Evaluating the Utility and Validity of the Representational Redescription Model as a General Model for Cognitive Development

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A thesis submitted in partial fulfilment of the requirements of the University
of Hertfordshire for the degree of Doctor of Philosophy

October 2007

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Acknowledgements

I would like to take this opportunity to thank my exemplary supervisory team – Professor Karen Pine and Professor David Messer, who have been extremely supportive, providing advice, insight and inspiration throughout.

To my family I also owe a debt of thanks for supporting me through my drawn-out student years. My parents have been particularly supportive, as have my brothers – thanks! To my Auntie Helen and Uncle Henrik I also owe much gratitude for providing a home away from home.

I am ever grateful to the numerous schools, headteachers, and teachers who allowed me access to conduct my experiments, and of course the children for their participation, their interest, and their patience!

I would also like to thank my colleagues at the University of Hertfordshire, for providing a pleasant environment in which to work – to Sarah (for tea and biscuits!), Liz, Avie, Martin, and everyone else, thanks for putting up with me!

Finally, for those with whom I have worked these last 5 years – thanks to

Jo, Edward and Andrew.

Abstract

A series of studies were conducted with the aim of showing that the Representational Redescription (RR) model (Karmiloff-Smith, 1992) can be used a general model of cognitive development. In this thesis, 3 aspects of the RR model were explored.

The first set of experiments involved analysing the generalisability of RR levels across tasks in a domain. In an initial study, the levels of the RR model were successfully applied to a balance scale task. Then, in a subsequent study, children’s RR levels on the balance scale task were compared with their RR levels on a balance beam task (see Pine et al, 1999). Children were seen to access the same level of verbal knowledge across both tasks. This suggests that it is verbal knowledge which provides the basis for generalisation of knowledge. The second set of experiments involved a consideration of the RR model in relation to the domain of numeracy. The levels of the RR model were applied to children’s developing representations for the one-to-one and cardinality principles. The RR levels were shown to have utility in predicting children’s openness to different types of “procedurally based” and “conceptually based” teaching interventions, with pre-implicit children benefiting from procedural interventions, and children who have implicit and more advanced representational levels benefiting from conceptual interventions. The final study involved a microgenetic analysis of children’s representational levels on the balance beam task. The findings from this study indicated the importance of a period of stability prior to a cognitive advance, and demonstrated that cognitive advances can be driven by changes in the verbal explanations that are offered, rather than changes in successful performance. This provides support for the mechanism of change proposed by Karmiloff-Smith, 1992.

Together, the findings indicate that the RR model provides a useful perspective about the cognitive development of children. In particular, the thesis highlights when children can use the same representations for different tasks in a domain and suggests the mechanism that brings about representational change.

Chapter 1: Literature review

The object of cognitive developmental psychology

"We shall not cease from exploration
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time" (T.S. Eliot)

Psychology can be thought of as an attempt to explore and describe the human mind. Like poetry, psychology often depends on metaphors. The above quote from T.S. Eliot for example can be thought of as capturing the course of development as described by the Representational Redescription model (Karmiloff-Smith, 1992) which will be described at length in this thesis. The human mind as being like a computer processor is currently a predominant metaphor, both in psychology in general, and in developmental psychology. It provides a blueprint for how we think about the mind, by providing use with a set of structures and computational processes which may be able to describe how humans think.

Moving on from metaphors, there is a desire in psychology to present fully described theories or models which depict the mind in greater detail – to elucidate and build upon the metaphor. In developmental psychology, Piaget's cognitive developmental model has been hugely influential in defining the developing mind. Piaget described the child developing through a series of developmental stages, which proceed from early sensory-based stages, to later stages where children can begin to engage in formal-logical thought (Flavell, 1963). Within this model, the stage of development is a crucial aspect of the theory – for example, the earliest stage involves "sensorimotor" thought. During this stage children respond to objects in their environment, In Piaget's model, cognitive development eventually results in "formal logical" thinking, whereby adolescents can internalise objects, and reason independently of the environment in a logical manner.

The importance of general theories such as Piaget's is that they allow for a comprehensive overview of developmental psychology, highlighting important developmental phenomena, describing the course of development, and the impetus for this development. It is in the context of Piagetian theory for example that the conservation task (Piaget 1952, Greco, 1962) gains its importance. Conservation in itself is a relatively small phenomenon, where children think that a longer row of objects contains more objects than a shorter, more densely packed row of objects. This demonstrates "pre-operational thinking", as children do not appreciate that the numeric value of the set does not change when it is more widely spaced out. Within Piagetian theory, this simple task takes on significance, and can be compared to similar findings in other areas (e.g. tasks on conservation of number, and volume) of pre-operational thinking. The general model provides a context in which to recognise similarities in the process of development across different domains. General models can also be used to provide general implications for teaching and learning – they can for example be used to provide a basis for designing school curricula.

Piaget's theory has been much criticized over the last 30 years (Donaldson, 1976, Siegler, 1996, Karmiloff-Smith, 1992). Critiques have ranged from problems with tasks which underestimate children's competence on tasks (e.g. Donaldson, 1976), a lack of emphasis on social aspects of development (e.g. the Vygotskian perspective), through to specific problems with aspects of stage theory (Karmiloff-Smith, 1992, Siegler, 1996). One of the biggest problems with the Piagetian model is that it is largely descriptive, rather then explanatory. The stages of thinking are described in great detail across a number of domains (e.g. Piaget 1928,1932,1958, 1960, 1974, 1976), but the process of how this change occurs through a process of "accommodation, assimilation and equilibration" is underdescribed. These criticisms leave a gap, a need for a modern developmental model which can add to Piaget's in providing key insights into children's developing mind. In the absence of a general model, there is the possibility of fragmentation occurring in cognitive developmental topic, as findings from one domain (for example on theory of mind; see Wellman, 2002), which is a much-studied current topic – general implications cannot be derived from studies on theory of mind in the absence of a general framework which described children's general mental processes. That is to say, in the absence of a general framework, it is impossible to draw any general conclusions about children's thinking in relation to the theory of mind, but only to say how children think specifically regarding the theory of mind.

In this literature review, some of the critiques of Piaget will be highlighted, and a more recent general model, Karmiloff-Smith's (1992) Representational Redescription (RR) model will be described, and its unique features highlighted. This literature review will include the following sections:

1. A description of the RR model

2. A contrast between the RR model and Piaget's developmental model, to highlight unique aspects of the RR model

3. Reaction to the RR model, and recent research involving the RR model, to highlight questions arising from the experimental literature

4. A description of contemporary cognitive models, and comparison of the RR model with another contemporary model, the Overlapping Waves model, and a description of modern research using the Microgenetic Method, arising from this model.

5. The research questions arising for the RR model for this PhD

6. A description of the issues for the specific domains (e.g. the domains of balance and numeracy) investigated in this PhD

The key underlying aim is to show that the RR model can provide unique insights into the developing mind – that it has validity as a general model of cognitive development, as it addresses key developmental issues, both generally (e.g. by addressing general developmental issues such as generalisability of knowledge and the process of change), and in relation to specific domains (e.g. by increasing our understanding of the development of knowledge in specific domains, such as children's developing understanding of number).

1. The Representational Redescription Model

The RR model describes the development of representations, along an implicit-explicit continuum. The representation acts as the unit of analysis, with children being assigned representational levels, as will be seen, on the basis of their ability to perform tasks. Initial implicit representations involve knowledge encoded in a procedural format (i.e. an ability to perform a task), but are eventually redescribed into more explicit formats. Children's final representations within this model involve having verbally accessible knowledge. Karmiloff-Smith (1992) describes a series of representational levels which children pass through, to emphasise that change along the "implicit-explicit" continuum occurs gradually. These representational levels capture the form and content of children's thinking in relation to their performance on a domain-related task. Karmiloff-Smith (1992) initially describes 3 recurrent phases that occur within the RR model. The first phase focuses on information from the external environment, to create what Karmiloff-Smith terms "representational adjunctions" which lead to Behavioural Mastery for tasks within a domain. The second phase is internally driven, during which "*system-internal dynamics take over such that internal representations become the focus of change*" (Karmiloff-Smith, 1992, page 19). This internally-focused phase can lead to downturns in performance; it is important to note however that this "downturn" is simply at the level of performance on a task, but not in representational level or children's level of knowledge within a domain. The final phase involves a reconciliation of internal representations and external data, as children achieve a complete representation in which children can perform a task successfully and have explicit verbal knowledge for the concept underlying the task.

These 3 phases are linked to "*at least 4 levels at which knowledge is represented and re-represented*" ((Karmiloff-Smith, 1992, page 20). Karmiloff-Smith talks of implicit level, and 3 explicit levels, which reflect children's verbal access to knowledge. The implicit level maps on quite well to the first phase described by Karmiloff-Smith, with "representational adjunctions" which are "Implicit", or procedurally based only. The rest of the development is taken up with redescribing representations into increasingly explicit knowledge; though Karmiloff-Smith states that explicit verbal knowledge is not achieved until E3, the final phase. In her 1992 work, *Beyond Modularity*, Karmiloff-Smith described how this process may apply in the domains of language, physics, mathematics, psychology, and notation. It is important to note however that Karmiloff-Smith's (1992) interpretation of experimental findings were usually applied to her model in a post-hoc manner.

1.1. Applying the levels of the R-R model to a balance beam task

An example will now be given to describe the path of development for children's representations according to the RR model. Pine et al (e.g. 1999, 2001, 2003) applied the levels of the R-R model to a balance beam task similar to that used by Karmiloff-Smith and Inhelder (1974). Children were given a series of symmetrical and asymmetrical beams, with blocks of wood at either end of the beam, acting as weights. The children were asked to balance the beams along a fulcrum. Table 1.1 below shows the course of children's development as described by the RR levels. It is very important to note at this point that the RR levels described in Table 1.1, and descriptions of the level hereafter refer to the research of Pine et al (1999, 2001, 2003) in applying the levels of the RR model to the tasks, as Karmiloff-Smith (1992) has done relatively little experimental work in which RR levels are applied to children for a domain based on their performance on a task and their ability to verbalise the concept underlying that task. On the balance beam task, children are initially able to balance both types of beams, but cannot explain why the beam balances (e.g. behavioural mastery, accompanied by implicit verbal explanations, indicating no verbal access to knowledge). This behavioural mastery is interpreted as an implicit representation of balance, which is initially purely procedural in format. The next level in development is abstraction nonverbal, at which point children begin to focus on the most prominent variable; that of weight, and overgeneralise it. Because of this, children no longer balance asymmetrical beams. This gives rise to a U-shaped curve (Strauss, 1982) in development in terms of behavioural success in relation to verbal access to knowledge. At this point, the child does not have explicit verbal access to this knowledge, but at the next level, Abstraction Verbal, they begin to provide explicit weight-based explanations for why the asymmetrical beams do not balance

The final level within the model is called E3. When children achieve this level for a domain, verbal knowledge (for the domain of balance, this would include knowledge of the role of distance as well as weight in knowing how objects balance) is accompanied by behavioural mastery once more.

Table 1.1: Coding scheme for representational levels for the balance beam task

Representational Level	Expected Level of Performance	Levels of verbal Knowledge expected
Implicit	Balances both symmetrical and asymmetrical beams	No explicit explanations offered
Abstraction Nonverbal	Balances symmetrical but not asymmetrical beams	No explicit explanations offered
Abstraction Verbal	Balances symmetrical beams, but not asymmetrical beams	Explicit, weight based explanations offered
E3	Balances both symmetrical and asymmetrical beams	Explicit, weight and distance based explanations offered

From this description of the model, both Karmiloff-Smith's (1992) description, and the work carried out by Pine et al (1999) it is clear that the main impetus of the model is to describe children's increasing verbal access to knowledge. This is one of the key unique features of the RR model, as will be seen when comparing it with other theoretical models.

2. Comparing the RR Model with Piaget's theory

The representational redescription (Karmiloff-Smith, 1992) model marks an attempt to move away from stage models in the light of criticisms of Piagetian theory. Three key departures of the RR model from Piagetian theory will be focused on, to highlight the significant and innovative features of the RR model, though it is important to note that certain aspects of Piagetian theory (such as the constructivist focus) are maintained.
The first departure from Piagetian theory is that the levels of the representational redescription model are domain specific, rather then domain general stages. The RR model envisions the same developmental processes occurring in all domains, but does not state that children should display the same levels simultaneously across all domains. The process of redescription is thought to be applicable to all domains, though it does not necessarily occur simultaneously across the domains.

The RR model is not only based on aspects of Piaget's model, but also reacts to Fodor's views on Modularity. Fodor (1982) stated that the mind is composed of separate, and "domain specific" modules, which are "informationally encapsulated" – that is to say, information available to one particular module is not available to others. Karmiloff-Smith (1992) follows this path of reasoning, to a certain extent as the RR model states that children's representational levels will differ from domain to domain, rather than maintain general stages of development as Piaget does. If a child is at a certain representational level for one domain; the domain of balance for example, it does not mean that children will be at the same level for all other domains. In this way, representational levels are clearly differentiated from Piagetian stages – levels refer to discrete domains, whereas Piagetian stages were thought to be universal, and apply across multiple domains at any one time. A domain is "*the set of representations sustaining a specific area of knowledge*" (Karmiloff-Smith, 1992, page 6). Domains such as language, physics and mathematics are given as examples by Karmiloff-Smith, and she also speaks of micro-domains within these domains; counting for example would be a micro-domain within mathematics, and balance would be a micro-domain within physics. Domains as defined by Karmiloff-Smith (1992) differ greatly from Fodor's (1982) modules, as they are not thought to be informationally capsulated (e.g. the process of redescription renders the information in a domain more accessible to other operators).

The second departure from Piagetian theory is that children's representations develop gradually along an implicit-explicit continuum, rather than through the Piagetian stages which focus on developing logical thought. Representations therefore refer to children's knowledge for a concept, which may be implicit, and "*encoded in procedural form*" (Karmiloff-Smith, 1992, p 20), or in some other format, with the end-point being an explicit and verbally accessible format. This compares with the Piagetian model which focuses on changing structures of thinking, from sensorimotor through to formal logical thinking.

Conscious access to knowledge plays an important part in the development of representations. Nelson (1999) notes that Piaget "*viewed language as a component of the representational function, but not as an important contributor to cognition per se*" (p189). Within the RR model, verbal access to knowledge is an integral part of the developmental process. Indeed, there is some evidence that getting children and adults to provide self-explanations following tasks can produce better understanding of the concepts underlying those tasks (Chi et al, 1994) or bring about cognitive change (Siegler, 1995). Ability to verbalise concepts is therefore a central, potentially driving force for change within the RR model, rather then simply being a developmental outcome.

The third difference between the RR model and Piagetian theory relates to how cognitive change occurs. Piagetian theory states that cognitive change arises through cognitive conflict – failure on a task brings about a series of processes of "accommodation", "assimilation", and "equilibration". Karmiloff-Smith notes that Piagetian theory focuses solely on change being brought about "*when the system is in a state of disequilibrium*" (1992, page 25). Karmiloff-Smith on the other hand states the importance of stability in bringing about change; "*it is representations that have reached a stable state that are redescribed*" (1992, p 25). Furthermore, it is not necessarily failure on a task that brings about redescription. Rather "*positive feedback is essential to the onset of representational redescription" (Karmiloff-Smith, 1992, p 25).* This positive feedback, thought to be based on a stable period of success, allows for the redescription of knowledge – using the example of the balance beam task, it is thought that a stable or consolidation period of sustained success is needed in order for the internal-phase to occur; a stable representation must exist, in order for it to be redescribed into more explicit formats.

Representational redescription is thought to involve the redescription of knowledge which is initially procedurally based, into gradually more explicit and consciously accessible formats. Representational change is based on increasing conscious access to knowledge. Therefore, the RR model offers an alternative model for how cognitive change may occur, and states how stability may be necessary for change to occur, rather than negative feedback

3. Reaction to the Representational Redescription model

What response has the representational redescription model evoked? An article in the journal Behavioural and Brain Sciences (Karmiloff-Smith, 1994, 1997) brought about a series of commentaries from many major names (Bloom & Wynn, Campbell, Dartnall, Freeman, Goldin-Meadow and Alibali, Kuhn, Rutkowska, Smith, Zelazo). A number of the commentaries focus on the lack of detail in reference to the process of redescription, and in the model in general. Bloom and Wynn (1994) stated that "*mental mechanisms capable of generating new concepts or structures simply do not exist*" (p 708). Campbell (1994) focuses on the lack of a clear description of what representations are and how they get redescribed. Freeman (1994) similarly noted that assessing how representational change occurs is difficult "*in the absence of an explicit account of representation*". Olson (1994) downplays redescription as a form of reanalysis of data, rather then "*one of the human instincts for inventiveness"* (Karmiloff-Smith, 1992, p 193). Scholnick (1994) notes a need for a "*rich model of the early implicit and early explicit codes*" and a need "*to specify the limits of cognitive transfer and conscious access to cognitive structure*" (p 728).

