base stations	SNR	signal-to-noise ratio
FC	fusion centre	SR	spatial reuse
FSA	fixed spectrum access	SS	spectrum sensing
GA	grouping algorithm	ST-WF	space–time water-filling
GLRT	generalized likelihood ratio test	SU	secondary users
IA	interference alignment	SVD	singular value decomposition
IC	interference channel	SWF	spatial water-filling
LIF	leakage of interference	TBF	threshold-based beamforming
MBS	macrocell base station	TDD	time division duplex
MEB	maximum eigenmode beamforming	TDMA	time-division multiple-access
MIMO	multiple-input multiple-output	TMA	transmission-mode adaptation
MU	macrocell	TO	transmit opportunities
MUE	macrocell users equipment	Tx	transmitter

2. Interference Alignment

2.1. Principles

IA is a precoding transmission technique for interference channels and networks that linearly encodes signals over multiple signaling dimensions e.g., time slots, frequency blocks or multiple antennas. By coding over multiple signaling dimensions, transmissions are designed to align the interfering signals observed at individual receivers into a low dimensional subspace. Consequently, IA maximizes multiplexing gain, i.e., the amount of non-interfering symbols that can be concurrently transmitted over the interference channel [24]. Realising the channel's maximum multiplexing gain, otherwise known as DoF, indicates that the sum rates provided by IA can approach sum capacity at high SNR. Initial research on IA indicated that a system's ability to find such IA precoders is directly related to the number of signal dimensions it can encode [20]. The more multiple signaling dimensions that are accessible for precoding, the more flexibility a communication system has in aligning interference [14].

To illustrate the general IA fundamentals, we consider the system model in Figure 1 where real-valued signals are coded over three dimensions and transmitted over real-valued channels. Individual receivers observe all three interference signals, each of which are denoted as a vector in the real three dimensions space. All three signal dimensions at the receiver are occupied by the three interference signals and the signals do not accidently align. IA users are permitted to cooperatively precode their communications such that individual three interference signals are completely contained in a two dimensional space [24]. IA consequently leaves one dimension to enable users decode symbols free from interference by projecting the received signal onto the subspace orthogonal to the interference subspace.

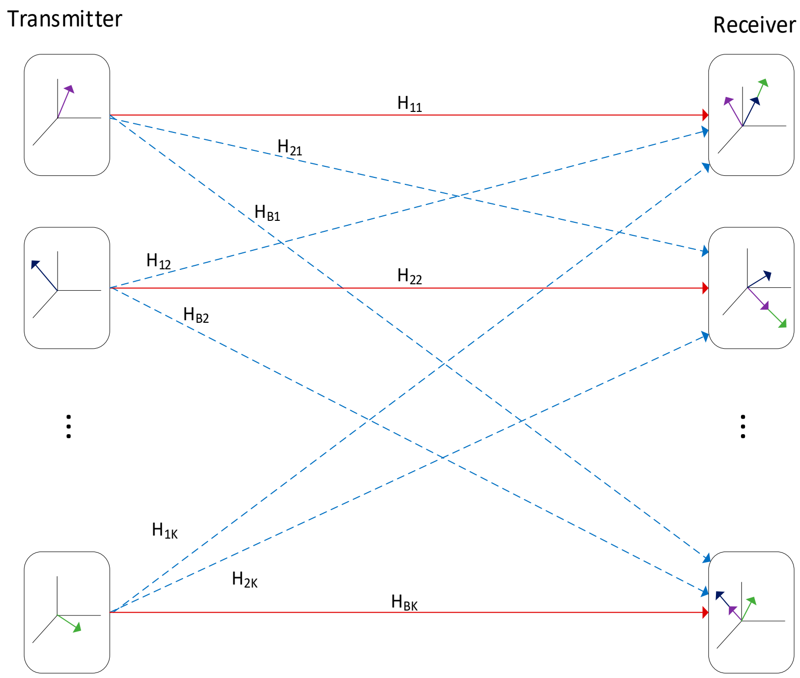

Figure 1. Illustration of the interference alignment (IA) principle.

While there are many IA schemes that take much more sophisticated forms, the origins of the idea lie in elementary linear algebra. The illustration in Figure 1 deciphers into an equally intuitive mathematical representation. Considering a system of linear equations:

$$\begin{aligned} y_1 &= H_{11}x_1 + H_{12}x_2 + \ldots + H_{1K}x_K, \\ y_2 &= H_{21}x_1 + H_{22}x_2 + \ldots + H_{2K}x_K\ , \\ &\vdots \\ y_B &= H_{B1}x_1 + H_{B2}x_2 + \ldots + H_{BK}x_K, \end{aligned} \tag{1}$$

where there are B observations $y_1, y_2, \ldots, y_B$, each in the form of a linear combination of K information symbols $x_1, x_2, \ldots, x_K$ with coefficients H_{ij}. K is the number of transmitters, each trying to send an information symbol, for a total of K independent information symbols $x_1, x_2, \ldots, x_K$. The coefficients H_{ij} represents the effective channel coefficients. B is the number of signaling dimensions accessible at a receiver through a linear channel. Each signaling dimension produces a linear combination of the transmitted information symbols due to the channel being linear. Consequently, a receiver has access to B signaling dimensions.

Assuming effective channel coefficients are taken from a continuous distribution, all the information symbols can be recovered, provided that there are at least as many observations as unknowns. Thus, if the receiver desires all symbols, at least K signaling dimensions at required. In IA networks just a subset of the symbols are sought after by the receiver and the remaining symbols carrying information for other receivers are undesired at the receiver, they solitary underwrite interference. For instance, suppose the receiver is only interested in symbol x_2, which carries information desired by this receiver while all other symbols carry only interference. Overall, K signaling dimensions will be required to determine the one symbol wanted by this receiver. Assuming there are K receivers, each interested in a different symbol, and individually they have access to a different set of K linear equations dictated by its linear channel to the transmitters, each receiver will be able to solve the system of equations and recover its desired symbol. Therefore, a total of K signaling dimensions are used so that each receiver is able to resolve its desired one dimensional signal.

2.2. General IA Techniques

Current IA techniques along with signal dimensions, and corresponding references are listed in Table 2. General IA techniques are concisely prsented in the subsequent paragraphs.

Linear IA is a fundamental technique of IA where the alignment of signal spaces is performed based on linear precoding schemes. Distributed IA is centred on the local channel knowledge rather than global channel knowledge. In subspace IA, the interferences are aligned to multidimensional subspace as opposed to a single dimension. Blind IA is a method that majority of the IA results are based on the assumption of perfect, and occasionally, global channel state information (CSI). In ergodic IA the CSI is partitioned into complimentary pairings for a wide class of channel distributions over which the interference can be aligned so that individual users are able to achieve marginally over half of its interference-free ergodic capacity at any SNR [25]. Retrospective IA techniques refer to the IA schemes that make use of delayed CSI knowledge at the transmitter. Lattice alignment adopts lattice codes in an IA network with the lattices structured in a way in which the interfering signals at an interfered receiver arrive on identical lattice, and the wanted signal stands separately [26]. IA and cancelation (IAC) is a technique developed when neither IA nor cancelation applies alone.

Opportunistic IA (OIA) is used to represent the IA-based opportunistic scheduling methods. The IA-based opportunistic scheduling methods aim to improve the performance of wireless communication networks by exploiting the channel fluctuations achieved through applying user selection or antenna selection [27]. Asymptotic IA is an opportunistic technique that exploits the presence of complementary channel states in equal magnitudes to achieve linear IA [28].

Table 2. List of general IA techniques.

References	IA Techniques	CSI	Signal Dimensions (Space, Time, Frequency, Time-Frequency)
[29–32]	Linear IA	Perfect/delayed	Single/Multi
[33–36]	Distributed IA	Local	Single
[37–40]	Subspace IA	Perfect	Multi
[29,41–43]	Blind IA	Absent	Single
[25,44,45]	Ergodic IA	Perfect/delayed	Single
[36,46,47]	Retrospective IA	Delayed	Single
[26,39,43]	Lattice Alignment	Perfect	Single
[40,48–51]	IA and Cancelation	Perfect	Single/Multi
[27,52,53]	Opportunistic IA	Perfect	Single/Multi
[28,54,55]	Asymptotic IA	Perfect	Single

2.3. Applications

IA is a common element of most multiuser communication network capacity problems. Indeed, IA comprises of wide-ranging techniques to manage interference among users, and it can be implemented in various multiuser communications networks to perform interference management. Owing to its capable performance, IA has been applied to CR networks, heterogeneous networks, ad hoc networks, 5G cellular wireless networks, satellite networks, underwater networks, etc., as shown in Table 3. With the advance and growth of wireless communication networks, innovative classes of IA techniques will certainly materialise to solve the interference problems in the advanced wireless systems.

In this paper, the focus will be on IA in CR networks. Our main objective is to provide a systematic overview on IA in CR communications and networking. We review the key system models/paradigms involved in the implementation of IA in CR networks and explain how these are crossly related.

Table 3. Applications of IA.

References	Applications
[56–60]	Cognitive Radio Networks
[30,41,47,61]	K User Interference Channel
[27,30,31,46,47]	K User $M \times N$ MIMO Interference Channels
[62,63]	5G Cellular Wireless Networks
[64–66]	Cooperative Interference Networks
[66,67]	Multihop Interference Networks
[68,69]	Ad hoc Networks
[70,71]	Physical Layer Security
[72,73]	Satellite Networks
[74,75]	D2D Networks
[50,76]	IoT Networks
[34,35,38,51]	Heterogeneous Networks

3. Interference Alignment in Cognitive Radio: First Paradigm

The first paradigm that considers a CR network having a number of SU pairs and a single PU-Tx and PU-Rx pair, where the PU-Tx is assumed far from its Rx and the SU-Rxs are considered not to be influenced by the interference from the PU-Tx was presented in this section.

