Time-keeping with Limited Clocks

Extended Abstract

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Abstract

We study the problem of measuring time given strongly limited resources using Markov chains interpreted as clocks. More precisely, we assume the existence of an accurate short-scale time tick and are interested how to measure larger time scales with informationally limited memory. We use Information Theory to uncover novel behaviour of clocks with a small state space.
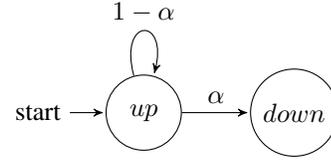
Introduction

Most living organisms have a biological clock (Bloch et al., 2013) called the circadian rhythm. In cyanobacteria, the circadian clock is only made up of three proteins and is so simple that it has been reconstituted in vitro (Nakajima et al., 2005). Inspired by this minimalistic clock, we build small machines that give an agent information about time. An obvious mechanism for the agent's clock would be a switch that flipped back and forth at each tick. This would be able to provide one full bit of information about current time; however, this is a purely local characterization, as it distinguishes only odd and even times. We are studying to which extent more global time information can be achieved with a limited clock. Because of this, our approach uses probabilistic machines and we study their effectiveness using Information Theory.

Previous Work

The Information Bottleneck Method (Tishby et al., 1999) is a method to filter information by relevance. Time measuring dynamics can emerge as side effect of other information processing in minimal agents (Klyubin et al., 2007).

Investigations

One of the machines which we study in most of our experiments is inspired by physical decay processes whereby excited particles have a probability to transition to a ground state. This machine, which is illustrated in Fig. 1 can be fully described by its state, either up or down, and by its decay parameter, α, which is the probability of transitioning from the up state to the $down$ state.

Figure 1: Drop machine

Fig. 2 shows $I(state; time)$, the amount of information that would be gained about time from measuring the state of the machine, given various α and T (lengths of time that are of interest).
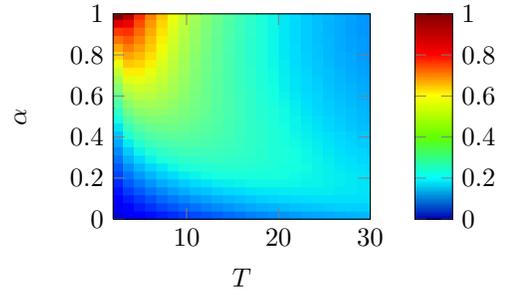
Figure 2: $I(state; time)$ for various α and T

Consider a vertical slice in Fig. 2 at $T = 20$. Surprisingly, the information curve does not always have a unique maximum (Fig. 3). Plotting $I(state; time)$ shows an inflection in the graph.

We can explain this by classifying time into *relevancy variables*. In this paper, we use relevancy variables as a kind of filter; to hide details in the probability distribution of t by partitioning its state space. One relevancy variable divides t into two halves, while the other divides the space into alternating parts. This breakdown is plotted in Fig. 3. The explanation is that the two maxima belong to different relevancy variables.

Consider now $\arg\max_{\alpha} I(state; time)$, the optimal α for various T (Fig. 4). This reveals two different regimes
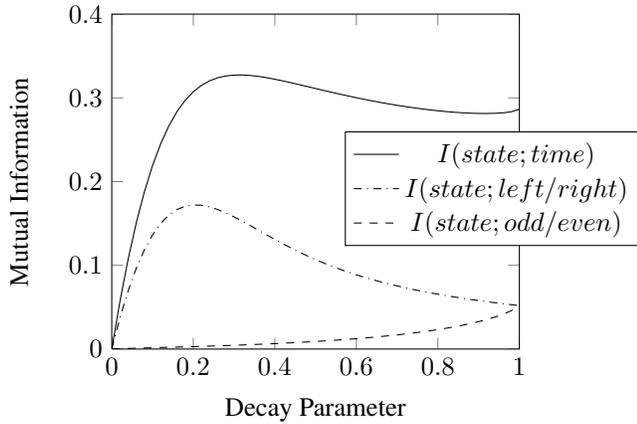
Figure 3: Mutual information between the Drop machine and time for a timespan of 20 ticks.

separated by a discontinuity at $T \approx 15$: for smaller T, $\arg\max_\alpha = 1$, while for larger T the optimal parameter follows the expected monotonically decaying relation. This discontinuity comes from the different maxima shown in Fig. 3.
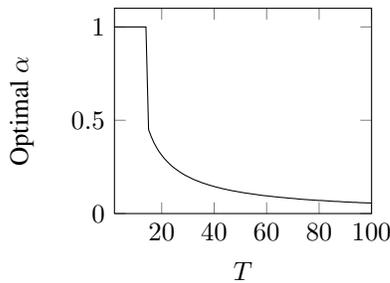
Figure 4: Optimal Decay Parameter for Drop machines

Finally, we "evolve" a cascade of independent clocks. The experiment is started without any clocks and the collection is built up one clock at a time. Before being added to the collection, each clock is optimised to provide the most amount of information given all clocks already in the collection and then committed permanently. The machine used in this experiment is the *Flip-Flop machine*, a generalisation of the drop machine which is allowed to decay back to the *up* state with a probability β. T is set to 5. As the collection grows, so does $I(states; time)$ (Fig. 5). The first clock is trivially the oscillator, but all subsequent additions turn out to be pure drop machines (Fig. 6). The first two add ~ 1.5 bits, while every subsequent clock adds significantly less.

Discussion

We investigated the characteristic properties of a minimal probabilistic automaton used to measure extended time inter-
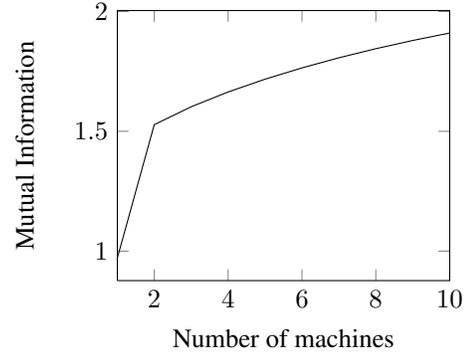
Figure 5: Amount of information as the size of the collection grows
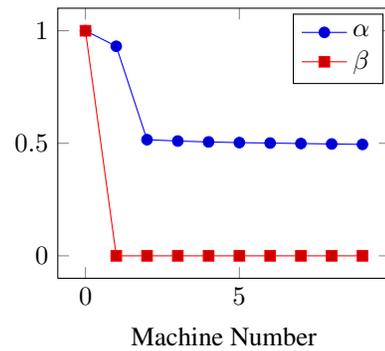
Figure 6: Parameters for the first 10 machines that are found for $T = 5$

vals. We find intricate dynamics already in the case of 2-state automata.

References

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