There were a number of negative responses. Foster-Cohen (1994) questioned whether language could be thought of as a "module" in and of itself. Zelazo (1994) also mentions the failure of the model to account for age-related domain general changes in reflection and the control of behaviour. There were also positive responses. Goldin-Meadow and Alibali (1994) make a positive contribution in light of their own research in noting that redescription can occur without mastery. Similarly, Kuhn (1994) highlighted the importance of explicit knowledge as well as implicit processes. There was also a great deal of debate about what the RR model can contribute to cognitive science (Grush, 1994, Dartnall, 1994, Hampson, 1994, Rutkowska, 1994, Shultz, 1994).

In summary, the main points arising from these commentaries were that aspects of the RR model were underdescribed, especially with regard to the process of change. What was lacking however amidst the welter of commentary, praise and questioning, was a drive to apply and work with the model on a practical level. There is also a need to validate the model as a general model by attempting to address the concerns raised in these commentaries.

3.1 Additions to the RR model following experimental research

Pine et al. (e.g. 1999, 2001, and 2003) have provided empirical support for part of the RR model, by replicating and building on the work of Karmiloff-Smith with reference to the concept of balance (Karmiloff-Smith and Inhelder, 1974). Their research has provided cross-sectional evidence for the representational levels at different ages as well as quasi-longitudinal (Pine and Messer's 2003 study which tested children's representational levels on the balance beam task across 5 days) evidence of children's movement through these levels in the order laid out in Table 1.1 above. By studying the performance of large groups of children modifications have been suggested to the details about the levels of representation. First, Pine et al (2003) introduced "transitional" representational levels which are thought to capture gradual change from representational level to representational level. They introduced an implicit transition level which denotes children who are thought to be moving between Implicit and Abstraction Nonverbal levels, and Explicit transition levels for children who seem to be moving between Abstraction Verbal and E3 levels. These transitional levels also allow for periods where children may show greater variability in performance than the RR model's levels would normally allow. The second addition to the RR model is based on the research of Pine et al (2003) which focused on the flexibility of children's representational levels beyond simply being able to balance beams and provide verbal explanations on the balance beam task. A prediction task (e.g. children were asked if they could balance a particular beam before being presented with it) showed that children may have had access to some form of knowledge even when displaying implicit representations for the balance task. Children clearly therefore have some access to knowledge, and indeed can verbalise this knowledge prior to achieving E3 representational levels.

3.2 Other research involving the RR model

The RR model has been used by a number of researchers, in a number of domains – these include Balance (Horst et al, 2005, Philips and Tolmie, 2007), training in pedestrian skills (Tolmie et al, 2005), literacy (Critten et al 2007a, Critten et al, 2007b) drawing (Barlow, Hollis and Low, Picard), and number (Dixon and Bangert, 2006, Chetland et al, 2007). These will briefly be summarised in turn, to demonstrate in part the domains within which the RR model has been applied, and the extent to which the 3 unique features of the RR model highlighted already have been investigated.

3.3. Balance

With regard to balance, Horst et al (2005) questioned whether redescription occurred in children's knowledge on the balance beam task. They looked at children's ability to balance beams across sessions, to see if children followed the U-shape curve as described by Karmiloff-Smith (1992). Children were given a set of 5 symmetrical and asymmetrical beams to balance. Horst et al (2005) looked for the U-shape curve in performance on the balance beam task in relation to age, rather than in relation to children's verbal knowledge, which is the basis for the U-shaped curve in the RR model. They did not record children's verbal knowledge. Horst et al (2005) found "quasi-linear" improvements in performance in terms of symmetrical and asymmetrical beams balanced across different age groups (4, 5, 6, and 8 year olds). In contrast to this however, Pine and Messer (2003) showed that children did move through the RR levels in the hierarchy laid out in Table 1.1, as they were tested each day across 5 days. It is important to highlight that the U-shaped curve arises in relation to children's verbal knowledge, which Horst et al (2005) did not measure within their study. It is also important to note that the "U-shaped" curve is a metaphor to easily describe the fact that children can show decreases as well as increases in performance as they show changes in verbal knowledge. It is not therefore surprising that the cross-sectional type of experiment carried out by Horst et al does not reveal a U-shaped curve in terms of performance on the balance beam task in relation to age.

Phillips and Tolmie (2007) on the other hand focused on children's explanations about balancing, and on the type of language used by those teaching them about the balance scale. They looked at the effect of parental input either through describing strategies for use on the balance scale task, or providing explanations for the principle underlying the concept of balance (e.g. the rule of torque). Children given this sort of input were found to show gains in integration of performance and understanding, and in the use of similar complex explanations themselves, though Phillips and Tolmie (2007) state that the effect of these interventions may be constrained by initial representational level.

3.4 Pedestrian skills

Tolmie et al (2005) looked at the RR model in relation to teaching children pedestrian skills. As with Phillips and Tolmie (2007), the focus was on the use of adult discussion and peer discussion. In this case the focus was on their use in aiding the development of children's pedestrian skills, to the point where they would have generalisable verbal knowledge, which is taken to be E3 type representations(e.g. verbally accessible and generalisable knowledge about pedestrian skills, with regard to what precautions need to be taken when crossing roads). They reported that the specific teaching methods used did help to develop E3 level representations, where verbal knowledge was generalisable. The study reported above assumes E3 representational levels as the point at which knowledge becomes generalisable. In this regard Shultz (1994) notes that research on the RR model has been restricted to individual tasks, rather then attempting to look in depth to see if knowledge, once consciously available, does become available for other tasks within that selfsame domain (but see Messer, Pine & Butler, 2007). There is a need to see at what point in development along the representational levels knowledge does in fact become generalisable.

3.5. Literacy

Critten et al (2007a) have applied the levels of the RR model to spelling and reading development. Critten's research has focused on children's overgeneralisations which lead to phonological and morphological errors (e.g. overgeneralising the –ed rule for verbs in the past tense). In particular, their findings have supported key aspects of the model with regard to literacy which have been described by Pine et al (1999) with regard to the concept of balance. Critten et al have shown that children do have explicit verbalisable knowledge prior to E3. A longitudinal study (Critten et al, 2007b) has also supported the hypothesised path of development through the representational levels set out by Pine et al (2003).

3.6 Drawing

A number of researchers have looked at the RR model in relation to drawing. As Picard notes, this area has been used to "*study internal representational changes and to reveal the constraints acting on such changes*" (Picard and Vinter, 2006, p 529). Their study focused on the role of flexibility in Representational redescription. This is based on

Karmiloff-Smith's (1990) study on children's drawing. The focus of these types of studies is on children's increasing flexibility in terms of their ability to draw objects, and their ability to deviate from strict procedures in drawing objects – drawing a human body without a head for example. Barlow et al (2003) provide evidence that children are not as rigid in applying their procedures as presumed by the RR model, though in this case the drawings in question were of familiar objects, and may therefore have been more advanced in terms of representational level then Barlow et al (2003) assume. Another study by Hollis and Low (2005) looks at generalisations within a domain – children were taught to draw "pretend" humans (e.g. drawings that deviate from the normal human body), and were later asked to draw "pretend" houses (e.g. including novel parts, or removing normal features such as doors, windows, etc.). No transfer between these two domains was found. This again draws attention to the question of whether children can show generalisation of knowledge across tasks within a domain.

3.7 Mathematics

The final domain in which the RR model has been used in research is mathematics. Chetland et al (2007) have shown how in basic cardinality tasks (e.g. the "give me x" task) children continue to show development in their use of more advanced strategies following behavioural mastery. This study discusses the Overlapping Waves model (Siegler, 1996) alongside the RR model, and will be returned to when the Overlapping Waves is being described. The work of Dixon and Bangert (2006) also includes the RR model, though in this case the process of "redescription" as a means of cognitive change is being contrasted with a "theory revision" approach. This approach is interesting insofar as it reduces the RR model to the process of change, rather then incorporating the levels. There is a need therefore to show that the levels of the model do in fact contribute to our understanding of our knowledge of mathematical concepts.

Summary of RR model and questions to be addressed:

3.8 Generalisation

A number of issues have been addressed with regard to the RR model. Clarification is still needed for several issues however. These can be linked back to the unique features of the RR model. The first feature related to the model's levels being domain-specific rather then universal. Two issues emerge from this based on the studies described above. First, within the RR model, children's representations are supposed to gradually become more verbally accessible, and more generalisable. At this point, one can look at generalisability in two ways – one can talk about the generalization of knowledge from one modality to another: therefore the RR model describes a process of how generalization comes about, as procedural information becomes redescribed into a verbally accessible format. A second point of view looks at when knowledge, in whatever format, becomes available to be utilized for another task within a domain. This is the aspect of generalization being focused on in this thesis. Is verbal knowledge, once it is achieved within a domain, generalisable? In other words, is verbal knowledge automatically accessible to other tasks?

There has been some focus on the generaliseability and flexibility of children's representations (Tolmie et al, 2005, Hollis and Low, 2005, Pine and Messer, 2003). There have been conflicting findings here – Tolmie's findings follow Karmiloff-Smith's RR model and presume that generalisation of verbal knowledge occurs at the E3 level. Hollis and Low (2005) found that children could not generalise abilities from drawing one type of object (e.g. humans) to another (e.g. houses). Finally, Pine et al (2003) did find evidence that children could access knowledge at the Abstraction Verbal level to be able to make predictions about the balance beam task. These different findings suggest a need to clarify to what extent children's representations are in fact generalisable, and more importantly what aspects are generalisable. There is a need to explore at what point knowledge, and or behaviours become generalisable to other tasks within a domain – when do children become able to generalise verbal knowledge /and or strategies from one task to another within a domain? Given that Pine and Messer et al (03) showed signs that children have access to the same knowledge across multiple tasks, do children show the same verbal knowledge, and the same ability to perform tasks on different tasks within a domain? This has important implications as if children show generalisable knowledge or verbaliseable knowledge prior to E3, it must be removed as an indicator of "E3" or that children have achieved the most advanced level of knowledge for a domain. If children's verbal knowledge is inaccurate or is an incomplete representation (which would seem to be the case from the example of the domain of balance of Pine et al, with children giving solely weight-based verbal explanations), and if this information is generalisable to other tasks within a domain it is important insofar as it shows that incomplete representations can generalise as well as complete representations. If children can and do show explicit verbal knowledge prior to E3, this also has important methodological implications, as experimenters should try to devise ways to access this verbal knowledge from children at a younger age then might be currently the case in studies.

3.9 Applying the levels of the RR model to different domains

Second, there is a need to apply the levels of the RR model, both to different domains, and to different tasks within a domain, to demonstrate that the validity of using the RR model as a general cognitive developmental model. With the exception of Pine et al (e.g. 1999) and Critten et al (2007), none of the research above has actively applied the levels of the RR model to tasks. The importance of applying the levels arises due to the fact that a number of the studies mentioned above (Dixon and Bangert, 2006, Tolmie et al, 2005, Picard and Vintner 2006) have merely cited the RR model, and made general predictions from the model .The work of Pine et al (2003) has shown that the levels of the RR model were underdescribed in Karmiloff-Smith's (1992) original description of the model. There is a need to apply the levels in different domains, to see first if they apply, to understand the characteristics of the levels in different domains, and then to see to what extent predictions from the model can be made. The RR model remains limited if it can only be applied to very rigid types of tasks, in only a small number of domains, as these tasks or may either be not representative of a domain, or the characteristics of the task may elicit the type of behaviour which is codable into RR levels, as was the case of Siegler's (1976) rules model.

3.10 The importance of access to verbal knowledge, and its role in cognitive change

The second unique feature of the RR model was its focus on the role of increasing access to verbal knowledge as a key component of development. The work of Tolmie et al (2005), Phillips and Tolmie, (2007), the work of Pine et al (1999), have explored this aspect, though some researchers have not focused on this particular aspect of change within the RR model (e.g. Dixon and Bangert, 2005). Given that increasing access to verbal knowledge is supposed to play a key role in representational change this is certainly an issue that needs to be addressed. Does increasing access in fact drive the process of development and change, as the RR model suggests? This links back to some extent to the discussion on generalization above, though here the focus is on change

The third unique feature of the RR model is its unique description of how cognitive change comes about. There is a need to explore how cognitive change actually occurs in this model. This is particularly important given that, as a number of commentators have stated (e.g. Campbell 1994, Freeman 1994) this particular aspect of the RR model is underdescribed. As stated in the previous paragraph, there is a need to explore how change occurs within the RR model, as this is an aspect of the model that has yet to be researched in any great detail – given that one of the key points of the model is that, as opposed to Piaget's model, it tries to give a clearer description of how change occurs, there is a need to test this process of change, and the roles of both stability and increasing access to verbal explanations driving this change.

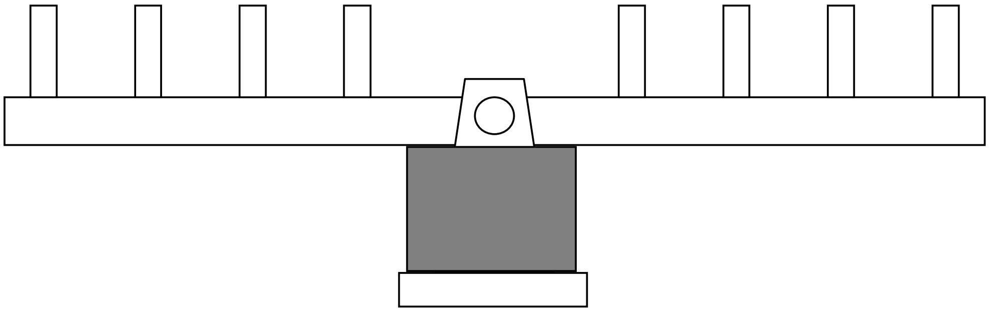
4. Contemporary cognitive developmental models:

Up to now a key point of comparison with the RR mode has been Piagetian theory. However, other significant and contrasting theoretical models have emerged since Karmiloff-Smith's RR model. The main theory with which the RR model will be compared is Siegler's (1996) Overlapping Waves Model, though a number of other models will also briefly be described, to show what the RR model has to offer as a general model in relation to other contemporary models of cognitive development. The Microgenetic Method, will then be described, as it is a context in which many relevant studies for these models have been conducted.

4.1 The Overlapping Waves Model

Siegler's (1996) book Emerging Minds criticises his own Rule assessment methodology (Siegler, 1976) and the general stage based approach to development. In 1976 Siegler described a rule assessment methodology, which describes a series of rules through which children were thought to develop in solving tasks on a balance scale (see Figure 1.1 for an example of the balance scale apparatus). Rules were typically presented as binary decision trees (e.g. in the case of the first rule for the balance scale task, children would simply look at the number of weights on either side of the scale; if there are the same number on either side of the fulcrum it balances, if not, it doesn't), akin to basic computer programs, with children applying specific rules depending on the situation facing them. The rules progressed from the initial simple ability to focus on one dimension of a task at hand (e.g. focus solely on weight in a balance task), to being able to integrate multiple dimensions, with the rule approximating the actual normative rule (e.g. the rule of torque in the case of balance). The rules were shown to apply to various types of task, including understanding of projection of shadows and fullness (Siegler 1978, Siegler and Vago, 1978), conservation of liquid quantity, solid quantity and number (Siegler, 1981), and time, speed, and distance (Siegler, 1983, Siegler and Richards, 1979). Siegler even found evidence that children were at similar stages in their development of rules across different tasks (Siegler, 1981).

Figure 1.1: The Balance Scale Apparatus used by Siegler (1976)



Siegler (1996) highlights criticism of this approach, and in particular, the tasks to which the rules were applied. The main criticisms of the rules based model were (a) that they arose on tasks that children typically would not have encountered prior to the experiment. (b) These tasks tended to have 2 dimensions to them (e.g. weight and distance in the case of the balance scale [e.g. Siegler1976]), one of which tends to dominate. This could lead to children being more single-minded in their thinking than is the case in more typical tasks.

The overlapping waves model on the other hand depicts children's thinking as involving "*a gradual ebbing and flowing of the frequencies of alternative ways of thinking, with new approaches being added and old ones being eliminated as well*" (Siegler,1996, p87). Children have at their disposal a number of different strategies to solve specific tasks. These strategies may be more or less sophisticated. For example, to perform a simple addition sum, a child may use a number of strategies: they may remember the answer, having previously done the sum, they may guess, they may add the 2 numbers (either starting from zero, or starting from the first number in the sum), or they may use more advanced strategies such as the min strategy, where the child starts adding on from the highest number, rather then automatically adding on from the first number in the sum (which is not always the highest number). In these everyday tasks, children have a variety of different ways of arriving at an answer. The introduction of new, more sophisticated strategies does not however mean that children stop using older, simpler strategies (Kuhn et al., 1988, Kuhn and Phelps, 1982, Schauble, 1990, Metz, 1985, Siegler and Jenkins, 1989). Rather, children show variability, using different strategies to solve the same task on different occasions. However, it is assumed that there is a gradual change in the frequency of strategy use, and the introduction of new ways of thinking.