3.1. System Model For Paradigm 1

Consider a K-user MIMO interference channel with a single PU link and K SUs as shown in Figure 2. Each Tx/Rx pair are fitted out with N and M antennas, respectively. It is assumed that the interference from the PU-Tx does not affect the SUs receivers. From the point of view of IA, a d_i dimensional transmit signal vector $d_i \in \mathbb{C}^{d_i \times 1}$ obtained at receiver j is given as:

$$y_i = H_{ii} V_i x_i + \sum_{l=1, l \neq k}^{K} H_{ij} V_j x_j + z_i, \; i = 1, \ldots .K, \tag{2}$$

where $H_{ij} \in \mathbb{C}^{M \times N}$ denotes the channel between jth Tx to ith Rx, $V_i \in \mathbb{C}^{N \times d_i}$ is the precoding matrix with columns comprising of d_i suitably selected linearly independent beamforming vectors and $z_i \in \mathbb{C}^{M \times 1}$ denotes the receiver thermal noise, modelled as complex additive white Gaussian noise vector, i.e., $z_i \sim \mathcal{CN}(0, \mathrm{I})$. The PU transmission is a point-to-point communication without considering the secondary links. It is also assumed that the primary channel matrix H_{pp} is perfectly known at PU-Tx and PU-Rx.

In designing IA in single-tier networks (without PU links) over MIMO block fading channel, the precoding matrices $\{V_i\}_{i=1}^{K}$ $k = 1$ and interference receiving matrices $\{U_i\}_{i=1}^{K}$ satisfy the following conditions [13]:

$$\begin{aligned} U_i^H H_{ij} V_j = 0, \;\; \forall_j \neq i \\ rank\left(U_i^H H_{ii} V_i\right) = d_i, \; \forall i \in \{1, 2, \ldots .., K\}. \end{aligned} \tag{3}$$

Condition (3) make certain no interference arises from other SU links at the output of the kth SU-Rx, and guarantees that the wanted signal space at the kth receiver achieves dk DoF when H_{ii} is full rank. The wanted message for kth SU can be decoded by projecting onto the orthogonal complement of U_k and zero forcing the interference. Notwithstanding, it is an ongoing open problem for general interference channels to determine closed-form precoding matrices and received interference subspaces. For three-user interference channel, the closed-form solution of V_i for any d_i has been found in refs. [7,12]. For the two-tier CR network, the goal of IA is to choose precoder matrices $\{V_i\}_{k=1}^{K}$ and interference receiving matrices $\{U_i\}_{k=1}^{K}$ such that each SU-Rx can decode its own signal by forcing interfering SUs to share a reduced-dimensional subspace while keeping an allowable interference level to the PU within the specified limit. Firstly, transmitter i adjusts V_i to make sure the most of its induced interference at other receiver falls into the subspaces $\{U_i\}_{k=1}^{K}$, and keep the interference to the PU below a pre specified level. Secondly, each SU-Rx chooses a subspace U_i to guarantee most of interference falls into the interference subspace when transmit precoding V_i is fixed. This system model serves as the foundation on which various IA algorithms have been applied towards implementing the solutions under this paradigm, as will be discussed in the preceding section of this overview.

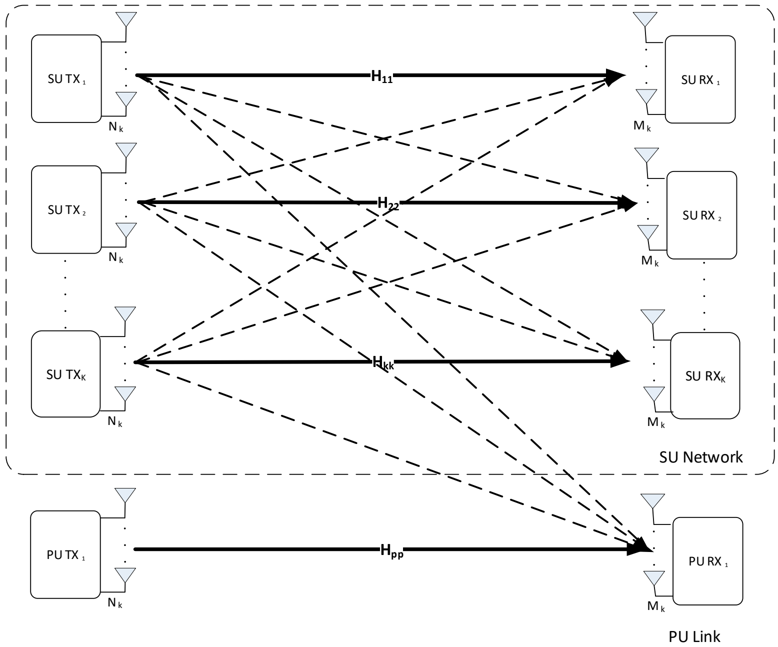

Figure 2. Multiuser cognitive radio (CR) network model consisting of one primary user (PU) link and multiple secondary users (SUs)—Paradigm 1.

3.2. *Techniques For IA in CR Networks*

In this subsection, state of the art IA in CR networks techniques, which fall under paradigm 1 were outlined in the following paragraphs.

3.2.1. Propagation Delay IA

IA can utilize the propagation delay difference among links to achieve theoretical DoFs. The research done in ref. [21] helped to further buttress IA as a general concept by establishing its applications in beamforming solutions and propagation delay based alignment [13,77]. The work in ref. [77] showed that IA could be achieved in a scenario where desired links constitute even delays and interfering links constitute odd delays. As such, IA is achieved over even time slots at each Rx. Desired signals can then be received without any interference. It was later proved in refs. [22,77] that even with random user positions, each user could still get half of the cake when propagation delay alignment is used on condition that symbol durations are made small enough or bandwidth made arbitrarily large.

This technique has been adopted in ref. [78] for two-tier CR networks, where a less opportunistic and more deterministic scheme for the SUs to successfully coexist with the PUs has been defined. This work makes use of an interference draining scheme, which is a specific extension of IA in CR networks. Assuming that IA has been computed on the SU network as expressed as follows:

$$y_i = H_{ii}(.)V_{i(.)}x_i + \sum_{j=1,j\neq i}^{K} H_{ij}(.)V_j(.)x_j + z_i(.), \tag{4}$$

where $(.)$ indicates the dimension, which could be frequency (f), time (t) or both. The received signals was re-written as:

$$y(.) = H_{sig}(.)x(.) + H_{int}(.)x(.) + z(.), \tag{5}$$

where $H_{sig} = diag\{H_{11}, \ldots\ldots H_{KK}\}$ is related to the desired signal space and $H_{11} = H - H_{sig}$ is related to the interference space. Now assuming that the SUs network is aligned internally, then from Equation (4), the desired signal space and interference space can be defined as $span[H_{sig,j}(.)V_j(.)]$ and $span[H_{int,j}(.)V_j(.)]$ respectively. The overlapping SUs interference is then projected to the PU-Rx to maintain a certain threshold and in the given circumstances, $span[H_{ii}(.)V_i(.)]$ is the PU signal space and $span[H_{ij}(.)V_j(.)]$ is the SU interference space. The draining scheme is achieved on a similar condition to the work done in ref. [22], that the interference from the SUs is at different time slots to the PU or spread evenly to every slot at the PU. Differing circumstances can lead to the availability of unused time slots, but this work crucially offers an idle time slot to accept the SUs interference. Therefore, assuming CR network aligns its own internal interference, this work adds another dimension to the SUs other than just matching the PU's transmission space, thus making it more feasible to achieve higher DoF.

3.2.2. Leakage of Interference Signals/Distributed Algorithm IA

Distributed IA is centred on the local channel knowledge instead of global channel knowledge. The work done in ref. [78] assumes that IA among the SUs has already been done and thus only focuses on subspace selection. While this will simplify computational complexities, it could also place a limit on finding novel solutions. As a matter of fact, the "half-the-cake" solutions [20,21] are actually sub-optimal because of the following reasons. Firstly, they present closed form expressions, which require global CSI, a requirement that can prove overwhelming in practice. Secondly, the feasibility of IA over limited number of dimensions is still very much an open problem [79]. Lastly, the more favourable performance of IA in the high SNR regimes is also not very feasible in practice.

The work done in ref. [79] proposes distributed cognitive algorithms that utilizes only local CSI, minimizes each transmitters interference to unintended receivers and utilizes the principle of reciprocity [79]. Typically, some interference power remains in the received signal after applying receiver interference suppression matrices, otherwise known as leakage of interference (LIF). The distributed IA algorithm achieves IA by progressively reducing this leakage interference at each receiver. The total leakage interference due to undesired transmission at each Rx is given as:

$$I_i = Tr\Big[U_i^H Q_{ij} U_j\Big], \tag{6}$$

where $Q_i = \sum_{j=1, j\neq k}^{d^{[i]}} \frac{p_j}{d_j} H_{ij} V_j {V_j}^{H\dagger} {H_{ij}}^{H\dagger}$ is the interference covariance matrix at receiver i. For the reciprocal channel, if $\overleftarrow{P}^{[k]} > 0$ is the power constraint at a SU, then the total leakage interference can also be defined as:

$$\overleftarrow{I}_i = Tr\Big[\overleftarrow{U}_i^{H} \overleftarrow{Q}_{ij} \overleftarrow{U}_j\Big], \tag{7}$$

where $\overleftarrow{Q}_i = \sum_{j=1, j\neq k}^{d^{[i]}} \frac{p_j}{d_j} \overleftarrow{H}_{ij} \overleftarrow{V}_j {\overleftarrow{V}_j}^{H\dagger} {\overleftarrow{H}_{ij}}^{H\dagger}$ is the interference covariance matrix at receiver j.

The algorithm is modelled to alternate between both the original and reciprocal network and only the Rx's update their interference suppression filters to minimize leakage interference. The significance of this solution is about two things: Firstly, each SU-Tx is only required to learn the effective channel of their desired SU-Rx. This lessens the burden of global CSI of all available matrices. Secondly, these algorithms can be used for further analytical study of IA.

For a two-tier CR network, the distributed IA algorithms can be applied with some enhanced algorithms, the goal being to concurrently reduce the leakage of interference signals while preserving interference to the PU-Rx below a satisfactory measure.

Such an enhanced algorithm has been aptly described in the work in ref. [80], which introduces the concept of matrix distance so that at receiver i, the distance between the subspace spanned by the interference signals $H_{ij}V_j$, $i \neq 1$ and its interference receive subspace spanned by U_i is kept as close as possible. Unlike the work done in ref. [78], this work has no constraints on the number of users. Thus by assuming perfect local CSI, the distributed algorithm iteratively solves the optimization problem. Each SU-Tx updates its precoding matrix to minimize the total interference leakage from interference subspace to signal subspace as well as guarantee its interference to the PU below a certain level. The matrix distance, used as a measurement metric, can be defined as the distance between two orthonormal matrices A and B, such that the following expression holds $\left\|A - BB^H A\right\|_F$ [80].