Chen and Siegler (2000) state that the pattern of use of strategies "*arises through the working of five component processes*" (Chen and Siegler, 200, page 8). These processes are: (1) acquiring new strategies, (2) mapping the strategy onto novel problems. (3) strengthening the newly acquired strategy. (4) refining choices among competing strategies, and (5) increasingly effective execution of strategies. Strategies are acquired, are used in different tasks, and are increasingly used accurately, or to put it another way, children become increasingly good at choosing which strategy is most appropriate for a specific task. In the study described by Chen and Siegler (2000), children are trying to reach toys which are beyond their reach. One clear finding is that older infants (children ranged from 1 ½ years old to 2 ½ years old) were increasingly consistent in their choice of strategy (using a toy cane or rake to reach for a toy beyond reach). Therefore, one of the outcomes of the overlapping waves model is a movement towards a kind of stability in strategy use.

The main starting point for the overlapping waves model is variability, which forms the main argument against stage models – these static models fail to capture children's variable strategy use across time. By assigning a child a simple "stage", variability is ignored. These stage models also fail to describe change accurately or in any depth, as changes are usually depicted as occurring suddenly. Stage models have also been referred to as staircase models (Siegler, 1996), where change is thought to occur through sudden upwards shifts, whereas the overlapping waves model shows that changes occur more gradually. Indeed, the overlapping waves model in some ways dispenses with change as a discrete concept, as children's thinking is regarded as being more or less in a constant state of flux.

Siegler (2005) also identifies five dimensions along which cognitive change can be analysed within the overlapping waves model. These dimensions are: source, path, rate, breadth, and variability. The latter two, breadth (e.g. the extent to which learning generalises) and variability (e.g. in contrast to the focus of the RR model on stability preceding representational change) tie in with 2 of the themes that have been highlighted with relation to the RR model – generalisability and the role of stability in cognitive change. This issue will be returned to when describing the Microgenetic Method.

It is important at this point, however, to highlight the differences between the RR model and the Overlapping Waves model in terms of the unit of analysis. The RR model focuses on representational levels, which encompass the ability to perform a task and conscious access to this knowledge. The Overlapping Waves model on the other hand focuses on "strategies". These strategies are not considered in relation to access to thinking in relation to the implicit-explicit continuum. The Overlapping Waves model focuses exclusively on strategy use, which may be thought of simply as a child's behaviour, whereas the RR model tries to look at the link between children's performance on a task with their verbal knowledge. The verbal knowledge components provides a specific element which can be thought of as being more "cognitive" than the Overlapping Waves model which is more oriented towards behaviour. Indeed, Siegler's (2005) review of microgenetic studies discusses the contrast between "learning" and "development". Siegler suggests that new strategies are "learnt", moving away from the notion that children are "developing" new and more sophisticated ways of thinking about concepts. Studies involving the Overlapping Waves method will be described in relation to the microgenetic method, after considering another important contemporary model of cognitive development, the dynamic systems model. The other key difference between the RR and Overlapping Waves model is that the RR model emphasises the importance of stability in representational levels, which is thought to be important for bringing about redescription. While Chen and Siegler (2000) state that stability may be an endpoint for development in the overlapping waves model, a period of stability is not necessarily the endpoint in the RR model, where a period of stability is thought to be necessary in order for the type of qualitative change in knowledge envisioned within the RR model (e.g. the development of explicit verbally accessible knowledge from an initially procedural format). The Overlapping Waves model emphasises variability in children's thinking, in the form of different strategies used to solve a task, but does not go beyond looking at strategy use to try and provide any detail of the cognitive system that the child has, which underlies or compliments the strategies they use.

The fundamental issue to be addressed however when trying to compare the RR model and the Overlapping Waves model, is to ask whether children best depicted as being variable in their thinking as the overlapping waves model states, or are they generally stable, as the RR model implies?

A final important point to mention in relation to the RR an Overlapping waves models' is the nature of the models. Karmiloff-Smith (1992) described her model as being "soft-core", as it is merely described verbally the model, rather then specifying it in such a way as to be implementable as a programme, or connectionist framework. The work of Pine et al (2003) have gone some way towards moving the model to a more "hardcore" set-up, where the different levels are clearly specified, and one can begin to make more easily testable predictions from the model. The Overlapping Waves model would also be thought of as being "softcore", as its 5 sequential components of strategic change, along with the 5 dimensions which are important for change are descriptive rather than explanatory. The RR model, as currently envisioned in this thesis, and following the additions to the model from the work of Pine et al (e.g. 2003) is a more "hardcore" and clearly specified model of cognitive development than the overlapping waves model.

4.2. Other contemporary developmental models

The Overlapping Waves model has been focused on as the key contemporary model for comparison. Other contemporary models will now briefly be described. In the 2002 edition of the Blackwell handbook of cognitive development, it is interesting to look at the section covering models of cognitive development. It is hardly surprising to find a chapter on Piaget, and a chapter on Vygotsky. Other models of normal development are summarised in one chapter on "information-processing models of cognitive development" (Halford, 2002), which includes a very brief description of the RR model. This chapter covers Neo-Piagetian models (McLaughlin, 1963, Pascual-Leone, 1970, Case, 1992) which focus mainly on development being driven by an increasing processing power. This is contrasted with the RR models which is described as a "levels of processing" approach. These models do not provide much scope for further comparison with the RR model, given their general adherence to Piagetian thinking – Case's (1992) model states that children pass through 4 neo-piagetian stages (sensorimotor, inter-relational, dimensional, and vectorial stages) which are comparable to Piaget's stages. The criticisms arising from stage models Karmiloff-Smith made in relation to Piaget would equally apply to these neo-piagetian models.

One modern developmental model not covered in this Blackwell handbook of cognitive development is the dynamic systems model (Thelen and Smith, 1994).This model (Thelen and Smith, 1994), alongside both the RR model and the Overlapping Waves model, involved a critique of stage models. Thelen and Smith contrast a view of development "from afar", where "*the grand sweep of development seems neatly rule driven. In detail however development is messy. As we turn up the magnification of our telescope, we see that our visions of linearity…break down*" (p xvi). The notion of general stages of development, based on broad cross-sectional studies are contrasted with more in depth views following an individual child's development across time, which is shown to be much more variable than stage models would indicate. The dynamic systems model borrows ideas from contemporary chemistry and physics. Rather than having a "closed" system with specified endpoints as classical stage models do (e.g. formal-logical thinking as the end-point in Piaget's stage theory), Dynamic Systems theory focuses on open systems, and their properties, which, like the Overlapping Waves model, focus on change, rather then stability. Equilibrium is an important concept for Thelen and Smith – at points of change "*the system loses its ability to maintain these patterns and the fluctuations become enhanced. At these points, the system is dominated by these fluctuations and may display transient behaviour where no stable pattern can be discerned*" (1994, p 63). For a system to change, it must first become unstable (Thelen and Corbetta, 2002). Because dynamic systems theory is rather complicated, this section will limit itself to pointing out this distinction between the RR model and Dynamic Systems theory – the fact that dynamic systems theory states the importance of instability in bringing about cognitive change. Indeed, it serves to reinforce points made in relation to the overlapping waves model – the fact that both models emphasise the importance of variability, and move further away from a stage-based model than the RR model.

Another approach not mentioned in the Blackwell Handbook is the work of Katherine Nelson (1996). Her focus is on the role of language development in children's thinking. In her book "language in cognitive development" she touches on the RR model, but takes a more social rather then a purely cognitive approach to development, looking at the emergence of language as an important "mediating" factor, which is importantly driven by social factors. The approach owes more to Vygotsky then to Piaget in this sense.

The final research to be touched on here is the work of Carey (1985). Carey focused on the types of change that occur in children's thinking. Does change involve simple enrichment of already present structures, or does it involve significant and conceptual change? Carey has described children's thinking in a number of domains (e.g. children's concepts of animal, plants, alive, eat, breath sleep, etc, Carey 1985, 1988, as well as the physical concepts of matter, material, kind, weight and density (Carey et al 1991, Smith, Carey and Wiser, 1985). The question of enrichment or change is very relevant to the RR model, in terms of how "redescription" is thought of. It may be thought of as "enrichment", as knowledge that was available in one format is simply redescribed so that it is available in another format. The resulting change from this "enrichment" is qualitative in nature, with the knowledge being in a new format which is thought to allow for greater generalisability. In this way redescription within the RR model emphasises that the distinction between enrichment and conceptual change may in fact be a false dichotomy.

4.3 The Microgenetic Method

As new theoretical approaches such as the Overlapping Waves model and the dynamic systems model emerged, so too has a new methodology has emerged which involves looking in detail at variability, and its relation to cognitive change; the microgenetic method. The microgenetic method is used to look at change in great detail. This approach has been championed by major figures within cognitive development (e.g. Karmiloff-Smith, 1984, Kuhn et al, 1982, Siegler and Crowley, 1991). Despite it being a fairly recent phenomenon, it does in fact have roots in the works of both Werner (1948) and Vygotsky (1930). The microgenetic method involves looking in detail at periods of change. A high density of testing is done in relation to the rate of change, in order to capture the immediate precursors of change, and to delineate the progress of change.

As has already been stated, within the Overlapping Waves model, 5 dimensions have been identified which can be used to analyse change; source, path, rate, breadth, and variability. The latter two are directly related to two of the unique aspects of the RR model highlighted so far – generalisability and the role of stability (as opposed to variability) in cognitive change. Microgenetic studies focusing on these aspects will now be described to provide a basis for the development of methodologies to investigate these two features of the RR model.

4.3.1 Microgenetic studies on the breadth of knowledge – can children generalise knowledge

Siegler's (2006) review of microgenetic studies identified a number of instances of failure to show generalisation from one task to another – in learning to walk for example (Adolph, 1997), and in transferring strategies from one mathematical task to another (Alibali, 1999, Alibali and Goldin-Meadow, 1993). Children have also been shown to over-extend strategies – applying a new strategy to a superficially similar task or domain in maths tasks (e.g. Siegler, 2002), or in overextension or overuse of a newly learned word (Bowerman, 1982). There has been some evidence of transfer of learning in more general studies of acquisition of a "control of variables" study (Chen and Klahr, 1999). These types of study focus on whether children can manipulate a number of variables within an experimental situation (an example would be if children had control over a number of different variables on a car in a computer simulation, and tested to see which combination of variables was quickest). Children were found to be able to transfer this newly acquired skill to structurally similar types of task (Chen and Klahr, 1999).

It seems that children's strategies do not generalise to different tasks – this is not surprising, as strategies can often be unique to a specific task. By using this level of analysis, it may be difficult to assess whether children are in fact accessing the same type of knowledge and applying it to different tasks within a domain. Therefore, this approach may in fact mask the generalisability of children's knowledge. An attempt will be made to address this issue by looking at children's RR levels with relation to different tasks within a domain.

4.3.2. Microgenetic studies on the role of variability in cognitive change

One of the key findings of microgenetic studies has been the high incidence of variability in children's thinking across a range of different tasks – in terms of strategies used when learning to walk (Adolph, 1997), in maths tasks (Grupe, 1998, Alibali, 1999), control of variable tasks (Schauble, 1990), logical inference task (Kuhn et al, 1992), amongst others. There have been findings indicating that high initial variability in strategy use can predict later learning (Coyle and Bjorklund, 1997, Siegler, 1995, Perry and Elder, 1997). There have also been findings indicating that variability may be cyclical in nature – with children beginning by systematically using one type of incorrect strategy, through a period of variability, followed by a later shift to consistent use of a more accurate strategy (Siegler and Chen, 1998, Siegler and Svetina, 2002, Van der Maas and Molenaar, 1992, Hosenfeld et al, 1997). There has been much focus on the role of variability in bringing about cognitive change – either based on the notion arising from the Dynamic System theory which states that for a system to change, it must become unstable (Thelen and Corbetta, 1997). Another theory arising is that this variability may reflect children beginning to activate multiple conflicting representations, which may for example be operating in different modalities (e.g. a child utilising one representation verbally, whilst displaying a different representation in hand gestures, Goldin-Meadow, 2002).

The large amount of data gives a strong indication that variability does play a key role in cognitive change, though it is not clear yet what role(s) this variability may play. It is this variety of different findings emerging that indicates the need for a model to try and explain the role of variability in change. The overlapping waves model does not seem sufficient here, due to its lack of description of how the 5 variables which are thought to be important for describing change (e.g. path, rate, breadth, source, and variability). There is no clear theoretical link between these 5 variables and the fundamental phenomenon of the ovberlapping waves model. Whether or not children show stability in maintaining a representational level prior to change has not been addressed within any of these studies. One of the aims of the PhD is to address this gap in the literature.

4.4 The Representational Redescription model in light of the overlapping waves model:

The description of the overlapping waves model and the microgenetic method, in which many studies based on this model has been carried out, give a clearer picture of the questions which need to be addressed with regard to the RR model.

A key distinction can be made between the units of analysis for these different theories. The RR model maintains a hierarchical structure of levels, based on performance and verbal knowledge which are not apparent within the overlapping waves model. Indeed, the Overlapping waves model seems to focus only on strategies, rather than looking in greater detail at the contents of children's knowledge (e.g. they look at what children can do, rather than what they know). There is therefore a need to show that these levels have validity and utility, if the RR model is to be of use as a general model for cognitive development. In this case, validation of the RR levels means showing that children are relatively stable in their performance and access to verbal knowledge for a domain across time, rather then being as variable in these measures as the Overlapping Waves model may suggest. In light of findings on variability with regard to strategy on various tasks, and lack of generalisation of strategies, there is a need to demonstrate that this does not also apply to representational levels.

It is also important to note the variety of tasks and domains that have been studied using the microgenetic method. The levels in the RR model on the other hand have thus far only been actively applied to 2 domains – that of balance (Pine et al, 1999), and literacy (Critten et al, 2007). There is a need therefore to study the levels of the RR model to different domains, in order to show that it can apply, and can add to our knowledge of different domains within cognitive development. This PhD will address some of the issues raised so far with regard to the concept of balance. Other issues will be addressed by applying the RR model to another domain, that of number. A more thorough review of relevant work within the domain of balance will follow, alongside a review of literature about number. These will help to show how the 3 unique features within the RR model that have been highlighted so far will be addressed as specific research questions.

5. Summary of questions for the RR model

In this literature review, the RR model has been described, and by focusing on (1) contrasts arising from a comparison with the RR model (2) Issues arising from experiments involving the RR model, and (3) A comparison with the Overlapping Waves model, a set of unique features arising from the RR model are translated into more specific research questions.

The first key point of the RR model was its use of domain-specific levels of thinking, rather than universal stages. These levels are thought to capture children's thinking for a domain. The research described so far has only focused on single tasks within a domain (e.g. Pine et al, 1999). There have been questions about when children may begin to generalise knowledge within a domain (Tolmie et al, 2005). The evidence from microgenetic studies indicates that children may be very poor at generalising across tasks, bringing into question what knowledge children can generalise to other tasks within a domain.

The levels of the RR model are domain specific. In spite of the research cited above, the levels have only been actively applied in 2 domains; balance (Pine et al, 1999), and literacy (Critten et al, 2007). An important part of the process of validating the RR model as a general model for cognitive development is by showing that the levels apply across different domains, in the same manner that the overlapping waves model, and Piaget's model in particular has been applied in different domains. One task of this thesis therefore is to show that the levels of the RR model are applicable to different domains, and to different tasks within a domain.

The second issue mentioned in relation to the RR model is its focus on the role of verbal knowledge as a key driving factor in children's developing representations. This brings about a need to look at the generaliseability of verbal knowledge

This links well with the third key feature of the RR model, and its unique perspective not only on the role of increasing access to verbal knowledge, but also the role of stability in relation to cognitive change. Given the fact that a number of commentators note that the processs of change in the RR model is underdescribed, and that a number of microgenetic studies (along with both the Overlapping Waves model and in particular Dynamic systems theory) have highlighted the fact that variability plays a major role in cognitive change, there is a need to study variability in the context of the RR model.

The final part of this literature review will give a general overview of the specific domains which will be covered in this thesis – the domains of balance, and numeracy, and the issues with regard to the RR model that will be addressed in relation to these domains.

6. Review of the Domains in which the RR model is applied in this thesis

6.1. Studying children's representations across tasks within a domain

In studying cognitive development there are advantages in considering more than one task in a domain. This will obviously give more insight into children's thinking about the domain (See Messer, Pine and Butler, in print), as well as removing the possibility that findings apply to only one task, rather then an entire domain. This is particularly important given that all the work on the RR model within the domain of balance, with the exception of the work of Messer et al (in print) has used only one task, the balance beam task. In addition, the use of more than one task will answer some of the criticisms that Siegler made of his own work within this domain.

The balance scale task used by Siegler (1976), which formed the basis for his Rule Assessment Methodology, involved asking a child whether or not a scale (see Figure 1.1) with weights on either side of the fulcrum, would balance, or tip down on one of the sides, if both sides of the scale were unsupported. This task was criticised by Siegler (1996) for being a novel and unusual task, rather then something that children would be exposed to in day-to-day life (though interestingly, for the balance beam task employed by Pine et al, children regularly note the similarity between the beams on the fulcrum, and a see-saw, indicating that balance might be a relatively commonly experienced domain for children). Wilkening and Anderson (1982, 1992) also criticised the task because it did not give children an opportunity to interact with the scale itself. By simply asking children whether or not the scale would balance unsupported, this approach may mask knowledge, which could be displayed if children were given an opportunity to balance the scale themselves.

In spite of this, Siegler's balance scale task has been used by a number of researchers, in relation to a number of different developmental theories (see Andrews and Halford, 2002, who applied the Relational complexity theory to the concept of balance, and Zelazo, 2002, who applied the cognitive complexity and control theory to this domain). Piaget (Piaget and Inhelder, 1958) used a balance scale task in support of his stage based developmental framework. The fact that a single domain has been used to support various different theories raises questions about generalising to a whole domain from a single task.

In light of these criticisms of Siegler's balance scale task, one of the aims of this thesis is to look at children's representations for the domain of balance using a number of balance tasks. This will help to address 2 specific issues for the RR model, which arose in relation to the research on the RR model already described in this literature review; (1) whether the levels of the RR model measure a child's knowledge for a domain, or just for a task, and (2) the generalisability of knowledge across tasks within a domain. Given the problems in using Siegler's (1976) balance scale task, there is a need to identify different types (though these criticisms do not stop Siegler from still using this task in experiment [Siegler and Chen, 1998]) of balance task, to which the RR model may be applied.

6.2 Studying cognitive change using a balance task

The second issue that will be addressed within the domain of balance is cognitive change. This domain was chosen due to the fact that the levels of the RR model have been comprehensively described (e.g. Pine et al, 1999, Pine et al, 2003), making it easier to study change within this domain. A number of studies have looked at change occurring within this domain – Pine and Messer (2003) have provided evidence that children do pass through the levels of the RR model in the sequence laid out in Table 1, and also shown that gesture-speech mismatches may indicate a readiness to learn and show cognitive change. Siegler and Chen (1998) have also studied children's developing Rule use, using the Rule Assessment Methodology, on Siegler's balance scale task, looking at the role of distal and proximal variables in the development of new and more complex rules. Siegler and Chen's (1998) study does not use the RR model, and does not therefore aid us in relation to studying the process of change within the RR model. Pine et al (2003) use the RR model, and look at children's increasing access to knowledge, which is one of the key features for representational change highlighted in this literature review. There is still a need however to look at the role that stability is supposed to play in representational change. This is particularly important in the light of the introduction of transitional representational levels by Pine et al (2003) – do these transitional levels indicate that stability is not necessarily present immediately prior to representational change occuring?

6.3. Beyond Balance

The domain of balance serves as a neat example of a domain in which cognitive developmental models can be applied. Developmental models have been applied in this domain by Siegler, 1976, Inhelder and Piaget, 1954, Halford et al, 2002, Zelazo et al 2002. It is however a relatively discrete domain, which is not as important for children as other domains. For educational purposes, the domains of literacy and numeracy for example are much more important. Karmiloff-Smith (1992) claims that the RR model is a general theory of cognitive development, and she cites examples of the way it can be used to explain development in a number of domains (balance, drawing, etc). For the RR model to be more generally relevant, it must be shown to apply to more central domains in cognitive development. Examples of such central domains are given by the chapter headings of Karmiloff-Smith's (1992) Beyond Modularity – "The child as a linguist", "the child as a physicist", "the child as a mathematician", "the child as a psychologist", and "the child as a notator". There is a need to show that the RR model can be actively applied to other domains - indeed the validity of the RR model as a general model for cognitive development rests to some extent on showing that it can be actively applied to these different domains. There is a need to show that the same can be said for the RR model. Karmiloff-Smith (1992), as already noted, marked out a number of territories; language acquisition, notation (writing and drawing), physics, and maths. Mathematics seems a conducive area of study, for a number of reasons. First, a central domain in mathematics can be identified; number. Mathematics is a very accessible domain for the purposes of child development. Number is a very visible and accessible part of our environment. From a very early age, number forms an important part of our environment, with examples of number liberally strewn across our visual and aural environment. Maths is taught from a much earlier age in schools then physics, and is considered a core part of the modern curriculum (one of the three r's no less!).

6.4 Children’s developing knowledge of number: Counting

A basic issue about maths/number is defining what number is. This is in itself a difficult task, as number is an elusive concept. Frege (1893) noted that number could not be defined – therefore it was not a concept which could be taught. The inference drawn from this is that number must be an innate concept. Numerous studies have attempted to prove this, by showing that animals have strong concepts of number (see Dehaene, 1997, in particular, the accumulator model). Infant studies have also been conducted (Starkey and Cooper, 1980, Spelke, 1991, Antell and Keating, 1983, Loesbroek and Van Smitsman, 1990, Starkey et al, 1990, Clearfield and Mix, 1999), to show that children are sensitive to aspects of number, from birth onwards. These studies form a substantial body of work which heavily influence the field of study of children’s developing knowledge of number. Research in older children (e.g. verbal children) has centred on children’s knowledge of Counting. This area has been dominated by Gelman and Gallistel’s (1976) 5 principles which aim to define a valid counting system. These principles are thought to provide the logical underpinning for the counting system, and, as will be seen, show how and when counting should be used. Counting is thought to provide the basis for an understanding of number.

6.5 Gelman and Gallistel’s Counting Principles

Gelman and Gallistel’s principles are generally considered in 2 groups – the first three principles relate to how one may count, followed by 2 more general principles which constrain how and when this counting system may be applied. The first counting principle is the 1-to-1 principle, which assumes that one counting word/symbol is used for each object in the array being counted. This involves two processes; first the child must partition each individual item, and then each item must be individually tagged with a number word. These two processes must of course be coordinated. One method for co-ordinating these processes involve pointing, and some work has been done in looking at the utility of pointing gestures in learning to count (e.g. Alibali and DiRusso, 1999). The second principle is the stable order principle, which states that the number words used to count should be arranged in a stable, unchangeable and repeatable order. This order can be arbitrary, and need not necessarily (though normally it does) match the conventional number system used in society, as long as they are applied in an invariant manner; a child may even count “one, three, four, two”, as long as they always use this particular sequence in this particular order. If they do so, they comply with this principle. The third principle, that of cardinality, states that the value of any set or array of objects should equal the value of the last number tag associated with a member of the array. This principle builds on the previous two, to begin to provide a use for counting, as they provide a meaning for the value which numbers are thought to entail.

The final two principles do not apply directly to counting as the previous three have done. The fourth, abstraction principle states that the three principles discussed so far can be applied to any array of objects, be they real or imagined. The final principle is the order-irrelevance principle, which states that the order in which we count an array of objects does not affect the numeric value of the array. We may count from left to right, from right to left, from the middle outwards, etc, but the cardinal value of the array remains the same. The number tag is the most important aspect for numeric purposes. This principle states clearly the arbitrariness of counting, and in some ways echoes Piaget’s views on post-formal logic, in understanding the uses of systems, in this case the counting system.

6.6 Research on the counting principles

This review has already mentioned some of the research which purports to support the theories of Gelman and Gallistel (e.g. infant number studies, as it has been theorised that these principles are innately specified). The original work was based on the findings of their “magic” experiment (Gelman and Gallistel, 1976). In this experiment, children were shown 2 sets of arrays, and told which one is the “winner”. They were then shown a number of different sets of arrays, and asked in each case to choose which of 2 examples was the “winner” and why. The game becomes more complex as the arrays are hidden and shuffled around, and sometimes magically changed, and the child must explain why the chosen array is not in fact the winner. This particular test was found to produce sufficient evidence of counting in order to support the theory that children show evidence of use of the principles outlined above. It is important to note the restricted numerosities used in this experiment; between 2 and 5 objects in any array. Because young children have more difficulties as larger amounts of objects are presented to them, this narrowing down represents a search for competency, rather then looking at whether children can continue to apply these principles in larger arrays at this early age. The sets used were homogenous and arranged in a linear manner (e.g. children may not count non-homogenous objects as one set, and may have problems keeping track of objects if they are not linearly arranged).

Children as young as two years old for example showed 80% error free performance in terms of the 1-to-1 principle, with performance rising to near-perfect in 3 to 4 year olds. The 2 year olds were the main group who did not use the conventional word lists for number. Similar findings of competence were found for the other four principles in young children, suggesting at least an early level of skill-based competence in counting. Whether this translates to having strong or explicit conceptual knowledge is another matter however.

6.7 The principles within a theoretical framework

Gelman and Gallistel claim (1983, 1984, 1986) that the principles for counting are innate. Based on this, they have predicted that the ability to detect errors in others' counting should precede counting skills; young children should be able to detect errors, even though they cannot themselves accurately apply the principles in their own counting. Thus, children are supposed to have the capacity to deal with number before they can accomplish this with words. This provides a very different developmental path to that proposed in the RR model, as their approach states that children detect deviations from the normal counting procedure prior to their being able to perform this procedure themselves. A number of error detection studies have been carried out over the years, with varying results. Gelman and Meck (1983) performed a study where children were asked to watch puppets counting arrays of objects. Some puppets counted correctly, and some puppets made errors. Gelman and Meck (1983) found that the majority of 3 and 4 year olds could in fact detect these errors. What they did not look at however, was children's abilities to explain the errors made in the error detection task, which for the R-R model is the most important aspect of conceptual knowledge, and allows us to measure whether their conceptual knowledge for a specific counting principle is implicit or explicit approach.

Briars and Siegler (1984) took a slightly different approach. They showed children a wider array of counting examples, displaying correct counts, a number of different counting errors, and "pseudo errors" (see chapter 4 for greater details.). Briars and Siegler found that older children were more likely to detect errors in counting (4 and 5 year olds were much more likely to detect errors where they occurred than the 3 year olds). Younger children detected the pseudo errors as true errors a significant proportion of the time. These results suggest that younger children were much less likely to regularly reject real errors in counting than was found in the studies of Gelman and Meck (1983). Gelman and Meck (1986) argued that these findings were due to procedural differences, with children given much more opportunity to detect errors by repeating the trials in the method of Gelman and Meck (1983). The difference between the findings of Briars and Siegler (1984) and Gelman and Meck (1983) remain unresolved, leaving unanswered the basic question as to whether or not children's conceptual knowledge precedes counting skills. Both Siegler (Rittle-Johnson and Siegler, 1998) and Gelman (Gelman and Meck, 1986) maintain their stances.

The RR model would state that children's ability to count should precede their ability to detect and explain errors' in others' counting, in line with the findings of Briars and Siegler (1984). It is important to note however that the RR model does include the possibility of "innate predispositions", which may be in line with Gelman and Gallistel's (1992) later work which talks about children being born with innate "constraints on counting". The question in this case becomes what is the nature of children's early knowledge of the counting principles – do they begin with early implicit type representations which enable to apply the counting principles, or do these develop at a later stage.

The approach of the RR model, with its focus on developing access to verbal knowledge alongside an ability to perform a task is relevant to current theorising in this area. This approach is similar to Baroody's (1998) mutual development approach – instead of looking at which aspect of counting knowledge, be it procedural or conceptual knowledge, we should look at how they develop together (1998). This is also in accord with the views of Sophian (1998) who stated that with regard to developing concepts for number, the research focus should move beyond looking at when basic competences emerge, and look at how it actually develops throughout childhood. Another good reason for the use of the R-R model in general with regard to counting, given its focus on verbal knowledge is that as Ginsburg (1976) notes, development of initial number knowledge is in fact largely verbal (e.g. children learn to count aloud, on their fingers, etc. before they are formally introduced to written number). Therefore, there is a need for a model which emphasises that a large part of early learning with regard to counting is verbal in nature.

6.8 The role of verbal knowledge in the development of children's representations for counting principles

Applying the R-R levels to children's knowledge of counting principles is an important first step for this domain. The R-R model must also have some utility; it must provide predictions for the course of development. Pine et al (e.g. 2000) have looked at the effect of various types of intervention on children's representations for the concept of balance. In Pine et al's (2000) study the effect of explaining another person's actions on the balance beam task on children's representational levels has been analysed. A study by Muldoon and Freeman (2003) has focused on a similar effect (e.g. looking at children's explanations of other people's actions) with regard to the concept of cardinality. They studied children's ability to compare sets of objects, and found that children's ability to detect and reason about someone else's miscounts was the best predictor of performance on the comparison task. This suggests, as Pine’s (2000) work does, that being able to explain other people’s performance on a task can play an important role in development. Muldoon et al’s study was not intervention-based however, so it still needs to be shown that giving children the opportunity to detect and explain others’ miscounts may bring about improvements in children’s ability to perform a comparison task.

Another study of interest here was conducted by Rittle-Johnson (2001) on children’s knowledge of fractions. Rittle-Johnson was testing an iterative model which focused both on children’s ability to verbalise concepts, and their ability to perform the task, which she argues are linked. She predicted that improvements in what she terms “conceptual knowledge” leads to improvements in “procedural knowledge”, and vice versa. Pine and Messer (1998) on the other hand noted that the effect of interventions on children’s representations can be mediated by their prior representational levels (see also Tolmie et al, 2005). This leaves open the question as to whether these different types of intervention are equally effective at different points in time.

Summary

The aim of this PhD is to demonstrate that the RR model can be adopted as a general model for cognitive development. Three sets of research questions are addressed to show that the RR model may be useful as a domain general model for cognitive development: (1) The first set of questions focus on the applicability of the RR model to different tasks within the domain of balance to assess the model as a domain-based rather then a task-based model, and to assess the generalisability of knowledge within a domain. (2) The second set of questions focus on applying the levels of the RR model to the domain of counting, and showing that the model can add to our theoretical understanding of this domain, and provide input on how to bring about cognitive change within this domain. (3) The final set of questions focus on clarifying the processes of "representational redescription" (Karmiloff-Smith, 1992) which are thought to play a key role in cognitive change within the RR model.

Chapter 2 Exploring children’s representations of balance using different balance tasks

Introduction

How generalisable are children’s representations? According to Karmiloff-Smith (1992), children’s representations as they develop should become more consciously accessible and generalisable across tasks and domains. In this thesis, it is thought that gaining verbal access to the data is the key in being able to generalise knowledge across task. The aim of the current study is to begin to assess whether or not this is in fact the case. An attempt will be made to apply the levels of the RR model to different types of balance tasks. This will serve 2 purposes: (1) To begin to show that children’s representations are generalisable, as the levels of the RR model can be applied to different types of balance task, and (2) to show what types of task the levels of the RR model can be applied to.

This introduction will begin with a brief recap on studies focusing on generalisability of knowledge within the RR model. Following on from this will be a description the research carried out by Pine et al. on the balance beam task. Other research on balance tasks will be discussed, with the aim of setting out balance tasks to which the levels of the RR model may be applicable.

1. Generalisability and the RR model

An important aspect of the RR model (Karmiloff-Smith, 1992) is that as children’s representations develop, they become more “cognitively flexible”, and consciously accessible. It is this access and flexibility which allows for generalisation of knowledge. Given that the key aspect of development described by the RR model is increasing verbal access to knowledge, it is likely that the process of increasing verbal access to knowledge plays a large part in this flexibility, and ability to generalize knowledge. This means that children should be able to begin to apply knowledge learnt from one task in one domain, to another task within that domain, or indeed to other domains. It is thought that verbalisable knowledge is what allows for generalisation; as this is the format in which information is redescribed into, and which is thought to be openly accessible in format; to other tasks in a domain, or indeed to other domains. The current focus however is on generalisation within a domain. The question of how and when children develop the ability to generalise knowledge is key. Piagetian theory for example states that children are at any one stage in time broadly applying the same form of thinking universally (Flavell, 1963). In his review of microgenetic studies, Siegler (2006) found relatively little evidence of generalisation – though the focus of these studies were mainly on strategies, which may not be easily generalisable, as "strategies" may only work on one type of apparatus or one type of task, and not be transferable to other tasks within a domain. For example, on the balance beam task used by Pine et al (1999), a strategy often used is for children to place the place the centre of a beam on the fulcrum. This strategy can not be used on a balance scale task, where children are asked to place weights on either side of the scale. Therefore, the focus for generaliseability is on explicit verbal knowledge, which may be used in relation to different tasks within a domain.