The work in ref. [56] is shown to improve sum rates of the CR network and employs a similar enhancement to ref. [80] to protect the PUs. The distance between the subspace of the received signal from each SU-Tx at the PUs and the set of interference subspaces at the PU is minimized. However, this particular solution consists of multiple PUs and presents an argument for when individual SUs attempt accessing the spectrum one after the other. Assuming local CSI is present at the PUs and global CSI is available at the SU, an optimization problem was defined so as to cause non or minimal interference to the PUs.

The IA schemes discussed so far all assume perfect local CSI. In practice however, it is inevitable that the CSI is corrupted by estimation errors, which will likely lead to diminished the system performance [13,81]. Equally potentially impeding to practical implementation of IA is finite SNR [81], because while the performance of IA at high SNR is adequate, it is far from optimal at moderate to low SNR [82]. It is based on these two drawbacks that the work in ref. [82] described a technique to counter CSI errors as well as providing the motivation for the work in ref. [83], where a more vigorous IA strategy for CR network comprising of CSI ambiguity is presented. It also proposes adopting a weighted optimization method related with the leakage interference to increase the system performance at low and moderate SNR [84]. Recalling the condition of distributed IA algorithms, ref. [83] proposes a robust joint signal IA design that transforms the transmit optimization and receive subspace selection limitation into a limited number of linear matrix disparities that are both optimal and solvable by interior point methods [85]. The proposed design of ref. [83] decreases the leakage of interference signals from the SU-Tx while preserving interference to the PU-Rx below a satisfactory level. The drawback of this solution however, lies in its intricacy making it impossible to prove that the iterations diverge to the universal ideal [23].

Solutions discussed so far have made several assumptions on network resources. This makes it necessary to consider allocation, utilization and performance of these resources in such a way that they do not negate ultimate goal of improving network performance. The work in ref. [86] presents distributed resource allocation algorithms to maximize the sum-rate of any-to-any links under per-node transmit-power and CR interference constraints. The first algorithm is developed when the CR interference constraints are absent. The issue thus amounts to weighed sum-rate maximization for MIMO ad hoc networks with per-node power constraints when other user interferences are treated as noise. The CR interference constraints are then added for the second algorithm, which incorporates a decomposition technique that ensures the constraints are enforced even during the iterative procedure. Thirdly, an alternate centralized algorithm was then developed based on network duality so that existing network duality results were extended to the ad hoc network configuration under the CR interference constraints. Therefore, by viewing this optimization as a multi-beam beamforming problem without loss of optimality, these novel algorithms iteratively updates receive beamformers, transmit-powers and transmit-beamformers in cycle to attain an optimal solution. Similarly, the work in ref. [87] presented a performance analysis of MIMO IA with user selection, where a transmitter sends the data stream of the selected user providing the maximum effective SNR, in order to derive the effective SNR of combining IA with minimizing LIF. By deriving exact closed-form expressions of the bit error rate (BER) and outage probability of the small-cell system, this resource optimization procedure work shows a marked gain in performance compared with conventional iterative solutions [79,88].

3.2.3. Symbol Extensions IA

The idea of using symbol extensions was introduced in refs. [20,21] for the two user MIMO X channel where all nodes equipped with $M > 1$ antennas achieved enlarged DoF by using linear beamforming across multiple channel uses [89]. Single-antenna networks are known to have unlimited dimensions as the channel symbol could be extended as long as possible. This provides drastic convenience for the design of IA as well as showing enhanced sum rates for both PUs and SUs [90,91]. However, the typical setup of wireless networks is the MIMO scenario that is based on setting the signal space over limited dimensions. Even though research endeavours based on symbol extensions such as refs. [92,93] show higher performance gains, the extent to which interference can be aligned over a limited number of dimensions still remains an open problem [23,80].

The work on finite symbol extensions have been extended to cellular systems in ref. [93], which makes use of a 2D space time spreading code system model containing linear coding design. The achievable sum DoF is formulated by firstly minimizing the dimension of the interference subspace as a rank minimization problem and then subsequently solving the problem by using a grouping algorithm (GA), which aligns interfering message streams into a low-rank inter-cell interference subspace as a group with a packing ratio as extensive as possible. Alignment rules for obtaining a feasible solution to the rank minimization problem are meticulously formulated to achieve sum DoF that is significantly better than prevailing results [94] and attains the upper bounds obtained in refs. [95,96], thus taking full advantage of the limited number of dimensions provided by space and time.

The work done in ref. [97] differs from the refs. [92,93] as it considers not only multiple SUs in the CR network, but multiple PUs as well that are cooperatively utilising IA. As is usually the case, the SUs are trying to gain access to the licensed spectrum, without degrading performance of the PU network. The unique condition that avoids degrading the sum rate of the PUs is a minimal-impact threshold for the quantity of SU-Tx antennas, that is set in such a way that those SU-Txs with greater or equal antennas than this threshold can utilize the licensed spectrum. When the SUs satisfy the zero-impact threshold, the optimum and suboptimum SU precoders are calculated, such that these precoders will not degrade the sum rate of the PU's. A specific successive IA precoding is also proposed and presented to be optimum for several network setups determined by the number of PUs, SUs and antennas at each node. Thus this piece of work makes a case for enough antennas at the SUs as well as providing new analysis for the case of inadequate antennas at the SUs.

3.2.4. OFDMA IA

Turning attention to classical OFDMA transmission due to its popularity and simple signal model, the self-sustainability of block transmission systems is analysed in ref. [98]. In this work, the cyclic prefix that is usually discarded at the Rx of legacy OFDMA systems is used to scavenge useful energy as a novel energy-harvesting Rx, thus enabling the received signal to carry both energy and data. Most importantly though is the fact that an enhanced Tx strategy where the Tx signal consists of an OFDMA signal and a cognitive IA signal, is also proposed to increase flexibility and generality of the whole scheme. Particularly interesting directions of this work is the practical design and implementation of an energy harvesting OFDMA receiver. The allocation and utilization of resources has also been analysed for OFDMA systems in ref. [98] based on IA in order to increase the spectral efficiency of CR systems without having a detrimental effect on the quality of service (QoS) of the primary system. Based on a frequency-clustering algorithm for multiple antenna scenarios IA plays a role in the proposed algorithm, which considers the channel qualities, power budget limitations as well as the induced interference to the PU band. The work in ref. [99] achieves a significant sum rate gain in comparison with the work in ref. [100], which leads to a considerable increase in the spectral efficiency.

3.2.5. Other Endeavours

There are also other research endeavours that do not fall within the confines of the more general IA techniques, but are equally significant and obviously worth mentioning. One of such endeavours is the work done in ref. [101], which researches a unique underlay MIMO CR system, in which the geometric or immediate CSI to the PU-Rx of the interfering channels is totally unknown to the CR. This paper initially shows that minimal ranked CR interference is desirable for increasing the efficiency of the PUs in comparison to spreading a lesser amount of power over additional transmit signal dimensions, followed by a water-filling solution (most of which will be discussed in the second IA in CR paradigm) that uses a negligible sum of power to attain the rate limitation with a low-rank transfer of covariance matrix. A combination of a rank minimization, CR transmission strategy as well as an analysis of the interference temperature at the PU-Rx is presented, the results showing higher PU sum rates with a trade-off of higher interference temperature compared with the likes of say refs. [101,102]. Of more significance is the work done in ref. [103], which successfully applies IA in a practical network setting, leading to a dramatic reduction in message transmission times. By using a vehicular traffic scenario, this work successfully analyzes optimal times to undertake local spectrum sensing and how to ensure correct packet receptions among the multiple base stations (BS) and CR vehicles.

Inadvertently, the sum rates of the SUs in a CR networks fall short particularly in low SNR conditions. The work in ref. [104] in order to further improve its performance considered power distribution in IA-based CR networks. While each SU is trying to decode its own signal by choosing appropriate transmit precoder and interference receiving matrices, three separate PA algorithms are proposed to also increase the energy efficiency, throughput of the SUs, as well as the requirements of SUs of the system, while maintaining the goal of maintaining an adequate interference level to the PU within the specified limit. As for the PU, a transmission-mode adaptation (TMA) scheme is proposed to further improve its performance to cater for low SNR conditions. The combined PA and TMA algorithms show the effectiveness of these algorithms for IA-based CR networks. Another performance metric that usually falls short of theoretical expectation is the signal-to-interference-plus-noise ratio (SINR) of the desired signal. To improve this situation, the work in ref. [105] proposed an IA scheme based on antenna selection (AS). In the proposed scheme, multiple antennas at each SU-Rx are selected to achieve optimal performance. Furthermore a CSI filtering scheme is proposed to counter the effects of imperfect CSI. Given the huge computational complexity of the IA–AS scheme, an optimization algorithm is introduced that can simultaneously converge with the precoding matrices to improve performance.

3.3. *Applications*

Given the volume of work that has gone into this research question, it is expected that it will find its applications in practical network scenarios. One of such applications is in Femtocell deployments. It is known that femtocells may coexist with macrocells in the same band and significantly improve the spatial reuse (SR) of spectrum. As such, the work in ref. [106] tries to avoid high interference to the scheduled macrocell users (which in this case are the PUs) from the femtocells acting as the SUs. The femtocells, just like typical SUs will sense and access the channel opportunistically. Using massive MIMO [107] to form guard zones, this work proposes spatially aligning the guard zones so that more SR opportunities can be provided to the femtocells. By proposing a scheme for user scheduling, the guard zones are aligned by taking into consideration the geographic locations for massive MIMO use scheduling as well as the interfering femtocell information. Simulations results show significant improvement for femtocell throughput without degrading the macrocell performance.

Another application of this paradigm can also be found in refs. [57,108,109], which proposes to employ IA for aligning the signals from the macrocell (MU)/SU that tend to increase interference at multiple femtocell base stations (FBS)/PU which serves as an adequate interference management technique from base station to user equipment. The proposed solution includes a selective IA (SIA) algorithm that comprises of a cautious selection of the macrocell interferers to align at several FBSs, and identify the subspace where cross-tier interference signals can be aligned. Subsequently, a distributed technique used for identification of the precoders required at the selected interferers is employed. The interference from FUs which is the intra-tier interference, is then overseen by making use of MMSE interference suppression [18,110].

4. Interference Alignment in Cognitive Radio: Second Paradigm

The second paradigm that is the same CR network as the first paradigm, but the SUs are in closer proximity to the PU-Tx was considered in this section.