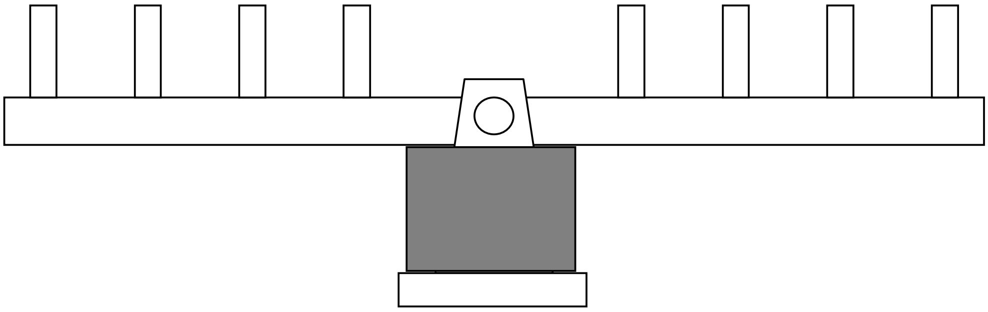
Studies by Pine et al. (2003) and Tolmie et al. (2005) have looked at generalisability within the RR model. Pine et al. (2003) looked at children's ability to predict whether or not they could balance beams, and found evidence that at an early developmental level (e.g. implicit transition), children showed access to knowledge in terms of being accurate in their predictions. Tolmie et al. (2005) on the other hand focused on generaliseability as occurring at the E3 level, as per Karmiloff-Smith's (1992) original description of the model. Given the findings of Pine et al (2003), there is now an open question as to when and how children begin to generalise knowledge. As Shultz (1994) notes, one of the problems with the RR model is that it has generally only been applied to one task within a domain.

Therefore, it is difficult to assess how "generalisable" representations are, and indeed to talk about the levels that emerge from a task as "representations" which cover an entire domain. The ideal way to assess generalisability therefore would be to apply the levels of the RR model to 2 tasks within a domain, and then to compare children's representational levels on these 2 tasks. A secondary aim is to see to what extent children use the same types of verbal explanations on different tasks within a domain. The aim of the current chapter is to apply the levels of the RR model to different balance tasks, to begin to assess generalisability in this way.

2. Applying the levels of the RR model to the domain of Balance

Pine et al. (1999, 2001, 2003) have successfully applied the levels of the RR model to a balance beam task, based on a task first used by Karmiloff-Smith and Inhelder (1974). This task involved asking children to balance beams of wood, with blocks on either end acting as weights, on a fulcrum. Children were asked to explain for each beam why it did or did not balance. Children's performance on the task and their verbal explanations provided the basis for coding Representational levels (see Table 1.1 in the literature review). Pine et al. (2001) have shown that for the balance beam task, children can display verbal knowledge at the first stage of abstraction (e.g. they can show explicit knowledge of weight, rather then having to wait until the last level before children display explicit verbal knowledge). As children's knowledge of the concept develops, this knowledge becomes available for conscious access, and can readily be applied to different types of task.

Figure 2.1: the balance scale apparatus



3. Other research in the domain of balance

So far, the levels of the RR model has only been applied to one type of balance task – Pine et al.'s (1999) balance beam task. Are these patterns of performance and verbal explanations unique to this specific task? This question is important given the variety of different theories which have used the concept of balance to illustrate their theories – e.g. Piagetian stage theory (Piaget and Inhelder, 1958), Siegler's Rule assessment methodology (Siegler 1976), relational complexity theory (Halford et al., 2003), and cognitive complexity and control theory (Zelazo, 2002). For the RR model to be judged a valid model, it should give a comprehensive description of the course of development of the concept of balance, across more then one different type of task.

In this experiment an attempt will be made to apply the RR model to different types of task. It must be clearly stated that the levels of the RR model can not be applied to every task. For example the levels of the RR model are not applicable to Siegler's balance scale task, as this task does not include a performance element. Messer et al. (2007) have also shown that task characteristics can play an important role in children's access to their knowledge. In light of this, the current experiment, an attempt will be made to apply it to different types of balance scale task, whose backgrounds are described next. One aim of the experiment will be to see how children's pattern of performance and verbal explanations differ on these different types of task.

3.1 The balance scale production task

The Balance scale task (Siegler, 1976) and Balance beam task (Pine et al., 1999) are not the only tasks that have been used to investigate children's concepts of balance. There have been a few other studies involving the balance scale which have utilised different methodologies. Messer et al. (in print) for example attempted to apply a similar methodology as used in the balance beam task used by Pine et al. (e.g. 1999) to the balance scale. The experimenter puts a number of weights on the pegs on one side of the scale. The child is given a specific number of weights to put on their side of the scale in order to make the scale balance and stay straight.

The scale used by Messer et al. (in print) had only 4 pegs on each side, and children were only given 2 weights at most. Ceiling levels of performance were found for this task, with a majority of children succeeding across all 6 trials given them. Children's performance was consequently not codable into R-R levels. This ceiling affect may be due to the limited number of pegs and weights given to the child, or to the small number of trials involved in the task. One aim of this experiment is to expand on this task – using a scale with more pegs on each side, and giving children a greater number of weights, to see if this eliminates the ceiling effects, and allows for the application of the RR levels.

3.2 The fixed-peg balance scale task

A second type of task which deviates from Siegler's balance scale task was introduced by Surber and Gzesh (1984). Surber and Gzesh (1984) carried out a balance task similar to a production task using the balance scale, with one subtle difference. For some of the trials, instead of being given a number of weights to place on their side of the scale, they are shown a specific peg, and asked how many weights would have to be placed on that peg in order to make the scale balance. This places a greater emphasis on the variable of distance from the fulcrum, and acts as a possible bridge between the balance scale and balance beam tasks.

One could therefore predict from the RR model that children with Abstraction verbal representational levels (see Table 2.4) should not be able to successfully balance the trials where they are asked how many weights should be placed on a specific peg, as they do not yet have any explicit knowledge of the role of distance in the concept of balance. A comparison with the normal type of production task could elucidate whether this type of task configuration either elicits different verbal data or different behavioural success levels. Unfortunately, Surber and Gzesh (1984) do not report any verbal data, or behavioural data in an easily interpretable format. One aim of the current study is to replicate this task, in order to see what patterns of performance and verbal knowledge emerge from this task, to see if the levels of the RR model can be applied.

3.3 The Unconstrained Balance scale task

Finally, a number of experiments have been conducted involving unconstrained access to the balance scale. Kliman (1987) for example used a loose interview format, which provided opportunities for free exploration and for some direct questioning. Children were free to place weights on both sides of the scale to explore different ways to make the scale balance and stay straight. A key advantage with a free or unconstrained format of course is that it shows what children will produce of their own volition. The direct questions of course also highlights areas that children may not necessarily explore themselves. This could also highlight the process of implicit knowledge – which is of course by Karmiloff-Smith's definition information that they do not know that they know (e.g. a balance configuration they can produce themselves, but which they can not explicitly explain). Kliman's study showed that children's knowledge was built on local observations, on individual cases rather then on generalisations. Oshima and Okada (1996) used a test involving a "hypothesis testing" element, where children were asked to test what the variables they thought were involved in balancing. Thus Oshima and Okada (1996) emphasise how goal-orientation played a part in knowledge acquisition, emphasising the utility of unconstrained tasks as offering the opportunity to develop new theories or test old notions. In the current study, an unconstrained balance scale task will be used, in an attempt to apply the levels of the RR model to this task.

Summary

The current study aims to apply the levels of the RR model to different types of balance task. This serves 2 aims: (1) To begin to show that the levels of the RR model are generalisable within a domain, by demonstrating that they can be applied to more than one balance task. (2) To look at what task characteristics are required in order for the levels of the RR model to be applicable.

Method

Participants

A convenience sample of 39 children from years 1 to 3 (see Table 2.1 for demographic information) in a mainly middle class Hertfordshire primary school participated in this experiment. Table 2.2 shows how many children participated in each of the 3 types of balance scale task.

Table 2.1: Demographic data

Year Group	Number of participants	Age range (months)	Mean Age (months)	Sex (Boy / Girl)
Year One	11	66-77	73.8	5 / 6
Year Two	14	80-89	85	8 / 6
Year Three	14	90-101	96.4	6 / 8
Total	39	66-101	86.1	19 / 20

Table 2.2: Number of children who performed each task:

Task	Number of children
Balance scale Production Task	13
Fixed Peg balance scale production task	12
Unconstrained balance scale task	14

Materials

A wooden scale was used for the 3 different balance scale tasks (see Figure 2.1). The scale consisted of a 60 cm long beam of wood, centred on a wooden fulcrum. There were 6 wooden pegs on each side of the beam at regular intervals of 5 cms. Seven identical metallic rings, which could be placed upon the pegs, were used as weights. No rests were used to stop the movement of the scale as children placed the rings on the pegs. All experimental sessions were recorded with a digital video camera to allow for later coding of the data, using the computer-based Noldus Observer system.

Procedure

Children were brought from their class to a quiet room where the apparatus was set up. The child sat in front of the balance scale, with the video camera facing them. The child was shown the balance scale apparatus, and the weights, which were to be placed on the scale. The child was then introduced to the puppet. The children were told that they would be asked to show to the puppet how the scale works, and explain it to the puppet. The experimenter then explained to the child the specific task they were going to be asked to do. Each subject was randomly assigned to one of the 3 experimental tasks used. The sessions were all videotaped, with the recording commencing prior to the children starting the task.

1. Balance Scale Production Task

This task was based on the procedure used by Messer et al. (in press). The experimenter explained to the child that the he would place rings on one side of the scale. The child was given a specific number of rings to put on the other side of the scale, to try and make the scale balance and stay straight. There were 2 types of trial; (1) Symmetrical trials, where children were given the same amount of weights as are placed on the scale by the experimenter. It is possible in this trial to balance the scale by copying the figuration laid out by the experimenter. (2) Asymmetrical trials, where children were given either more or less weights then the amount the experimenter places on his side of the balance scale. The children must compensate for this weight difference by considering the distance from the fulcrum on which to place the weights. Children were allowed make as many moves of the rings as they wanted in order to make the scale balance. A move is defined as occurring when all the rings the child has have been placed on their side of the fulcrum.

Children were asked to balance 6 symmetrical and 6 asymmetrical trials, ranging from a simple symmetry of 1 weight on each sides, to a complex asymmetrical state where there are 3 weights on the experimenter's side, and the child is given 4 weights to place on their side of the beam in order to make the scale balance and stay straight.
For each trial, the child is asked to explain to the puppet why the scale did or did not balance.

2. Fixed Peg Balance Scale Task

This task was based on the experimental procedure used by Surber and Gzesh (1984). There were 2 types of trial. The first type of trial used exactly the same procedure as described in the first task, using a limited number of 3 symmetrical and 3 asymmetrical trials. In the second set of trials, instead of being given a specific number of weights to place on their side of the scale, the child was directed to a specific peg, and asked how many weights would have to go on that peg in order to make the scale balance and stay straight. There were 3 symmetrical and 3 asymmetrical trials; symmetrical trials involved the experimenter pointing at the same pegs on the child's side of the scale as the pegs on which the weights were on his side. Asymmetrical trials, involved the experimenter pointing to different pegs on the child's side of the scale than the pegs on which the weights were on his side. For each of these trials, children were asked to explain to the puppet why the scale did or did not balance and stay straight.

3. Unconstrained balance scale task

For this task, children's manipulation of the balance scale was unconstrained. Children were encouraged to place weights on both sides of the scale in order to find out all the different ways in which the balance scale could be made to balance and stay straight. Children could produce both symmetrical and asymmetrical balance combinations. As they went along, the child was encouraged to explain their movements to the puppet – questions such as *"why is the scale balancing now?"* and *"why is that side going down?"*

were asked to ascertain children's verbal knowledge for the concept of balance. When the child had exhausted the balance possibilities, they were asked 2 specific questions, based on specific balance situations set out by the experimenter (e.g. with weights placed on both sides of the scale by the experimenter). First they were asked about what would happen if there were more weights on one side of the scale. They were asked to explain their answer. They were then asked what would happen if the same number of weights were on both sides of the scale, but placed on different pegs. Again, they were asked to explain their answer. These questions helped to address situations which the child may not have shown or explained during their exploration of the scale.

For all three types of task, when they were finished, the children were thanked for their participation, and asked whether they had any questions. The camera was then switched off and the child was brought back to class.

Measures

Performance was measured along a number of dimensions: verbal explanations offered, types of successful balance combinations produced (e.g. ability to balance symmetrical, asymmetrical configurations), and number of moves used (in the two constrained tasks).

Verbal Explanations

In all 3 balance tasks, children were asked to explain why the scale did or did not balance. Their verbal explanations were coded into categories (see Table 2.3). In the unconstrained balance scale task, children were encouraged to produce explanations and provide a commentary as they were manipulating the scales. In the constrained task the children were asked to provide an explanation following their success or failure in balancing a specific trial. For all tasks, the children were asked to explain themselves so that a puppet, who was trying to learn about balance, might understand better how to make things balance. The coding scheme used by Pine et al. (2000) was used as a basis for the current study. The weight and distance categories denote an explicit level of explanation. For the balance scale tasks, a number of adjustments were necessary. First was a superficial change of the “middle” category to a “same” category (e.g. instead of saying that the beam was in the middle for the asymmetrical beams, children note that the scales have the same configurations on both sides of the fulcrum). Second, a large number of explanations were offered which involved mention of either weight or distance which did not seem to go beyond procedural knowledge – in other words explanations that did not seem to merit being classified as “explicit conceptual knowledge”. Table 2.3 below shows examples of these types of implicit weight and distance explanations. These implicit types of explanation seem more procedural in nature, rather than demonstrating explicit knowledge of the role of weight and distance in the concept of balance.

Table 2.3: Coding scheme for verbal explanations offered in the balance scale tasks

Coding category	Examples of explanations
Implicit	“I don’t know” “just a guess”, doesn’t offer an explanation
Implicit (weight)	“I put one on”, “there are 2” (explanations that are purely numeric, without reference to weight, or weight-based terms)
Implicit (distance)	“I put it on that one” “on the fifth one” “I put it there” (reference to the pegs, without reference to relative distance in relation to weights on the other side, or relative to the fulcrum)
Weight	“it’s a bit heavy”, there aren’t the same amount of weights”
Distance	“it’s closer to the start”, “I put it on the end”,
Middle/Same	“it’s in the middle”, “same on both sides” “it’s symmetrical”

Type of successful balance combinations produced

Of particular interest is children’s ability to balance symmetrical and asymmetrical balance configurations; the amount successfully balanced is important in determining levels within the RR model, with an inability for example to produce any asymmetrical balance, along with weight based explanations, indicating performance a level of abstraction (abstraction verbal). Table 2.4 sets out the criteria for coding performance and verbal explanations into RR levels, using a simplified 3 levels of representations.

Table 2.4: Criteria for coding RR levels on the balance scale tasks

Representational Level	Balance Performance	Verbal explanations offered
Implicit Performance	Balances both symmetrical and asymmetrical configurations	No explicit explanations offered
Abstraction Verbal	Balances symmetrical, but 2 or less out of 6 asymmetrical trials	Weight based explicit explanations offered
E3	Balance symmetrical and asymmetrical configurations	Weight and distance based explicit explanations offered

Number of Moves

Because children were given more than one chance to make the scale balance and stay straight, it is of interest to see how many moves it takes a child to make the scale balance, in order to check whether children's successful balances are due to trial and error, or due to purposeful and systematic placement of the weights on their side of the scale. A move is defined as being completed when the child has placed all his/her weights on their side of the scale. This measure is of course not possible within the unconstrained task, due to the fact that the child may not necessarily place all the weights on the scale, leaving it impossible to define a "completed" move.

Reliability of the coding schemes for verbal explanations and RR levels

For the balance scale tasks, a selection of nine children, three for each of the different types of balance task were chosen. For each of the three tasks, the second coder coded the verbal explanations into the categories described in Table 2.3, and for the unconstrained balance task, the second coder coded children to RR levels based on the criteria set out in

Table 2.4. For the verbal explanations, there were agreement levels above 80% for all three balance tasks (see Table 2.5). There was agreement for RR levels on all 3 of the unconstrained balance scale tasks.