4.1. System Model for Paradigm II

The system model for this paradigm is a MIMO CR network that consists of a single PU link (PU-Tx and PU-Rx) and k SUs (SU_1, ..., SU_k) as shown in Figure 3. Every user is assumed to have M transmit and N receive antennas. The PU link is a point-to-point MIMO link, while the SU network can be a point-to-point or multi-user MIMO network. The following assumptions are made for the purpose of the system model as follows:

- The PU and SUs operate in the same frequency band and all channels are Rayleigh flat-fading.
- The PU link is a single user MIMO channel, which is represented as a $N_{pu} \times M_{pu}$ matrix, H_{pu} with channel gains h_{ij}.
- CSI is perfectly known at both the transmitter and receiver, thus H_{pu} is also perfectly known.

The capacity of this channel can thus be defined as:

$$C = \max_{Q:Tr(Q)=P} log\left|I_N + H_{pu} Q H_{pu}^H\right|, \tag{8}$$

where Q is the $M \times M$ input covariance matrix. The SU setup is assumed to be either a single user MIMO SU link or a multi-user MIMO channel. While the PU has reserved rights to the spectrum, the SU transmitters sense vital information about the PU to avoid causing interference at both the PU-Rx and other SUs. Each transmitter therefore transmits a sequence of Gaussian encoded symbols to its corresponding receiver by processing its symbols using a $M_i \times d_i$ precoding matrix V_i to form the transmitted signal vector $V_i x_i$. The assumption is that the SUs possess perfect knowledge of the entire channel transfer matrices. Although unrealistic, this condition provides an upper bound on the achievable rate of each SU. The IA condition states that the primary and secondary received signals are represented by [24]

$$y_i = \mathrm{H}_{ii} V_i x_i + \sum_{j=1}^{K} \mathrm{H}_{ij} V_j x_j + z_i, \tag{9}$$

where y_i denotes the $N_i \times 1$ received signal vector at the j^{th} receiver; z_i denotes the $N_i \times 1$ zero mean unit variance circularly symmetric AWGN noise vector at the j^{th} receiver; x_i denotes the $M_i \times 1$ signal vector transmitted from the i^{th} transmitter; H_{ij} is the $N_i \times M_i$ matrix of the channel coefficients between the i^{th} transmitter and the j^{th} receiver; also, $P_i = E\left[x_i\, x_i^H\right]$, where P_i is the transmit power of the i^{th} transmitter. It should be noted that i and j are used as a generalization denoting each transmitter and receiver pair.

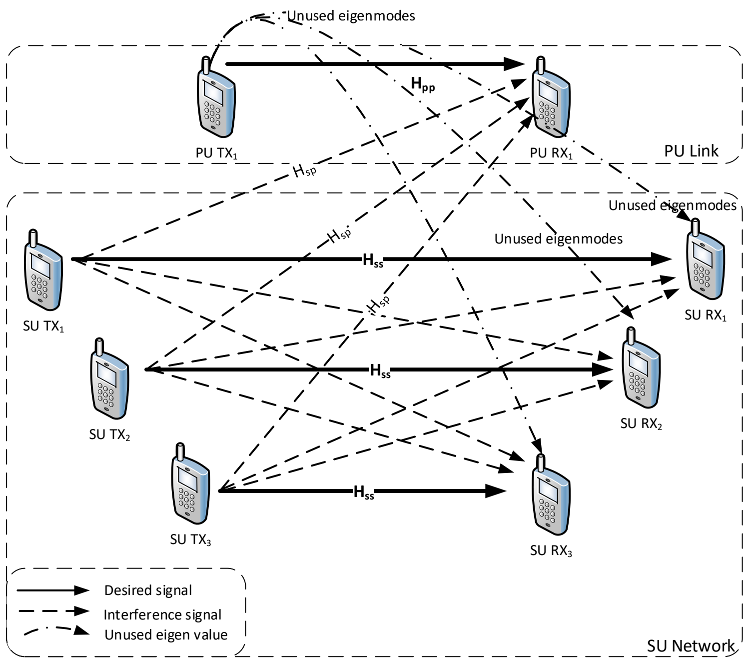

Figure 3. Multiuser CR network model consisting of one PU link and multiple Sus—Paradigm 2.

When CSI is known at the Tx and immediate variation is probable, the capacity gain distribution is evaluated by making use of the renowned water-filling technique where either the temporal or spatial domains have been presented to be optimal. There is also another optimization technique called maximum eigen beamforming that can be used in place of water-filling. This paradigm's operation is based on three main procedures that characterize the design of OIA.

Firstly, the PU maximizes its own rate on its MIMO channel singular values to leave unused spatial dimensions. Then, the SUs perform linear precoding, which aligns the SUs transmission with such unused spatial directions, thereby ensuring orthogonality between their transmissions and enables the SUs to share the PUs spectrum without causing any interference to the PU's transmission. Finally, the SU implements an optimization procedure to ensure useful data rates for their transmission. The next sections will therefore review each of the water-filling techniques as well as the various IA techniques that have been exploited under each of the water-filling solutions.

4.2. Water-Filling Techniques For IA in CR Networks

One of the attractive features of MIMO systems is spatial multiplexing gain and consequently a higher capacity performance over single-input single-output (SISO) system, achieved by the classical water-filling (WF) algorithms [111]. WF algorithms are known to provide capacity-achieving scenarios arising from MIMO systems taking advantage of the DoF offered by antennas to increase spectral efficiency as well as maximizing the mutual information between the input and the output of a channel composed of several sub-channels with the availability of global CSI at the Tx's [83].

Figure 4 describes how the classical WF works, where units of water per sub-carrier are filled into the vessel and μ is the height of the water surface. For some sub-carriers, the bottom of the vessel is above the water and no power is allocated to them, making them the unused spatial directions that could be utilized by the SU transmission. In these sub-carriers, the channel is too poor for it to be worthwhile to transmit on, thus allocates more power to the stronger sub-carriers.

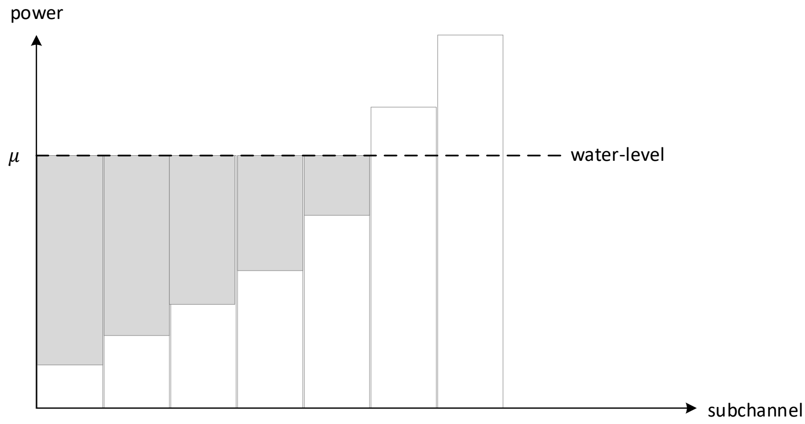

Figure 4. Classical water-filling power allocation.

4.2.1. Spatial Water Filling (SWF)

The work done in ref. [111] has shown that when CSI is available at the transmitter, the capacity achieving distribution can be found by using the well-known water-filling algorithm. Thus given the input–output relationship of a point to point MIMO system as

$$y = Hx + z \tag{10}$$

where y is the $N_r \times 1$ received symbol vector, H is the $N_r \times N_t$ MIMO channel matrix, x is the $N_t \times 1$ transmitted symbol vector, and z is the $N_r \times 1$ additive white Gaussian noise vector with variance $E\left[vv^{\dagger}\right] = \sigma^2 I$, where $(\cdot)^{\dagger}$ denotes the operation of matrix complex conjugate transpose [112].

The MIMO channel H is modeled as $H = \sqrt{s}H_w$, where H_w is an $N_r \times N_t$ Rayleigh fast fading MIMO channel whose entries are i.i.d. complex Gaussian random variables, and s is a scalar log-normal random variable, that is, $10\ \log_{10} s \sim \mathcal{N}\left(0,\ \rho^2\right)$, representing the shadowing effect [112]. The maximization problem using spatial water-filling for MIMO channels where CSI is known at the transmitter and power adaption is implemented with a total power limitation for individual channel realization can be represented as:

$$\begin{aligned} &\max_{Q} log\left|I + \tfrac{1}{\sigma^2} HQH^{\dagger}\right| \\ &subject\ to\ tr(Q) \le P, \end{aligned} \tag{11}$$

where H is the MIMO channel, Q is the autocorrelation matrix of the input vector x, defined as $Q = E\left[xx^{\dagger}\right]$, P is the instantaneous power limit, $|A|$ denotes the determinant of A and $tr(A)$ denotes the trace of matrix A.

The function $H^{\dagger}H$ can be diagonalized as $H^{\dagger}H = U^{\dagger}\Lambda U$, where U is a unitary matrix, $\Lambda = diag\{\lambda_1, \ldots, \lambda_M\}$, and $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_M \ge 0$. The optimisation in Equation (11) can be carried out over $\widetilde{Q} = UQU^{\dagger}$ and the capacity-achieving $\widetilde{Q}$ is a diagonal matrix [113]. Let $\widetilde{Q} = diag\{q_1,\ q_2,\ \ldots,\ q_M\}$, then the optimal value for q_i is:

$$q_i = \left(\overline{\Gamma}_0{}^{(\sigma^2, \mathrm{M})} - \frac{\sigma^2}{\lambda_i}\right)^{+}, \tag{12}$$

where σ^2 is the noise variance, a^{+} denotes $max\{0,\ a\}$ and $\overline{\Gamma}_0{}^{(\sigma^2, \mathrm{M})}$ is solved to satisfy $\sum_{i=1}^{M} q_i = P$, which then signifies zero transmission in this eigenmode [112].

Single User MIMO SU Link

The idea behind opportunistic IA (OIA) in a CR network was properly introduced with a single-user MIMO SU link [24] as shown in Figure 5. Therefore, with the assumption of perfect CSI at Tx and Rx ends, capacity is gained by implementing a water-filling PA scheme over the spatial directions related with the singular values of its channel transfer matrix. Significantly, when the PU Txs maximize their transmission rates, power constraints generally lead the Txs to retain some spatial directions (SD) unused. The unused SD can consequently be reused by an alternate system operating in the same frequency band. Certainly, an opportunistic Tx can transmit its individual data to its respective Rx by processing its signal in such a manner that the interference originating from the PU link impairs only the unused SDs.

Therefore, the SDs which are calculated analytically and presented to be adequately high, are beneficial for the SUs when the available spectral resources are fully exploited over a certain period in a certain geographical area. The process is as follows:

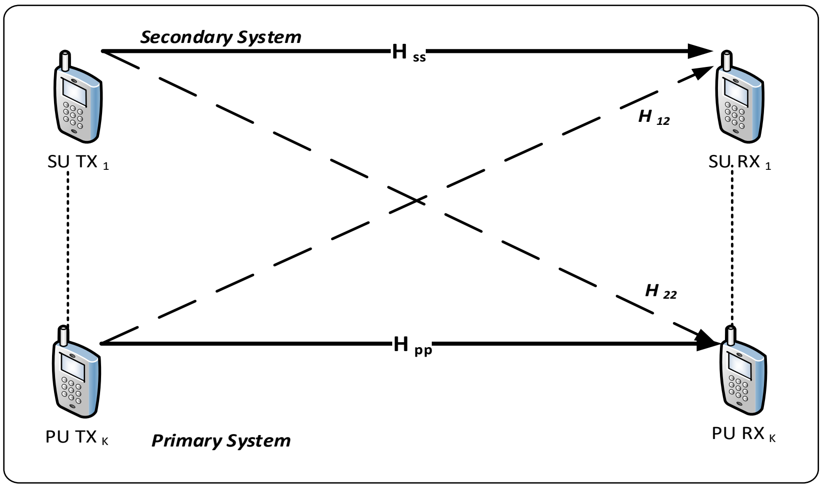

Figure 5. Single user multiple-input multiple-output (MIMO) link.

The PU-Tx follows a water-filling power allocation by choosing its pre-coding and post-processing matrices in such a way that their channel transfer matrix is diagonalised. The PU allocates it's transmit power over an equivalent channel, which consists of parallel sub-channels with non-zero singular values, referred to as the used transmit dimensions. If $m_1 \in \{1, \ldots.., M\}$ denotes the number of transmit dimensions used by PU, then the unused transmit directions can be computed.

There exist a total number of dimensions N described in the range as:

$$1 \leq m_1 \leq rank\left(H_{11}^{11} H_{11}\right) \leq N, \tag{13}$$

where H_{11} is the SVD of the $N_{pu} \times M_{pu}$ matrix. The PU equivalent channel can alternatively be denoted as the $N_1 \times M_1$ matrix whose main diagonal consists of m_1 non-zero entries and $N - m_1$ zero entries or $N_1 - m_1$ unused receive dimensions without any PU signal. It is these empty unused SDs (also known as transmit opportunities (TO)) that the SU will exploit to increase the DoF on the reporting channels. The opportunistic transmitter has to avoid interfering with the m_1 dimensions used by the PU Tx, which is achieved by aligning the transmission from the SU using linear precoding, with the $N_1 - m_1$ unused receive dimensions of the PU link. This can be described as more or less being equivalent to the SU-Tx placing its transmitted signal in the null space of the PU-Rx by pre-multiplying its transmitted signal with a matrix of size $M_1 - m_1 \times M_1$, which is also equivalent to a Tx with $M_1 - m_1$ antennas and no PU. Thus, the opportunistic link is said to satisfy the IA condition of Equation (3), if the opportunistic SUs transmission rate is equal to the PUs transmission rate.

Another point of interest is the calculation of the input covariance matrix P_{CM} that will maximize the transmission rates for the opportunistic SUs transmission link. Once P_{CM} has been solved; the optimal power allocation can be defined as a function of the power constraint $\overline{P}$ i.e., $P_{PA} = diag(\overline{p_1}, \overline{p_2}, \ldots\ldots, \overline{p_i})$. The work in ref. [24] used P_{PA} to determine how the opportunistic SU spreads its total power among the identified TOs, allowing the opportunistic SU to achieve higher transmission rates. Each transmit opportunity can also be seen as separate spatial signaling dimension such as those provided by time, frequency, antennas, etc. in order to achieve higher DoF on the MIMO Channel [93]. The work done in ref. [114] was an earlier attempt based on this model, where the PU leaves some of it eigenmodes unused to enable cognitive transmission. In this work, the unused eigenmodes are not only used for aligning the SU transmission to convey data as their transmission status is also used as a coding scheme to transmit cognitive codewords. Thus a jamming signal (non-information signal) that is known at the destination and can be easily removed is sent when the CR codeword indicates "transmission" for a spatial direction that is used for primary transmission. This ensures a minimum cognitive rate equal to the number of antennas and corresponds to a simple implementation. The outcome of the work in ref. [24] shows that a zero interference constraint to the PU-Rx has to be satisfied. This theoretically diminishes opportunities for some of the IA algorithms described in Section 2 under the first paradigm, such as interference cancellation (IC) and distributed IA, which are used to optimize IA in CR.

Given this fact, subsequent research such as the work done in ref. [115] on interference cancellation require unique circumstances for the solutions to be feasible. This particular work proposes the use of both interference cancellation and IA techniques to enhance the achievable degrees of freedom (DoF) for the MIMO CR network. In particular, an intermediate relay node is introduced in between the SU-Tx and Rx that is designed to cancel interference from both the SU Tx and the relay node, thus achieving maximum DoF. It is shown that with the appropriate number of antennas at the intermediate relay against the sum of DoF of the PU network and the maximum DoF available in the SU network without the presence of PU network, the maximum achievable DoF of the two-tier CR network with M transmit antennas and N receive antennas is $\min\{M, N\}$. When the PU network also uses SVD based encoding and decoding so that no interference signal is allowed to its signal space along with appropriate number of antennas at the relay node, the CR network achieves a higher DoF than when the two-tier CR operates without a relay node.

Similarly, the work in ref. [68] investigates both orthogonal and non-orthogonal transmission of the SU, with the aim of determining spectral efficiency gain of an uplink MIMO CR system. In this work, the SU is allowed to share the spectrum with the PU by using a unique space alignment technique along with an interference temperature threshold technique to ensure a non-zero SU rate. The proposed scheme adopts a successive IC (SIC) technique so that the SU is not limited to exploiting the unused TOs of the PUs transmission, but it is also allowed to exploit the used eigenmodes of the PU by respecting both total power and interference temperature constraints. Furthermore, this work analyses the SIC's operational inaccuracy as well as the CSI estimation imperfection on the SUs power allocation.

Further studies such as ref. [61], which ensures zero LIF is only feasible when the SU-Tx has at least the same number of antennas as the DoF of the PU system. As the success of the SU communication depends on the availability of unused TOs, this work focuses on two very specific contributions. The first is a fast coarse sensing method that detects unused TOs, based on the eigenvalues of the received signal covariance matrix. Secondly, a more accurate sensing method based on the generalized likelihood ratio test (GLRT) is applied after coarse sensing to fine tune detection of the unused TOs. The proposed solution provides a significant throughput while causing no interference to the PU-Rx, and that the sensing detects the spatial holes of the PU network with higher detection probability.

Multi User MIMO SU Link

A factor mutual with the above mentioned research is that their entire system models make use of a single-user MIMO SU link therefore disregarding the effect of multiple SUs on the performance of a CR network. Actually, a single SU is unlikely to reliably detect the presence of a PU due to factors such as multipath fading impairments, low SNRs and sensing time constraints [7]. The literature has thus introduced research on utilising and evaluating the IA technique in a CR network that contains multiple SUs (Figure 4).

The research work done in ref. [116], called opportunistic spatial orthogonalisation (OSO) is one of the earlier endeavours consisting of multiple SUs that permits both PU and the SUs to communicate simultaneously at the same frequency band and time slot. This particular work differentiates itself in two aspects: Firstly, the SUs do not require knowledge of the PU channel matrices. This leads to the second differentiator, which achieves the goal of forcing the signals from the SUs towards only a certain direction by utilising the randomness of the channel matrices and relying on multi-user diversity. Originally developed for single-input multiple-output (SIMO) scenarios, the OSO scheme has led to the interesting concept of OIA for the MIMO case, where it is shown that the ill-conditioned MIMO channel significantly increases the total throughput without much sacrifice of the PUs performance. The general concept behind OSO can be applied to a broad class of multi-user systems other than CR, and for some of them, the throughput gain will even be more prominent. Some of the algorithms such as symbol extensions, feasibility conditions of IA, LIF and IC, which are used to enhance IA in CR, have found reasonable opportunities to further exploit this multi-user scenario.

A space-alignment (SA) technique based on symbol extension with multiple PUs was proposed in ref. [92], which gives a much higher performance than the single PU scheme in refs. [20,24,117,118]. After spatial water filling (SWF) has been applied at the PU-Tx leaving some TOs unused, which the SU-Tx can exploit, this work avoids interference to the other PU-Rx's by demanding that the input power allocation matrix at the SU-Tx meets both the SUs total power constraints and a set of interference-power limitations at the other PU-Rx's. This work also describes a power allocation matrix at the SU-Tx that jointly maximises the transmission rate of the SU as well as ensuring the zero interference constraint at the PUs. The result is a higher transmission rate in the low SNR regime than ref. [24].

The multi-user scenario is extended further in the work done in refs. [119,120], which are both based on the feasibility conditions of IA. The schemes developed in ref. [119] extends the work done in ref. [24] by also designing the SUs precoding matrices in such a way that exploits the free spatial dimensions left by the PU. In other words, no interference is generated at the PU-Rx. For a multi-user MIMO SU network, this work first extends the results of ref. [24], which show that for the SU-Rx not to be affected by the PU-Tx, it would have to null the PUs transmission. This would be the same as removing a number of antennas from both the Tx and Rx of the SU. For multiple SUs, the IA problem is achieved by simply converting it into a single-tier cognitive IA problem. Secondly, an iterative algorithm that exploits channel reciprocity is derived, which only necessitates local CSI at each node. The process involves a closed-form solution for the precoding matrices at the Tx's with the same number of antennas at each node (to satisfy the feasibility condition) and constant channel coefficients. A regrouping of the cognitive IA conditions is done to facilitate its formulation into a mathematical intersection of subspaces.