Table 2.5: reliability of coding of verbal explanations on the 3 balance tasks, and RR levels on the unconstrained balance task

Task	Level of agreement on verbal explanations (Kappa statistic in brackets)	Level of agreement on RR levels (Kappa statistic in brackets
Balance scale production task	91.66% agreement (kappa = .8897, p < .001)	-
Fixed peg balance scale task	88.88% agreement (kappa = .861, p < .001)	-
Unconstrained balance task	84.21% agreement (kappa = .811, p < .001)	100 % agreement (kappa = 1, p = .014)

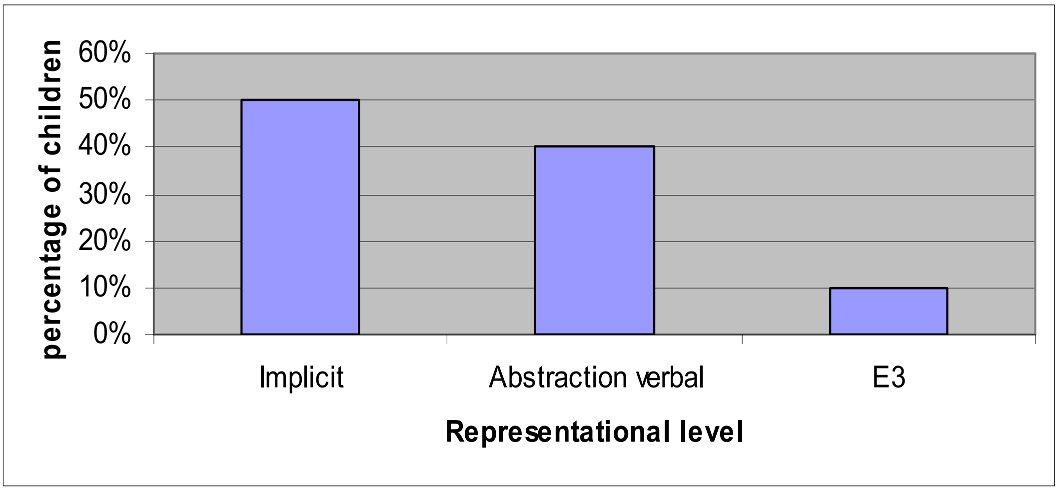
Results

The results are split into 2 sections: (1) Application of the levels of the RR model to the balance scale tasks. (2) Performance on the tasks to which the levels of the RR model were not applicable.

1. Applying the Levels of the RR model to the 3 balance scale tasks:

The levels of the RR model were only applicable to one of the three balance scale tasks – the unconstrained balance scale task. Figure 2.2 below shows the distribution of these levels, highlighting the fact that most of the children have not achieved an explicit understanding of the concept of balance. A substantial group of children could produce both symmetrical and asymmetrical balance configurations, without being able to verbally explain why the scale balanced. A second group of children could only produce symmetrical balance configurations, and provided weight-based explanations. 2 children who were given this task could produce symmetrical and asymmetrical balance configurations, and provided explanations mentioning the role of weight and distance in determining how a scale could balance.

Figure 2.2: Representational levels coded for the unconstrained balance scale task



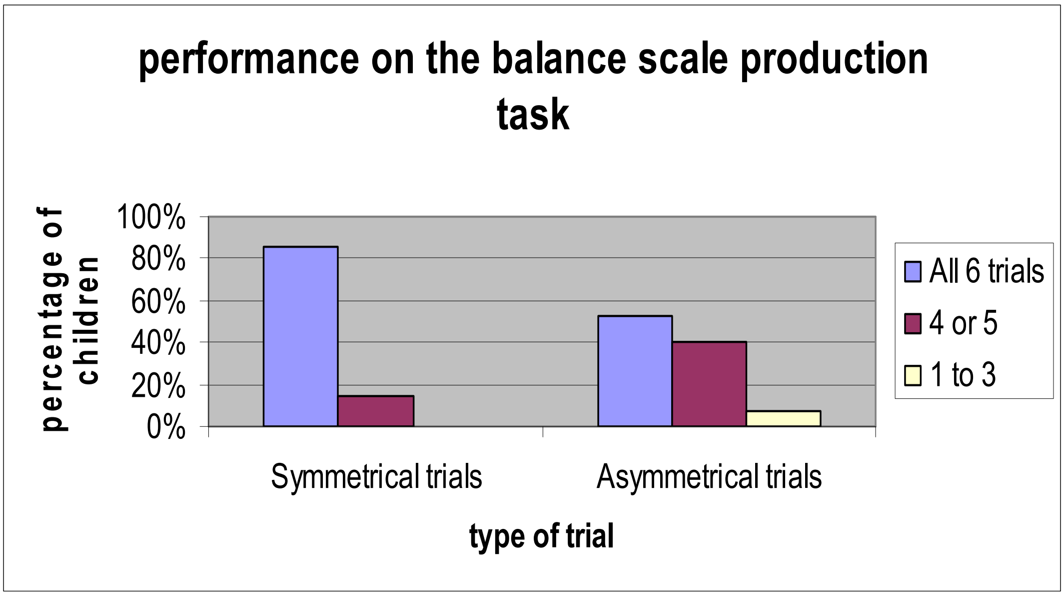
2. Performance on the tasks to which the levels of the RR model were not applicable

The 2 tasks were analysed in terms of children's performance on the task, and the verbal explanations used.

2.1 Children's performance on the Balance Scale Production Task and the fixed peg production task

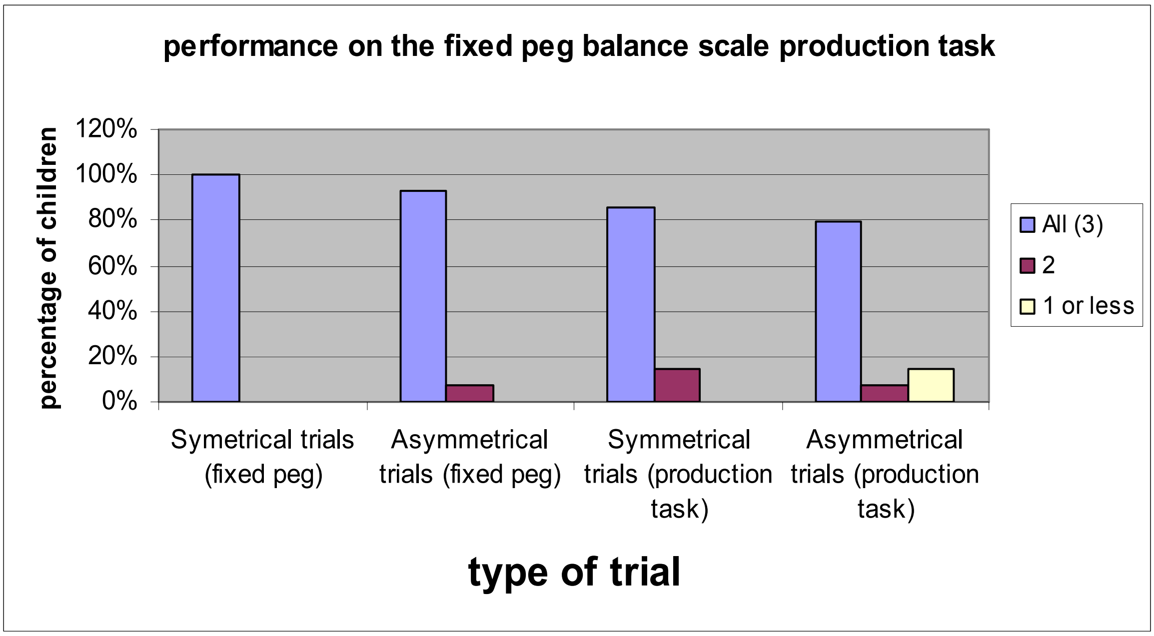
For the balance scale production task, children showed close to ceiling levels of performance in terms of being able to balance the scale in symmetrical trials, and is very high for the asymmetrical configurations (See Figure 2.3). Only one child showed any significant inability to make the scale balance, only managing to make one asymmetrical configuration balance. All other children balanced at least 4 of the 6 asymmetrical trials given to them.

Figure 2.3: Performance on the balance scale production task



For the fixed peg balance scale construction task, the child undertook 6 trials which involved the experimenter pointing at a fixed peg and asking how many weights would have to be put on that peg in order to make the scale balance. The other 6 trials followed the balance scale production task, above and these two sets of results are separated (see Figure 2.4). For the normal balance scale production trials, the same pattern is repeated as is seen in Figure 2.3, with a majority of the children managing to balance all the trials, both symmetrical and asymmetrical. The same pattern repeats with the fixed-peg trials, with a majority of children again being able to balance the scale on a majority of these trials.

Figure 2.4: Performance on the fixed-peg balance scale task



2.2. The number of moves used per trial

Children showed ceiling levels of performance in these 2 tasks. Was this due to children making multiple moves per trial, until they managed to get the scale to balance? The focus in particular is on the asymmetrical configuration trials. A move is defined as a situation in which all the weights given to the child have been placed on the scale. The aim in looking at the number of moves made is to try and pin down children who managed to make the scale balance through trial and error – at what stage though can we move from describing children's moves as purposeful through to mere trial and error? Figure 2.5 shows the number of moves children made on the asymmetrical trials on the balance scale production task. A majority of children successfully balanced these asymmetrical trials within 3 moves, indicating that children weren't merely continuing to guess and make moves until the scale balanced.

Figure 2.5: Number of moves taken to successfully balance asymmetrical trials on the balance scale production task

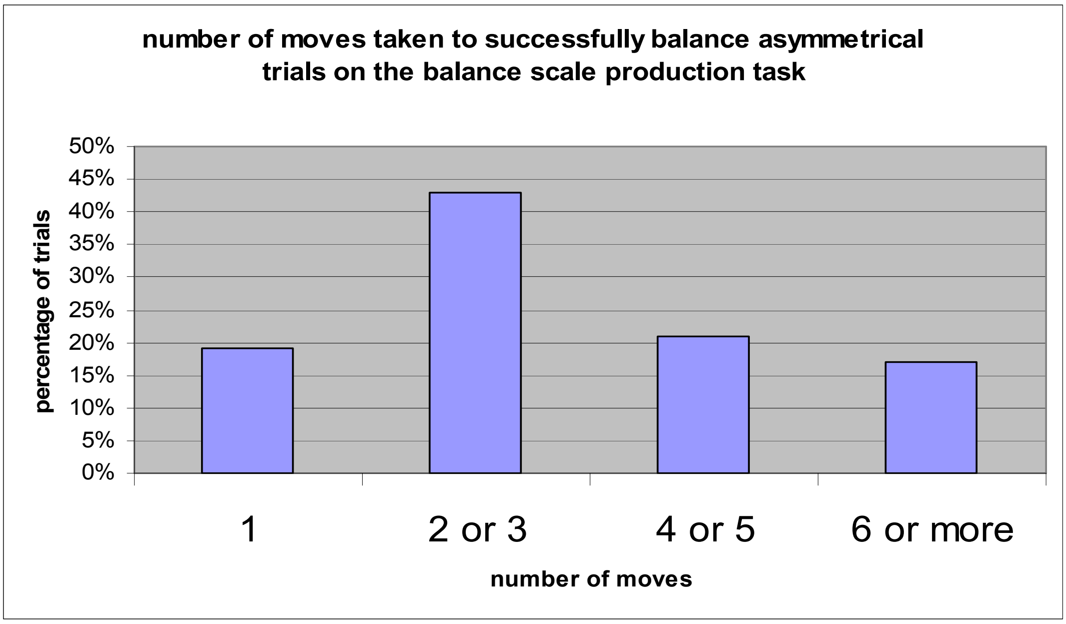


Figure 2.6 shows the same finding for asymmetrical the trials in the fixed peg production task that did not involve a fixed peg. Finally, Figure 2.7 shows the number of moves required to solve the 3 asymmetrical fixed peg trials. A clear majority of children managed to balance these trials within 2 moves, indicating that trial and error did not play a major part in their successful performance on these 2 tasks.

Figure 2.6: Number of moves taken to successfully balance asymmetrical trials on the non-fixed peg trials for the fixed peg balance scale task

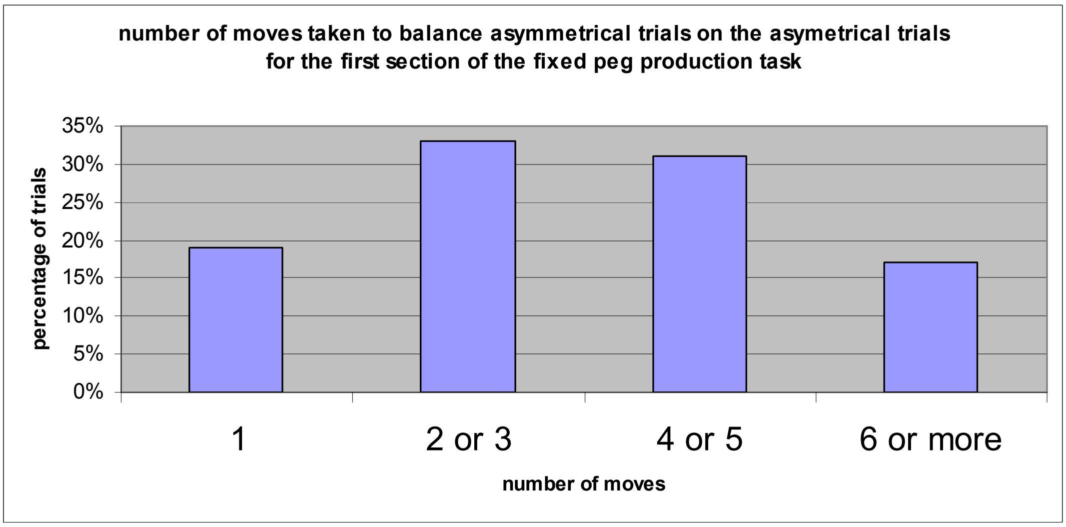
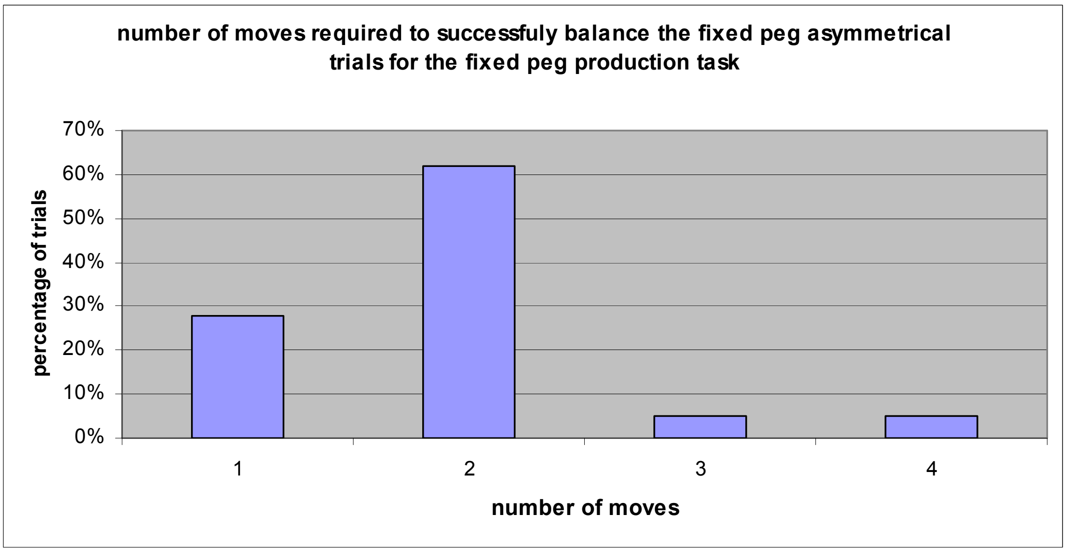


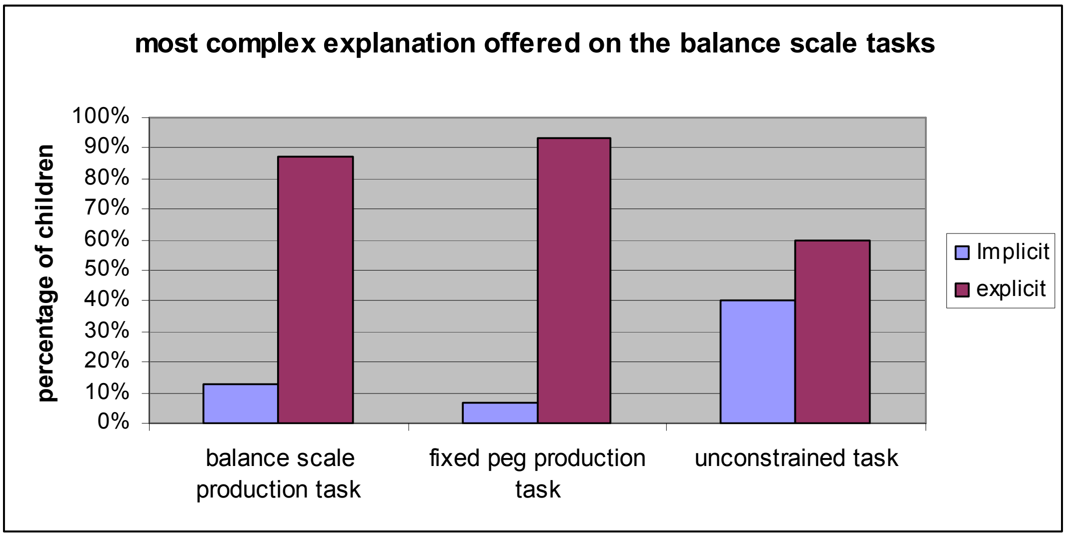
Figure 2.7: number of moves taken to successfully balance asymmetrical trials on the fixed peg balance scale task



2.3.Verbal explanations offered

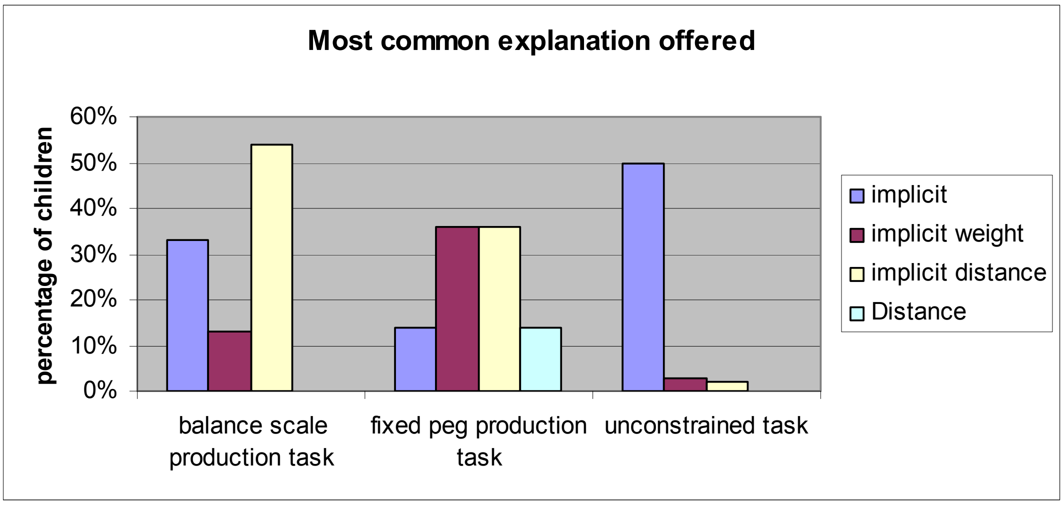
It is clear from above that children showed ceiling levels of performance on these 2 particular balance tasks. What patterns emerge for the verbal explanations offered? Figure 2.8 below shows the most complex form of explanation offered, focusing on whether or not children produced an explicit type of explanation (weight, distance, or same). Because of the relatively small occurrence of distance explanations generally given, and the fact that the main focus was on the occurrence of explicit verbal explanations rather than the content, a simple distinction was made between whether the most complex verbal explanation was implicit or explicit. A majority of the children showed some ability to produce an explicit type of explanation, regardless of the nature of the task. The largest occurrence of children who did not produce explicit explanations was in the unconstrained production task.