Similarly, the work in ref. [120] also explores the feasibility condition of IA. This work essentially challenges the low achievable transmission rates of the SU network, especially in the higher SNR regimes. By allowing the PU-Rx to perform interference suppression and also checking that the number of variables and the number of equations match the feasibility condition of DoF allocation of IA [121], it is shown that the SUs achieve considerable performance gain in terms of their transmission rates.

Leakage Interference

Despite the condition that a zero interference constraint to the PU Rx has to be satisfied by the SU, the performance of IA in realistic propagation environments still causes LIF [122,123] i.e., the interference power that remains in the received signal after applying receiver interference suppression matrices. With this in mind, the work in ref. [122] proposes the subsequent two categories of OIA: Singular value decomposition (SVD)-based OIA and antenna selection-based OIA. The suggested OIA works with merely local CSI at the transmitter, without iterative processing, dimension extension, and inter-node or inter-cell synchronisation. For the proposed schemes for the SVD-based OIA, individually SU designs the weight vector that minimizes the LIF making use SVD-based beamforming. For the antenna selection-based OIA, the best transmit antenna is selected at each SU, which shows results in sum-rate gain as the number of antennas at each user grows. However, the user-node scaling state in relation to SNR does not necessarily change when the number of antennas remains constant. An improved version of minimizing LIF is the work done in ref. [123], which considers a number of MIMO SU links in the presence of MIMO PUs as well. This work proposes an iterative algorithm to find the precoders and reception filters for both the PU and SU networks unlike the work in ref. [122], after which the transmission of the PUs is maximized by SWF. The cognitive IA problem is then formulated by defining the required iterative conditions for this paradigm to mitigate the harmful effects of the SUs transmission on the PU. The rates of the SU links are then maximized by employing water-filling algorithm similar to the PUs. It is observed that the proposed iterative algorithm converges faster than earlier schemes, while its complexity is not much more than ref. [79].

Interference Cancellation

The IC technique was initially used in refs. [124,125] that zero forces the intra-cell interference in the primary cell with the aid of the cognitive base station. The interference caused by the primary base station on the SUs are also cancelled using dirty paper coding (DPC), which provides outer bounds on the achievable DoF as well as showing that the achieved sum DoF is strictly larger than the case when cognitive message sharing is unavailable.

A similar strategy was then used in ref. [125] for the case of downlink of cellular networks to achieve outer bounds on the DoF through IA. IA helps the PUs and SUs using SWF to enable efficient utilisation of the available DoF, by cancelling intra-cell interference within the PU link with pre-coding matrices and zero-forcing the intra-cell interference within the SUs. The achievable sum rates within the PUs and the SUs were then maximized by applying SWF to calculate the optimal transmit power matrices Q^p and the source covariance matrix Q^s for the PU and SU respectively.

This concept is made that much clearer by the work done in ref. [126], which adopts a SIC technique similar to ref. [68], but also allows the SUs to not only make use of the unused eigenmodes left by the PU, but to also make use of the non-free eigenmodes. The SUs do so by using SIC to remove the effect of the PU signal in order to decode the transmitted SU signal. Although this work consists of multiple SUs, unlike the work in ref. [68], each PU channel is shared by only one SU to avoid co-interference between the SUs. Thus, after maximizing the PUs transmission in order to release some eigenmodes, a space alignment algorithm is used to study the effect of multiple SUs on the cognitive MIMO-MAC sum rates. After applying perfect or imperfect SIC, this scheme selects the best SUs based on their channels and their corresponding closed-form sub-optimal power solutions to transmit on the used eigenmodes of the PU, thus achieving higher sum-rates. The numerical results for Rayleigh fading channel indicated that the low-complexity selection structure performs similarly to the previous optimal solutions.

4.2.2. Space–Time Water-Filling (ST-WF)

Another the capacity achieving water-filling technique is the ST-WF for MIMO channels [112]. The comparative analysis done in ref. [113] has shown that the ST-WF achieves higher capacity per antenna than SWF at low to moderate SNR regimes. It is also simpler to enumerate the solution for space–time water-filling because it eludes the cut-off value computation for each channel realization, but as a trade-off, its spectral efficiency gain is also set out to be associated with a higher channel outage probability. Similar to Equation (10), the problem of space-time water-filling can be formulated as:

$$\begin{gathered}\max_{Q} E\left[log\left|I + \frac{1}{\sigma^2}HQH^{\dagger}\right|\right] \\ subject\ to\ tr(Q) \leq \overline{P},\end{gathered} \tag{14}$$

where $\overline{P}$ is the average power constraint; H and Q have the same meaning as in Equation (11), making Q a function of H. The expectation in $E[tr(Q)]$ is carried over all MIMO channel realizations. This notation can be assumed as the symbol rate is considerable quicker than the MIMO channel variation. Q is evaluated from all symbols within one channel realization. Solving Equation (14) as a function of all eigenvalues gives the following relationship:

$$E\left[log\left|I + \frac{1}{\sigma^2}HQH^{\dagger}\right|\right] = E\left[\sum_{k=1}^{M}\log\left(1 + \frac{p(\overline{\lambda}_k)\overline{\lambda}_k}{\sigma^2}\right)\right] = ME\left[\log\left(1 + \frac{p(\lambda)\lambda}{\sigma^2}\right)\right], \tag{15}$$

where $\overline{\lambda}_k$ is the kth unordered eigenvalue of $H^{\dagger}H$, and $p(\lambda)$ denotes the power adaption as a function of λ. Therefore, Equation (15) can be rewritten as a function of all eigenvalues as follows:

$$\begin{aligned} &\max_{p(\lambda)} M \int \log\left(1 + \tfrac{p(\lambda)\lambda}{\sigma^2}\right) f(\lambda) d\lambda \\ &subject\ to\ M \int p(\lambda) f(\lambda) d\lambda = \overline{P}\,, \end{aligned} \tag{16}$$

where $f(\lambda)$ is the empirical eigenvalue probability density function. Similar to Equation (12), the optimal power adaption can be formulated as:

$$p(\lambda) = \left(\Gamma_0{}^{(\sigma^2,\mathrm{M})} - \frac{\sigma^2}{\lambda_i}\right)^{+}, \tag{17}$$

where $\Gamma_0{}^{(\sigma^2,\mathrm{M})}$ is found numerically to satisfy the average power constraint in Equation (17). The power adaptation is zero for the channel eigenvalue λ smaller than $\sigma^2/\Gamma_0{}^{(\sigma^2,\mathrm{M})}$ implying zero transmission in this eigenmode.

As such, the technique in ref. [127] is based on a SVD parallel channel decomposition technique that makes use of this ST-WF algorithm for power allocation. Although, majority of the research done for IA in CR require that the SU post coding matrix be approximated with symbols sent by the PU, the goal of any CR network is for the SUs to have the minimum possible interaction with the PU and vice versa. This work proposes a solution in which the SUs would require no interaction with the PU system, by estimating the required CSI in a blind manner. This work proposes an OIA technique in which two IA constraints Q_t and Q_r ought to be adequate as opposed to the initial approach of ref. [24], after which the SUs transmissions are then maximised. Given that most of the work on IA assumes that the SUs have all the CSI regarding the PUs transmission, this work takes on the challenge of designing a blind CSI estimation scheme for Q_t and Q_r, which contain information on the PU.

Assuming that a primary user system utilizes a time division duplex (TDD) scheme and henceforth the possibility of channel reciprocity is considered between the onward (Tx to Rx) and the opposite (Rx to Tx) channels, then the PU-Rx will intermittently communicate data or feedback to the PU-Tx using the Rx to Tx channels channel in timeslots of symbols. Even though the blocks interval is moderately insignificant compared to the involved channels coherence time, the SU only needs to know which timeslot is used for a regular PU communication (i.e., when the PU-Tx forwards data to PU-Rx) and which one is used for a reverse PU communication (i.e., when the PU Rx transmits back to the PU Tx). This is because it is the only knowledge that the SU requires to compute the pre/post-coding matrices, resulting in a novel blind CSI estimation scheme that achieves better performance. Similar to ref. [127], the work done in ref. [60] extends SVD for the PU link based on the ST-WF technique to achieve better channel capacity from the PU. However, this work employs multiple SUs in order to take advantage of the gains in performance for using cooperative spectrum sensing (CSS) [57,64,75]. Each SU senses the absence or presence of the TOs by the binary hypothesis test [128], and then sends a summary of its own observations to a fusion centre (FC) in the form of probabilities of missed detection and false alarm respectively.

The FC uses the hard combination fusion rule [129] for making the final decision on the state of TOs, and relays these decisions to the SUs. The SUs perform linear pre-coding to provide orthogonality and ensure multiple SUs have no interference effect on PU-Rx. Unlike [127], this research considered that multiuser SUs use a time division duplex (TDD) scheme hence assumption of channel reciprocity between the onwards Tx and Rx channels. The work in ref. [79], specifies that reciprocity is essential towards increasing sum rates, thus, the SU receivers will periodically transmit feedback through reverse channel in timeslots with channel parameters indicating a loss of fidelity or a change in the PUs optimization parameters.

To attain the maximum multiplexing gain of a multi-user MIMO CR network, the work in ref. [73] proposed a distributed OIA (DOIA) technique using a threshold-based beamforming (TBF) algorithm. Instead of using the SWF or ST-WF, this scheme makes use of a maximum eigenmode beamforming (MEB) technique for optimizing transmission at the PUs to enable some of its eigenmodes for SUs. While its operation is very close to the ST-WF scheme, the key benefit of adopting the MEB protocol is so that the PU puts all its power on the transmit antenna corresponding to the maximum eigenmode of its transmission channel, thus by default, the rest of the eigenmodes are left idle for the SUs' transmission. The benefit of its slightly lower computational complexity when compared with the existing water-filling methods that is still very much debatable as it also suffers from this rigid allocation of power on the largest eigenmode because the largest eigenmode mode might not be the optimum for the SUs transmission. For this virtual cooperation in which local CSI is available, the SUs sense for the unused eigenmodes in the manner of CSS. After the fusion centre (FC) has decided which eigenmodes to use, the SUs align the signals transmitted to the SDs related with the PUs unused eigenmodes to make sure of orthogonality between the PU and the SUs.

The TBF algorithm, which is a distributed power-allocation strategy, was used to maximize the SUs transmission rates. It enables the SU links with a maximum eigenvalue above a certain threshold to transmit data at full power, while the rest remain silent, thus enabling the CR network to maximize the sum rate of both the PU and SUs. This MEB scheme has been extended to the work in ref. [69], for femto-cells, which consists of an overlay MIMO cognitive femtocell network with macrocell users equipment (MUEs), taken as the PUs, and femto-cell access points (FAPs), which are taken as the SUs. In this work, the MUEs base station puts all its power on the antenna corresponding to the largest eigenmode of the channel matrix, leaving the rest free. An effective CSS technique is then proposed, where the FAPs detect these unused TOs. Instead of using energy detectors at the FAPs (despite their ease of implementation as well as low complexity), this work suggests the use of the GLRT detector, predominantly beacuse the GLRT detector is very adaptable and does not require a static threshold. The main objective is to maximise the average sum-rate of the macrocell base station (MBS) and the FAP's rate with negligible reduction in the PU's rate. This is achieved by the OIA technique using the TBF algorithm. The suggested OIA-TBF protocol makes use of the same frequency band of a pre-existing MUE to assure that no interference is levied on the MUE's performance for such a network and permits the opportunistic FAPs to transmit data for FUEs.

4.3. Other Endeavours

As mentioned earlier, there are quite a number of other equally significant algorithms to implement IA in CR that do not fall under the scope of both paradigms that have been discussed. The work done in ref. [130] using vandermonde-subspace frequency division multiplexing (VFDM) is quite similar to the second paradigm. Rather than using unused eigenmodes left by the PU, this work exploits the extra dimensions of the cyclic-prefix (CP) of orthogonal frequency-division multiplexing (OFDM), so that the SU's transmission is projected onto the null space generated by the CP. This enables the SUs to transmit without causing any interference to the PUs. It then goes on to minimize inter-cell interference by developing a chordal-distance scheme called exhaustive search algorithm, in order to maximize the transmission rates of the SUs. The intra-cell interference is also minimized by using a heuristic algorithm. Compared to the traditional time-division multiple-access (TDMA) scheme, the proposed scheme can support more interference-free symbols to be transmitted by SUs simultaneously. Since IA has been established as an effective approach to eliminate the interference of SUs in CR networks, the work in ref. [12] proposes a time resource auction scheme based on game theory to exploit IA for an overlay CR network. This work uses the given related equilibrium functions to balance the benefits of using IA as an effective approach of eliminating the interference of SUs on the PU. Simulation results are presented to show the effectiveness of the proposed scheme.

Furthermore, the work in ref. [131] breaks the restriction that SUs can only transmit on the idle sub-channels of the PU system by introducing IA based spectrum sharing into general CR networks and more specifically for distributed multi-user multi-antenna CR networks. Rayleigh quotients of channel matrices are utilized to predict the maximum interference from each SU and based on these predictions, the SUs with less interference to PUs are permitted access. As a result, the number of accessing SUs, the transmit power of each active SU-Tx as well as the sum rate of SUs are all increased. Given the nature of the time-variant channel condition between SUs and PUs, an adjustable transmit power baseline is selected to achieve a better trade-off between power gain and spatial DoF. This helps to solve the problem of low interference constraints that hinders SUs from being able to access spectrum holes.

The work in ref. [59] does not only focus on effectively eliminating the interference among users in CR networks, it also proposes to increases the transmission rate of the PU that guarantees its priority over the SUs. This is achieved by proposing an optimal transceiver design (OTD) strategy, in which two partial-IA based techniques are suggested to implement the precoding matrices of SUs, in which the interference is aligned at the PU-Rx with a reduced overhead and complexity. Then the PUs precoding matrix is implemented to increase its throughput consequently. Additionally, their decoding matrices are re-designed to increase the performance of SUs. Simulation outcomes showed the effectiveness of the proposed scheme. The work in ref. [132] proposed that the PUs trade spectrum for secrecy to enable SUs access the licensed spectrum as long as they help to secure the PUs transmission in the presence of an eavesdropper. Both PUs and SUs are allowed to cooperate with each other by exchanging CSI (but not information data). Achievable DoF of both PU and SU pairs are characterized in the presence of an eavesdropper that intends to intercept the PUs data.

5. Comparison of Research Literature and Analysis

This section present some of the results obtained from the research literature, and their analysis was based on achievable sum rates (bits/s/Hertz) of the SU systems and SNR (dB).

The first sets of results shown in Table 4 are for the achievable sum rates of the first paradigm. The sum rates for the leakage LIF algorithms are actually average values obtained from refs. [80–87]. These results are strikingly similar despite the fact that the work in refs. [80–83] assume perfect CSI while refs. [82,83] take CSI estimation errors into consideration. This is an indication that LIF, is still very much a mitigating factor to achievable sum rates in the first paradigm, and as will be shown later, in the second paradigm as well. Even though not reflected in Table 4, the values for sum rates when symbol extensions [93,97] are used are much higher than the other techniques.

It should be noted that for ease of simplicity and analysis, this work only considered results with the minimal number of antennas at the PU and SU as well as the minimum DoF. Obviously, the higher the number of antennas, the higher the DoF and therefore the higher the achievable sum rates. Thus from the comparison of results in Table 4 and Figure 6, the enhanced algorithms associated with the LIF technique perform better than the others enhancements, despite its effects on achievable sum rates.

Table 4. Average sum rates against typical signal-to-noise ratio (SNR) values for the first paradigm [80–87,99–101,103–105].

	Average Sum Rates of the SU Network Based on the IA Techniques (bits/s/Hertz)		
SNR λ (dB)	**Leakage of Interference [80–87]**	**OFDM [99–101]**	**Other Endeavours [103–105]**
5	5.5	9.0	1.0
10	11.0	10.0	5.0
15	15.0	11.0	9.5
20	20.0	12.0	16.0
25	28.0	13.0	24.0
30	32.5	23.0	33.0

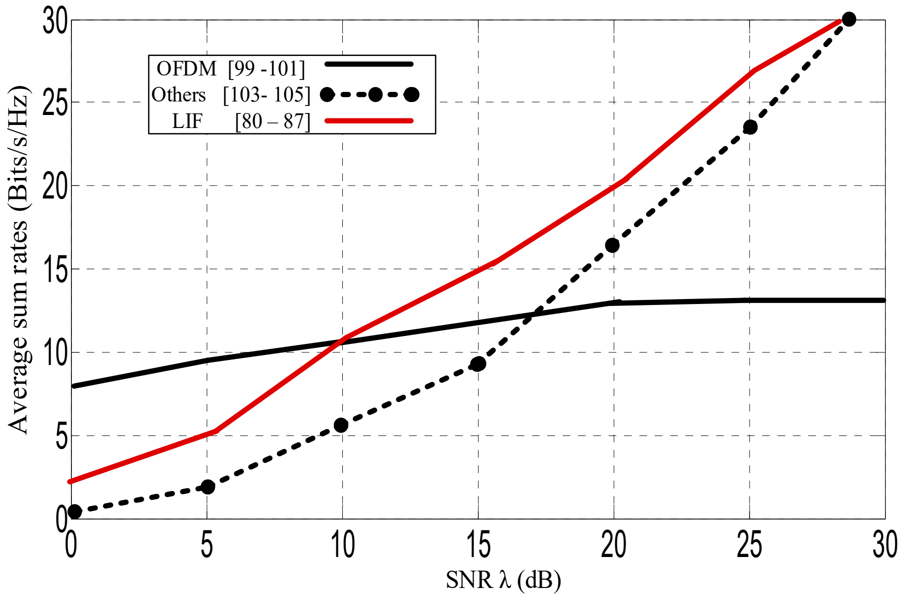

Figure 6. Average sum rates against typical SNR values for the first paradigm [80–87,99–101,103–105].

Table 5 considers the IA sum rate values for the second paradigm with SWF for the single SU scenario. Even though the sum rate results from the work in ref. [24] are quite low, they do provide the basis for this paradigm and shows just how critical it is to not only identify TOs, but useable TOs for the system model to work. Thus, a lot depends on the methods adopted for optimizing the SU transmission rates. Of all the results obtained, the work in ref. [115] that makes use of an intermediate relay was shown in Figure 7, to achieve maximum sum rates, thus corroborating the benefit of incorporating relays in wireless systems [74].

The work done on symbol extensions was a stark contrast to the first paradigm because of the sum rates achieved. The use of symbol extensions did not produce greater performance as it did with the first paradigm even though it had slightly marginal performance than ref. [24]. This could be attributed to the fact that the power allocation method used for optimizing the SUs transmission is similar to ref. [24], which has been established to be sub-optimal. Whilst the LIF technique was shown to have greater performance to other IA techniques in the first paradigm, it did not have quite the same effect in the second paradigm as seen in Figure 8. This is due to the fact that the PU transmission is completely orthogonal to that of the SU, making the effects of LIF very minimal. It could be argued that the results comparison for LIF was similar across both paradigms, implying that however minimal, interference plays a significant role in wireless communications.

Table 5. Average sum rates against typical SNR values for the second paradigm with spatial water filling (SWF) for the single SU MIMO link [24,61,68,115].

	Average Sum Rates of the SU Network Based on the IA Techniques (bits/s/Hertz)			
SNR λ (dB)	**OPA/UPA [24]**	**Relay [115]**	**Spectrum Sharing [68]**	**Fast Sensing [61]**
5	2.5	7.5	3.0	4.0
10	2.0	13.5	6.0	7.0
15	1.8	19.0	10.0	10.0
20	1.5	25.5	14.0	13.0
25	0.5	32.5	14.0	16.0
30	0.1	38.0	13.5	20.0

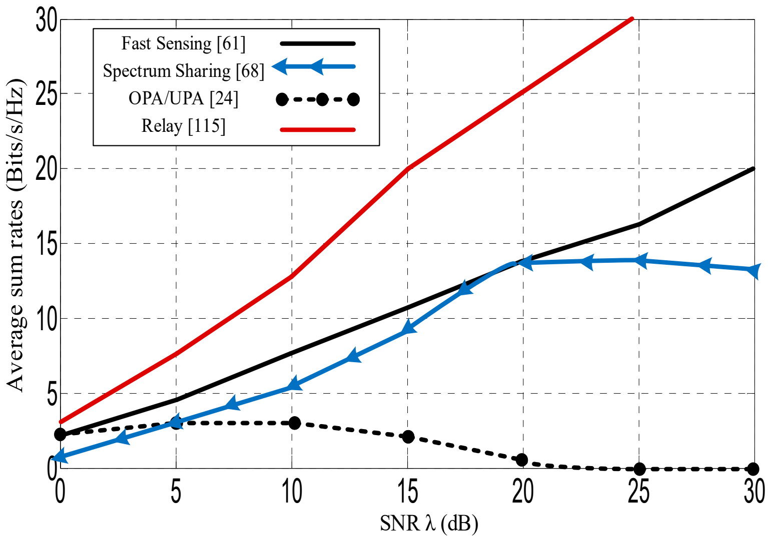

Figure 7. Average sum rates against typical SNR values for the second paradigm with SWF for the single SU MIMO link [24,61,68,115].