Figure 2.8: Most complex explanation offered by children on the 3 balance tasks



While children were capable of producing explicit explanations, they were by no means always the most common type of explanation offered, as Figure 2.9 below shows. For the 3 different tasks, different patterns arise. For the first, balance scale task, all the most common explanations are implicit, with a large number of children giving implicit distance explanations most commonly. A similar pattern holds for the fixed peg production task, though in this case, implicit weight explanations are as common as implicit distance. It is interesting to note for 2 children, explicit distance was the most common explanation offered. For the third, unconstrained balancing task, the pure implicit explanation was by far the most common type offered.

Figure 2.9: Most common verbal explanations offered by children on the 3 balance tasks



Clear differences can be seen in performance on these 2 balance tasks – but there are no strong clear differences in terms of the verbal explanations offered.

Discussion

The aim of the current study was to begin to apply the RR model to different types of balance task. This served 2 purposes – first, to show that the levels of the RR model could be applied to different tasks within a domain, and to begin to assess to what extent children's representations may generalise. The second aim was to look at the task characteristics necessary in order for the levels of the RR model to be applied.

1. Applying the levels of the RR model to different balance tasks

The levels of the RR model were applicable to the unconstrained balance scale task. The same patterns as seen by Pine et al. (1999) on the balance beam task seem to also apply to this particular balance scale task. This shows that these levels are NOT unique to the balance beam production task. The same patterns of performance are seen, alongside the same types of verbal explanation. This implies that the representations for balance can emerge following the pattern set out by the RR model on more than one type of balance task. This provides an important first step in showing that representations are generalisable. Children may therefore be less variable then the overlapping waves model implies – i.e. they may show the same types of RR level, and therefore the same levels of performance, and in particular the same levels of verbal knowledge across tasks within a domain. It is important to note however that the current study focuses on generalisation of the concept across tasks, rather then generalising to different domains, in order to avoid an interpretation of the current research stating that the RR model is a domain general rather then a domain specific model (though Pauen and Wilkening [1997] report some evidence of children being able to make analogies between a balance task and another physics based task).

This work marks an important step in validating the RR model as a potential model for general cognitive development. It is not constrained by being applicable to only one single type of task. The criticisms levelled by Wilkening and Anderson (1982, 1992) against Siegler's balance scale prediction task, and the rule-based model are not applicable, as children are given multiple opportunities, through providing verbal explanations, and through interacting with the scale as part of the trials, to show their levels of knowledge about the concept of balance. Similarly, the fact that the levels arise from 2 very different types of task (e.g. Pine et al.'s balance beam task and the unconstrained balance scale production task) are a strong defence against the arguments levelled by Siegler against the balance scale task he used (Siegler, 1976) being levelled against the RR model.

Therefore, there is some evidence for the validity of the levels of the RR model as a potential general model, due to their being applicable to a number of different types of task, rather then simply being a pattern unique to one specific task. The next step in this process of validation is to analyse children's representational levels across the unconstrained balance scale task and the balance beam task (Pine et al., 1999). If children's representational levels capture the development of knowledge across a whole domain, children should the same representational levels across these 2 tasks. This will be the focus of the next chapter.

It is worth noting in relation to generalisation that the same types of verbal explanations, derived from Pine et al (1999) was applicable to all 3 tasks, with a majority of children giving explicit explanations in all three tasks. This implies that the same verbal knowledge may well be applicable across different tasks within a domain. It is also worth noting that in the unconstrained task, once again children showed access to verbal knowledge prior to reaching E3; therefore, they showed an ability to generalise and access verbal knowledge prior to achieving the final level of development – meaning that children can generalise inaccurate knowledge.

2. Task characteristics necessary for the application of the RR model

Although the levels of the RR model were applicable to one of the 3 balance tasks used here, they were not applicable to the other 2 balance tasks used. In order to understand better the characteristics necessary for a task for the levels of the RR model, two comparisons will be made – first a comparison between the unconstrained balance scale task, and Pine's balance beam task, and second a comparison between the unconstrained balance scale task and the other 2 balance scale tasks used in this experiment.

There are clear differences between the unconstrained balance scale task and the balance beam task used by Pine et al. (1999). First, there are obvious differences between the apparatus used. There are also differences in the methodologies, with children being given free access in the unconstrained balance task, whereas in the balance beam task, children are given specific beams which they attempt to balance. There are clear differences in terms of perceptual features and task characteristics (Messer et al., 2007). Nonetheless, the levels of the RR model are visible in both types of task.

Comparing the unconstrained balance scale task with the other two balance scale tasks used in this experiment, the only clear differences are in terms of methodology. Children displayed behavioural mastery on both the balanced scale production task, and the fixed peg construction task. This was not the case with the unconstrained balance scale task, with a number of children failing to produce asymmetrical balance configurations on the unconstrained task. What is the reason for this? The key difference seems to arise from the methodologies – in the unconstrained task, the child must attempt to produce asymmetrical configurations spontaneously, whereas on the other 2 tasks, children were given a clear number of opportunities to balance asymmetrical configurations. Another key factor is the introduction of "implicit weight" and "implicit distance" verbal explanation categories. These categories captured children who gave purely numeric explanations (e.g. "I put 2 on" or "I put it on the second peg"), which seem to be more procedural in intent, rather then actually focusing on the role of weight and distance in causing the scale to balance.

This may be the key difference – on the other 2 tasks, children seem to be able to successfully create symmetrical and asymmetrical balance configuration, without necessarily having any explicit knowledge of the concept of balance. On the other hand, children’s performance on the unconstrained balance task is coloured by what they think is possible – some children for example do not even attempt to create asymmetrical configurations. The unconstrained balance task seems to be the type of task where children’s verbal knowledge can play a part in constraining their performance on the task – this seems to be the key similarity between the unconstrained balance scale task and Pine’s balance beam task. This characteristic also seems to arise in the spelling tasks used by Critten et al. (2007). Therefore, there is a need within this thesis to investigate the applicability of the levels of the RR model to tasks which do not have this characteristic.

While Siegler (1996) notes that the Rule Assessment Methodology may be based on specific characteristics of the task, this need not necessarily be looked on as purely a negative thing. The levels of the RR model may only be applicable to certain types of task, where performance is directly effected or constrained by their knowledge, whether verbalisable or not. These types of task may however be the most suitable for developing thinking within that domain. Certainly it seems logical to state that children are most likely to develop fully explicit representations using the constrained balance scale task used in this study rather than either of the other 2 balance scale tasks, even though children showed higher levels of performance on those 2 tasks. There is a need to investigate further the types of task to which the levels of the RR model can be applied.

Summary

Two important conclusions emerge from this study. First, the levels of the RR model are not unique to the balance beam task within this domain. This goes some way to showing that the levels of the RR model can be used as a general model for the development of a domain, rather than simply showing the development of competence for a particular task. This acts as a first step in looking at whether or not children’s representations are generalisable to other tasks within a domain. Verbal knowledge in particular is shown to be similar across the 3 different tasks with a majority of the children providing similar explicit type explanations across both tasks. The second finding is that for the levels of the RR model to be applicable to a task, children’s performance may have to be in some way constrained by their levels of knowledge (e.g. tasks where ceiling levels of performance do not become apparent).

Chapter 3: Generalisability within the RR model: Comparing children's representational levels across balance tasks

Introduction

Do children's representations capture a child's level knowledge for a domain? Do children show generalisation of knowledge across tasks within a domain? In this experiment, the aim is to look at children's representations on 2 balance tasks in order to address these questions.

Children's representations for domains

The RR model (Karmiloff-Smith, 1992) describes the development of children's representations for domains. These representations are thought to capture children's knowledge for a domain, rather then a single task within that domain. It is important to note that these representations initially develop through interaction with a single task – this model describes a route of development whereby behavioural mastery precedes explicit verbal knowledge for a domain. Children's first implicit representations for a domain are initially tied to the task which children are introduced to. Implicit representations do not allow for "*intra-domain" or "inter-domain" representational links"*(Karmiloff-Smith, 1992, p 20). A child who has an implicit representation cannot use the approach learned from one task to another in the same domain, or in other domains. As children's representations become more explicit, and consciously and verbally accessible, they become generalisable, so that these representations may be thought of as capturing children's general knowledge about the domain. For example, if children show fully explicit representational levels on one balance task, they should similarly show the same level of knowledge on other balance tasks. This brings into question the status of children's representational levels – do children's representational levels arising from one task, a balance beam task for example (Pine et al., 1999) tell us about children's general knowledge for the domain of balance? From what has been outlined above, children should, once they have explicit representations on one task, be able to show that same explicit representational level on another task. If they display implicit representations for one balance task, one can infer from the lack of explicit generalisable knowledge from one task that they should similarly show only implicit types representations on the other task. It is important to note however that in the current experiment, explicit verbal knowledge is expected to arise at an earlier point in time developmentally than Karmiloff-Smith's RR model originally states. At the first level of abstraction, children have been found to give explicit verbal explanations on the balance beam task, and are likely to as well for the balance scale task.

Another issue, raised by Messer et al. (2007) is that perceptual and task characteristics may affect children's cognitions, so that they may display different cognitions depending on the tasks in front of them. One aim of the current study therefore is to compare children's representational levels on different balance tasks to see whether or not these representational levels may be used as a domain general measure of children's knowledge about a concept.

This question is especially important in light of the fact that other contemporary theories for development, such as the Overlapping Waves Model (Siegler, 1996) and Dynamic Systems theory (Thelen and Smith, 1994) have moved away from level-type models in favour of models which characterise children as being much more variable in their thinking. There is a need to establish the utility of representational levels, by showing that children do show the same representational levels across different balance tasks, establishing that children can and do show consistency in their thinking across different balance tasks. As Shultz (1994) notes, the levels of the RR model have not been applied across tasks within a domain, and therefore this particular aspect of the RR model has yet to be addressed experimentally.

Do children show generalisation in verbal knowledge across balance tasks?

If a child has verbalisable, explicit knowledge which they apply to one balance task, within the RR model they should be able to apply the same knowledge in other balance tasks. The opposite should also apply – if a child has not got explicit verbal knowledge in one balance task, they should not have explicit verbal knowledge for the other balance task, otherwise, they would be able to generalise it. Findings from other studies are not equivocal however. Pine et al. (2003) showed that children could access their knowledge by being able to predict whether or not they would be able balance the beams presented them. This can be interpreted as showing that they have access to knowledge, but is not, strictly speaking, being applied to a separate task. Tolmie et al. (2005) on the other hand focus on generalisation occurring when children reach E3 representational levels, as per Karmiloff-Smith's (1992) original description of the RR model. The current study also aims to look at whether children generalise verbal knowledge from one balance task to another – do they show similar levels of verbal knowledge on the 2 balance tasks prior to reaching E3, as Pine et al. (2003) imply, or does generalisation not occur until children achieve E3 representations, at which point they would provide weight and distance based explanations on both tasks

There are also the findings related to children's drawing to take into account. As described in the literature review, there are contrasting findings on the degree that children can generalise knowledge across tasks within a domain, with findings indicating both that children can (Barlow et al., 2003), and cannot (Hollis and Low, 2005) generalise skills used in one drawing task to another. The question arising from this is – what does actually generalise? Do children show similar types of procedures in tackling single tasks within a domain, or does only verbal knowledge generalise?

The previous study used an "implicit weight" verbal explanation category on the unconstrained balance scale task. One important aim of this study is to see if this same type of verbal explanation also arises for the balance beam task used by Pine et al. (e.g. 1999). This will provide further evidence about the generalisation of knowledge across tasks, especially in relation to differences in the perceptual and task characteristics between the 2 balance tasks to be used.

The question of generalisation is again important in light of the fact that Siegler (2006) reports that microgenetic studies conducted in relation to the overlapping waves model, whose findings tended to show that children do not show generalisation across tasks. Does this same finding apply in the case of the verbal knowledge that children access on different balance tasks?

Summary

In summary, the aim of this current experiment is to address the complimentary questions of whether children's representations capture their general knowledge for a domain, and whether children show generalisation in verbal knowledge across 2 balance tasks.

Method

Participants

A sample of 87 participants in years 1 to 3 was recruited from 4 primary schools in the Hertfordshire area. These schools were predominantly middle-class schools in towns surrounding the university of Hertfordshire. Children's ages ranged from 5 years 6 months to 8 years and 8 months, with a mean age of 87.4 months (standard deviation = 11.62 months). The sample comprised of 48 males and 39 females.

Materials

Two sets of balance apparatus were used, a balance scale, and balance beams. The balance scale consisted of a long wooden beam balanced at the geometric centre (see previous chapter for further details). The wooden beam had 5 equidistant pegs on either side of the fulcrum, on which a number of metallic weights (seven in total were available) could be placed (see Figure 3.1).

Figure 3.1: The Balance Scale Apparatus

The balance beam task involved a series of long wooden beams ranging in length between 25cm and 45 cm (see appendices for experimental sheets used, and a full description of the different beams used). Wooden or metallic blocks were placed at either end of the beams acting as weights. Nine beams in all were used, of which three were symmetrical (e.g. the same number of weights on either end of the beam, see Figure 3.2) and six were asymmetrical (e.g. there were more weights on one side of the beam than on the other, see Figure 3.3). These beams could be placed on a fulcrum which consisted of a raised plane of wood, 1cm above a wooden board, and 1 cm in width. Children were asked to try and place the beams across this fulcrum in such a way as to make the beam balance and stay straight.

Figure 3.2: example of a symmetrical balance beam

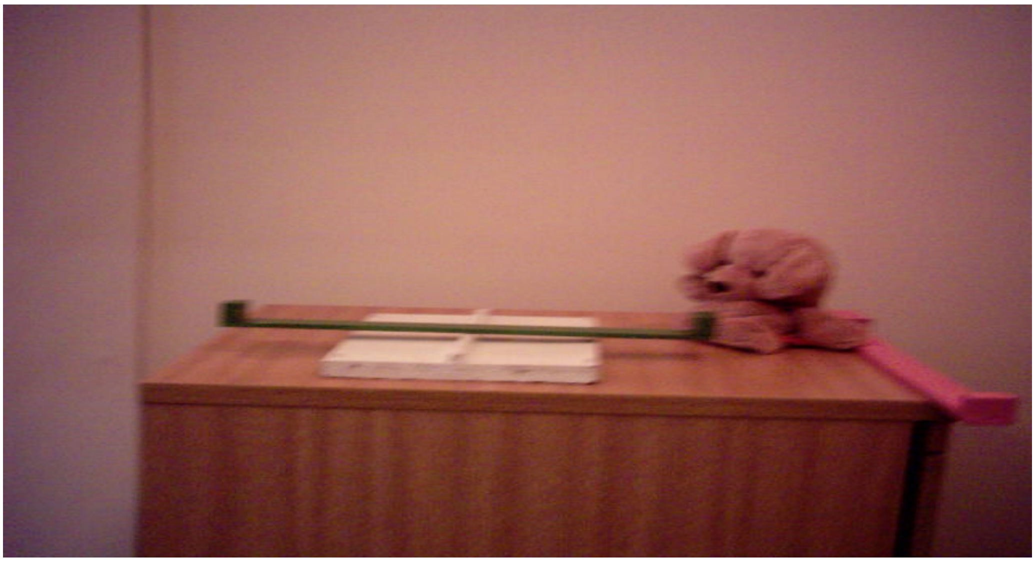


Figure 3.3: example of an asymmetrical balance beam

Procedure

Children were taken from their classroom to the room in which the experiment was conducted. They were asked to sit at a Table where the balancing apparatus and a video camera were set up. On the way, the experiment was explained in general term to the child – they were told that they would be playing a few “balancing games”. The child sat at the Table, and was shown the two types of balance tasks. The puppet sitting beside the camera was introduced to the child, and the child was told that the experimenter would like them to try and help the puppet learn about how to make things balance. The video camera was pointed out to the child, who was told that the experiment would be recorded so that the puppet would be able to watch it over again later to see how the child made everything balance. The child was asked whether he/she had any questions, and then the video camera was switched on and the experiment commenced. The order of presentation was counterbalanced, so half of the participants performed the balance scale task first, and the other half performed the balance beam task first.