Table 6 considers sum rate values for the second paradigm with SWF, for the multiple SU MIMO link. As was expected, the results from the feasibility condition technique of linear IA over constant MIMO channels were the most optimal sum rate values because of the feasibility condition defined in ref. [121], which states that by counting the number of equations and the number of variables in the IA condition, the DoF can be analysed. This flexible approach to determine the feasibility of a network makes it easy to define a system as proper or improper, and does so with an improved iterative algorithm that has a faster convergence rate [123].

Table 6. Average sum rates against typical SNR values for the second paradigm with SWF for the multiple SU MIMO link [80–83,93,97,119–123].

	Average Sum Rates of the SU Network Based on the IA Techniques (bits/s/Hertz)			
SNR λ (dB)	**Symbol Extensions [93,97]**	**Feasibility [119–123]**	**LIF Perfect CSI [80–83]**	**LIF CSI Estimation [82,83]**
5	5	16	4.5	7
10	4	24	6.5	14
15	4	30	8.5	20
20	2	38	10.0	30
25	1	46	11.5	38
30	0.5	56	12.5	47

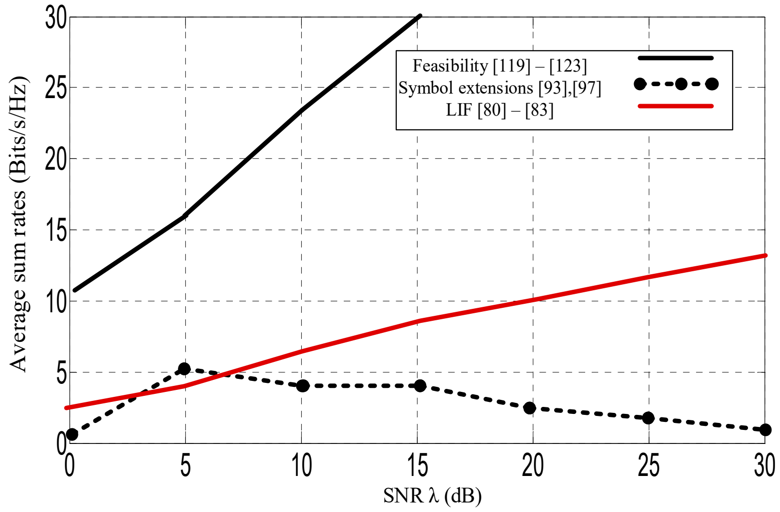

Figure 8. Average sum rates against typical SNR values for the second paradigm with SWF for the multiple SU MIMO link [80–83,93,97,119–123].

Table 7 shows the sum rate values for the second paradigm with ST-WF and MEB. It can be seen from both Table 7 and Figure 9 that the scheme used in ref. [127] performs better than the other techniques despite the single SU scenario. The OIA schemes in refs. [69,73] had similar performance in terms of sum rates largely because they both employed TBF to optimize the SU transmission, which is effective for saving the transmit energy in poor channel conditions and thus improving the spectrum efficiency of the SUs.

Table 7. Average sum rates against typical SNR values for the second paradigm with space–time water-filling (ST-WF) for the multiple SU MIMO link [69,73,99,127].

	Average Sum Rates of the SU Network Based on the IA Techniques (bits/s/Hertz)			
SNR λ (dB)	**Blind [127]**	**DOIA-TBF with MEB [73]**	**OIA-FAP with MEB [69]**	**OFDM-VFDM [99]**
5	6.0	3.0	3.0	3.0
10	8.0	4.0	4.0	5.0
15	13.0	5.0	6.0	8.0
20	17.0	7.0	8.0	12.0
25	22.0	9.0	10.0	16.0
30	26.0	12.0	14.0	20.0

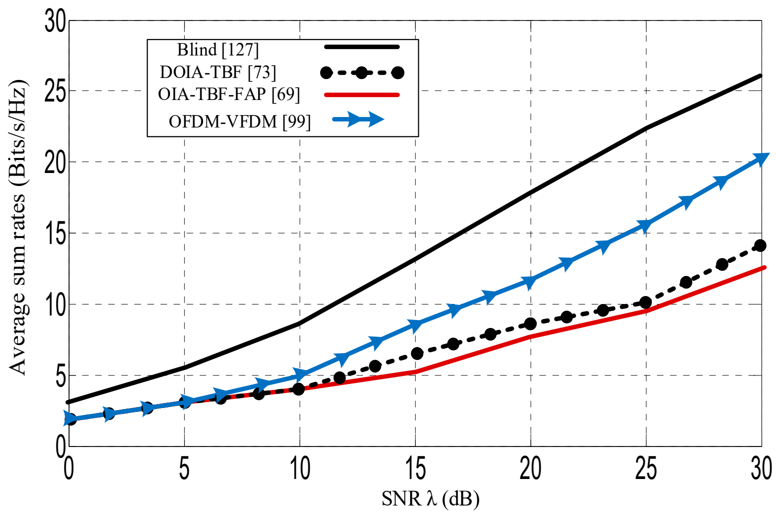

Figure 9. Average sum rates against typical SNR values for the second paradigm with ST-WF for the multiple SU MIMO link [69,73,99,127].

The proposed OFDM-VFDM scheme seems to increase the sum rate performance especially within the intermediate SNR range. On a wider scope, this scheme can simultaneously transmit more symbols for each SU than TDMA, and with the potential of extending the SU system to more than two cells, the spectral efficiency can only be improved.

6. Open Research Challenges

While much research has been carried out on IA in CR networks, there still exist some open challenges that are discussed in this section.

6.1. Channel State Information Knowledge and Feedback

In many cases, to carry out IA in CR networks, local CSI knowledge is required. It is a crucial to investigate suitable blind and semi blind IA techniques so that the burden for acquiring the channel knowledge at the CRs is minimized. CSI is key to evaluating IA precoders. Pilot transmission require adequate resources to be assigned, and in certain circumstances, to CSI feedback increasing the readiness of precise CSI. Subsequently, IA precoders need be recalculated during an appreciably change in channel conditions, in high-mobility fast-fading networks, the overhead of CSI acquisition can reduce the gains of IA [133]. Hence, minimal overhead signalling dimension techniques need be developed for an appropriately trade-off between CSI quality and CSI acquisition overhead. Temporal correlation can be used to both improve the precision of CSI and decrease the feedback overhead. The effects of feedback overhead and feedback delay in IA CR networks are a significant future research issue [88,90].

6.2. CR Network Synchronization and Organization

IA in CR networks by means of linear precoding is a transmission strategy for the coherent interference channel. Thus, IA requires tight synchronization to remove any timing and carrier frequency offsets between cooperating CRs [133]. In the absence of sufficient synchronization, additional interference terms are introduced to the signal model, rendering the IA solution ineffective. Synchronization strategies that could help fulfil this requirement are a vital future research topic. CRs cooperating via IA must not only synchronize, but also negotiate physical layer parameters.

6.3. IA in Relay Based CR Networks

Work on IA in relay based CR networks proposes that CR relays can greatly reduce the coding dimensions needed to achieve a network's DoF, and otherwise simplify the optimal transmission strategies [18,50,57]. The importance of practical precoding algorithms for the CR relay-aided interference channel is further amplified by the standards community's growing interest in deploying relays in future wireless systems.

6.4. Algorithms Optimization of IA in CR Networks

Algorithms optimization remains a hot topic for IA CR networks research as a number of features can be incorporated or further improved such as complexity, low SNR performance, CSS data fusion computation and robustness to CSI imperfections [53,67,110]. Emphasis on developing algorithms that will not only achieve IA by progressively reducing LIF at each receiver while keeping an allowable interference level to the PU, but will also be focusing on closer practical implementation in CR networks.

6.5. Practical Implementation of IA in CR Networks

Inadequate diversity of interference channels can possibly limit the relativity of the alignment. For instance, in the case where each CR has a single antenna and all channels are persistent across both time and frequency, the diversity of channels becomes limited. An open research problem is a practical achievable technique, which entails finite dimensions for the case of multiple non-intended receivers [134]. Hence, exploring innovative techniques for the optimization of linear precoders and alignment filters aimed at maximizing the sum-rate in low and moderate SNR regions is a vital future research challenge for practical implementation of IA in CR networks.

6.6. IA in CR Networks with Reinforcement Learning

The key to performing IA in CR without CSI is the use of reconfigurable antennas (RA), which are capable of dynamically switching among a fixed number of radiation patterns to introduce artificial fluctuations in the channel [41]. The radiation patterns used to realize blind IA have significant impacts on the overall performance of the system [42]. Hence, intelligent antenna pattern selection strategy is a crucial research issue for practical RA-based blind IA implementation using reinforcement learning techniques.

7. Conclusions

This article surveys the key concepts of linear interference alignment, surveys recent results on the topic and focuses on bringing the concept closer to implementation in CR networks, which promises to improve throughput. The IA algorithms/enhancements, as has been clearly described, remain the biggest challenge for this research. A number of enhancements have been incorporated in both paradigms of IA in CR to improve low SNR performance, enhance distributed computation and are more robust to CSI imperfections. To be more specific, under the first paradigm, enhancements applied to the LIF algorithm seem to yield the best performance in terms of sum rate capacity, regardless of CSI assumptions. Therefore, future research direction suggests more emphasis on developing algorithms that will not only achieve IA by progressively reducing LIF at each receiver while keeping an allowable interference level to the PU, but will also be focusing on closer practical implementation in CR networks. With the research done under the second paradigm, the combination of ST-WF algorithms at the PU optimization algorithms at the SU link achieved higher sum rate performance, owing to the ST-WFs ability to achieve higher capacity per antenna at low to moderate SNR regimes. While the sum rate results of the other endeavours where encouraging, it remains debatable whether the ST-WF algorithm with multiple SUs (incorporating CSS) will lead to achieving a more practical solution in CR networks. In addition, several practical challenges of these techniques have been identified in order to enable the future research in this domain.

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