Procedure for the Balance Beam Task

For the balance beam task, the child was shown the fulcrum, on which children were asked to place the beams in such a way as to make them balance and stay straight. It was emphasized that they would be trying to show the puppet how to make the beams balance and stay straight. Verbal explanations were also to be directed towards the puppet. For each beam, the child was told:

"I would like you to try and make this beam balance and stay straight on top of this piece of wood here (the fulcrum)".

The beam was handed to the child, and they were encouraged to place the beam on the fulcrum. Children were given 9 beams; 3 symmetrical and 6 asymmetrical (see appendices for full details of the beams). The order of the beams was partially counterbalanced. Half of the children were presented with a symmetrical beam as the first beam, and half of the children were presented with an asymmetrical beam as the first beam. The order of presentation of the rest of the beams was randomised. When the child successfully balanced or failed to balance a beam, they were asked to explain to the puppet why the beam balanced, or why it didn't balance. Children were given every opportunity to balance the beams, and were only asked to provide an explanation after they had stopped manipulating the beam for more than 30 seconds, or if they stated that the beam could not be made to balance.

Procedure for the Balance Scale Task

For the balance scale task, children were shown the scale apparatus, and the child was presented with the metallic rings, which they were told could be used as weights and placed on the different pegs, and told:

"I would like you to play with the scale, and show the puppet all the different ways you can make this scale balance and stay straight."

Children were asked if they understood what they were to do, and then allowed to start the task. Children were encouraged to explain to the puppet as they went along the different balance configurations, and why they did / did not balance. Children were encouraged to explore the different permutations until they could not think of any other way to make the scale balance. This was taken as having happened if the child said they could not think of any other ways to make it balance, or they stopped moving weights on the scale. They were asked:

"Can you think of any other ways to make the scale balance and stay straight?"

When the child gave a negative reply to this question, the experimenter removed the weights from the child, and said that he had a few questions to ask the child about the scale:

"To make sure the puppet understands about how the scale balances"

The purpose of these questions was to ensure that children had a further opportunity to provide verbal explanations, in case they had not supplied many explanations spontaneously, and to specifically look at children's knowledge for the variables for the concepts of weight and distance with regard to balance. The experimenter placed one weight on the peg nearest the centre, on either side of the fulcrum. The experimenter stated that the scale was now balancing and staying straight. The experimenter then asked what the child thought would happen if an extra weight was placed on one side of the scale. The child was asked

"Will the scale still balance and stay straight, will it go down this side (points to the side of the scale on which the extra weight would be placed), or will it go down this side (the other side of the scale)"

The child was encouraged to think about this question, and was not permitted to manipulate the scale in order to come up with an answer. The child was asked to explain their answer to this question to the puppet. They were then asked a second question. The experimenter asked the child what they thought would happen if the weight on one side of the scale was moved to the second peg from the centre. The child was again asked:

"Will the scale still balance and stay straight, will it go down this side (points to the side of the scale on which the extra weight would be placed), or will it go down this side (the other side of the scale)"

Again they were asked to justify their answer to the puppet. When the child had completed the two balance tasks, they were thanked for their participation and asked if they had any questions. They were then brought back to their classroom.

Coding children's performance across the balance tasks

Following the experiment, the videotapes taken of the session were subjected to intensive coding using the computer-based Observer system (See Table 3.1). For the balance beam task, children's performance in terms of the number of symmetrical and asymmetrical beams balanced was coded, alongside the initial placement of the beam (e.g. did they place the beam initially at its geometric centre, or did they tend to place the beam off-centre). Children's verbal explanations and gestures were also coded.

Table 3.1: the variables coded on the balance beam and balance scale tasks

	Balance Beam Task	Balance Scale Task
Performance	Number of Symmetrical and Asymmetrical beams balanced	Number of Symmetrical and Asymmetrical balance configurations produced
Strategy	Initial placement of the beams (e.g. did children attempt to balance beams in their geometric centre)	Number of asymmetrical non-balance configurations produced (e.g. did children attempt to produce non-symmetrical balance configurations)
Verbal Explanations	Children's verbal explanations coded using a coding scheme (see Table 2)	Children's verbal explanations coded using a coding scheme (see Table 2)

For the balance scale task, children's performance was coded in terms of symmetrical balance configurations (e.g. a configuration where the scale balances, where children have placed the same amounts of weights on the same pegs on either side of the scale) and asymmetrical configurations (e.g. a configuration where the scale balances, where children have not placed the same amounts of weights on the same pegs on either side of the scale), as well as the number of general non-symmetrical configurations (e.g. where the scale did not balance) produced by the child. Children's verbal explanations were also coded for the balance scale task.

Coding children's verbal explanations

Children's verbal explanations were coded using a revised coding scheme derived from the coding scheme used in the previous experiment. The implicit – explicit continuum used by Pine et al. (2001) is maintained, alongside a new set of "implicit weight" and "implicit distance", categories. These categories arose from findings on the balance scale task that children tended to give numeric explanations such as "*I put two on*" or "*I put it on the second one*", which shows some indication of knowledge of weight and distance, but does not supply enough explicit knowledge to justify being categorised as showing explicit knowledge of weight or distance. That is to say, an inference must be made about the statement made by the child, which means it should not be coded as "explicit", though there is some sign of knowledge on the child's behalf. Table 3.2 below gives examples of the types of explanations coded within these different categories. The categories are formed in a quasi-hierarchical manner – distance is placed after weight as a variable not because it is more complex, but because it is less common for children to give distance explanations, so therefore it is more interesting when children in fact do give distance-based explanations.

Coding the children into representational levels requires a focus on the categories of explanation offered, but for the purposes of further analysis within the representational redescription model, the main distinction is whether or not children produced explicit explanations. The most complex form of explanation given by the child is used as the basis for coding children's representational level, as long as the child has provided this type of explanation on more then one occasion (for example, children will have to give more then one explicit weight explanation in order to be coded to the Abstraction Verbal level, see Tables 3.3 and 3.4).

Table 3.2: Coding scheme for verbal explanations on the two balance tasks

Coding Category	Examples of explanations
Implicit	Explanations which do not show any signs of explicit knowledge of the variables of weight or distance. Examples: "*I don't know*" "*I guessed*" "*I thought really hard*" "*I just tried*" "*it's the same as the others*"
Implicit (weight)	Explanations which enumerate the set of weights placed on the scale, or the number of blocks on the beam, without any use of explicit, weight-based words. Examples:*" I put one on" " "there are 2"*
Implicit (distance)	Explanations which describe numerically the peg on which weights were placed for the scale, or vague distance-based explanations which lack any explicit mention of distance based terms (e.g. centre, far, or near). Examples*:" I put it on that one" "I put it there" "one of them's on the last one"*
Same/ Middle	Explanations which focus on how the two sides are the symmetrical, or how the geometric centre of the beam was placed on the fulcrum. Examples: *"in the middle" "they're symmetrical"*
Weight	Explanations which give explicit mention of the weight of the blocks, or the weights placed on the pegs, or which give some mention of weight-related terms. Examples: *"It's a bit heavy" "there aren't the same amount of weights" "It doesn't have a weight at the other end"*
Distance	Explanations which focus on the relative placement of the weights in relation to each other, or in relation to the fulcrum. Examples: *"It's closer on this side" "I put 3 near the end" "They're not on the same peg"" "It's further out"*

Coding Children's RR levels on the balance beam and balance scale tasks

Tables 3.3 and 3.4 below contain the criteria used for assigning a child to representational levels on the balance beam and balance scale tasks respectively. These criteria are derived from the work of Pine et al. (2002). For the balance beam task, the important criteria include the number of asymmetrical beams successfully balanced (e.g. the beams stay balanced around the fulcrum after the child has finished manipulating them), the initial placement of the beams (e.g. whether the child placed beams at their geometric centre to start with, or whether they initially placed them off-centre), and the explanations offered. For the balance scale task, the important variables are the number of symmetrical and asymmetrical balance configurations produced, whether or not they showed a tendency to only produce symmetrical weight placements, and the type of explanations offered. The answers to specific questions about weight and distance at the end of the balance scale task further elicit the extent of their conceptual knowledge, in the event of children not giving any explanations during the balance scale task, and measures the extent of their explicit knowledge about the concepts of weight and distance with regard to the balance scale task.

Table 3.3: Criteria for coding RR levels on the balance beam task

RR Level	Number of beams balanced	Initial Placement of beam	Verbal explanations offered
Implicit	Successfully balances 2 of 3 symmetrical beams, and at least 4 of 6 asymmetrical beams	No bias in initial placement across the beams (e.g. initially places the beams in the geometric centre for less then 5 of the 9 beams)	No explicit explanations offered
Implicit Transition	Successfully balances 1 of 3 symmetrical beams at most, and at best 2 of 6 asymmetrical beams	No bias in initial placement across the beams (e.g. initially places the beams in the geometric centre for less then 5 of the 9 beams)	No explicit explanations offered
Abstraction Non-Verbal	Successfully balances at least 2 of 3 symmetrical beams, and at best 2 of 6 asymmetrical beams	Initially places the beam in the geometric centre for at least 6 of the 9 beams	No explicit explanations offered
Abstraction Verbal	Successfully balance all symmetrical beams, and 2 or less asymmetrical beams	Initially places the beam in the geometric centre for at least 6 of the 9 beams	At least two explicit Weight / centre based explanations offered
Explicit transition	Successfully balances all symmetrical beams, and 3 or more asymmetrical beams	Initially places the beam in the centre, with adjustment to place heavier side closer to the fulcrum, for at least 3 of the 6 asymmetrical beams	At least two explicit Weight/centre based explanations offered
E3	Successfully balances all symmetrical beams, and 4 or more asymmetrical beams	Initially places the beam in the centre, with adjustment to place heavier side closer to the fulcrum, for at least 3 of the 6 asymmetrical beams	At least two explicit explanations involving both weight and two explicit explanations involving distance

Table 3.4: Criteria for coding RR levels on the balance scale task

RR level	Number of balance configurations achieved	Placement	Explanations
Implicit	Produces both symmetrical and asymmetrical balance configurations	Produces both symmetrical and non-symmetrical arrangements on the scale	No explicit explanations offered at all, weight and distance questions may be answered, but without explicit explanation
Implicit Transition	Produces 2 or less symmetrical configurations, no successful asymmetrical configuration	Produces 1 or less non-symmetrical configurations	No explicit explanations offered, can answer weight and distance question correctly
Abstraction non-verbal	Produces 2 or more symmetrical configurations, no successful asymmetrical balance configuration	Produces 1 or less non-symmetrical configurations	No explicit explanations offered, can answer weight but not distance question correctly
Abstraction verbal	Produces 2 or more symmetrical configurations, no successful asymmetrical balance configuration	Produces 1 or less non-symmetrical configurations	At least one explicit Weight based explicit explanation offered, can answer weight but not distance question
Explicit transition	Produces 2 or more symmetrical configurations, and more then 1 asymmetrical balance configuration	Produces more then 1 non-symmetrical configurations	At least one Weight based explicit explanation offered, can answer weight and distance questions
E3	Produces 2 or more symmetrical configurations, and more then 1 asymmetrical balance configurations	Produces both symmetrical and non-symmetrical configurations on the scale	At least one explicit Weight and one explicit distance based explanation offered, can answer both weight and distance questions

Reliability of the coding schemes

To ensure the reliability of the coding schemes for verbal explanations and RR levels, a second coder recoded the verbal explanations and RR levels on the two balance tasks. Table 3.5 shows significantly high levels of agreement both in terms of coding verbal explanations, and assigning RR levels for both the balance beam task and the balance scale task.

Table 3.5: Reliability of coding of verbal explanations and RR levels for the 1-to-1 counting principal

Task	Level of Agreement of Verbal Explanations (Kappa Figure in brackets)	Level of Agreement of Verbal Explanations
Balance Beam Task	91.35% agreement (Kappa = .896, p < .001)	88.88% agreement (Kappa = .864, p < .001)
Balance Scale Task	85.1% agreement (Kappa = .821, p < .001)	77.77% agreement (Kappa = .727m p < .001)

Results

The results are split into 2 sections – (1) Children's representational levels across the 2 balance tasks (2) Verbal explanation offered by children across both tasks

1. Children's representational levels across the 2 balance tasks

Figure 3.4 below shows the distribution of representational levels on the two tasks. An initial perusal indicates that children do not show the same patterns of representational level on both tasks, with children being most likely to show Explicit transition representational levels on the balance scale task, with a much more even distribution of representational levels on the balance beam task.

Figure 3.4: Percentage of children allocated to each RR level on the balance scale and beam tasks

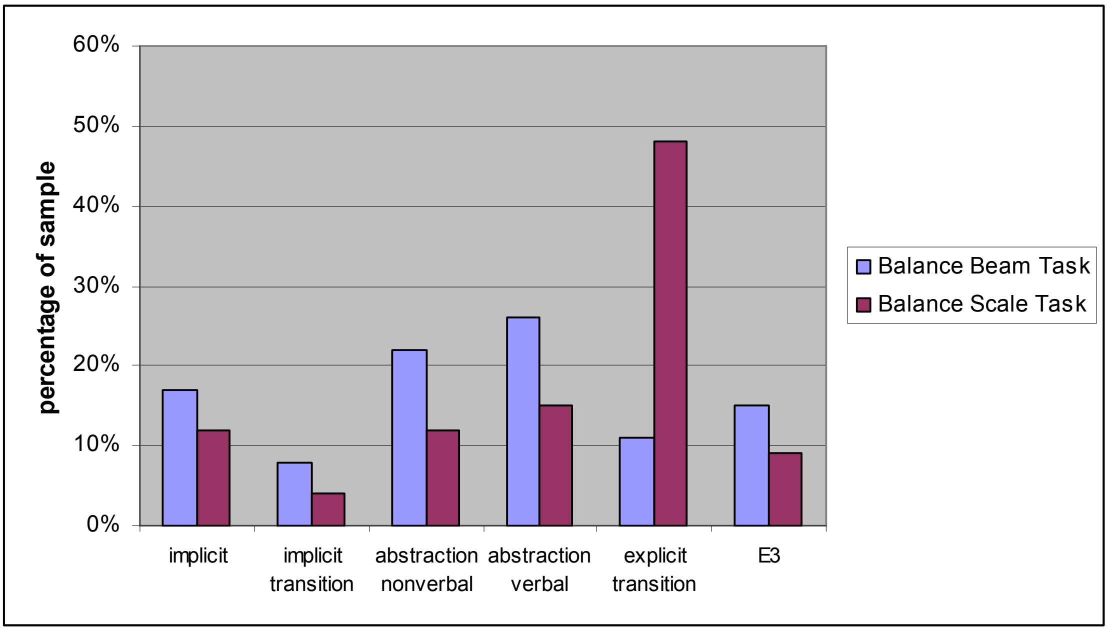


Table 3.6 below shows that very few children in fact showed the same representational level across both tasks. A chi square analysis χ^2 (25) = 52.12, p = .001, Cramer's V = .358, gives a significant result, but only 22 out of 84 children showed the same representational level across both tasks.

Table 3.6: Crosstabulation of children's RR levels on the balance scale and balance beam tasks

	Implicit (Balance scale task)	Implicit Transition	Abstraction Non Verbal (E1)	Abstraction Verbal	Explicit Transition	E3
Implicit (Balance beam task)	**3**	0	2	1	6	2
Implicit Transition	1	**0**	4	1	1	0
Abstraction Non Verbal (E1)	4	3	**2**	2	8	0
Abstraction verbal	1	0	1	**6**	13	1
Explicit Transition	0	0	1	2	**6**	0
E3	1	0	0	1	6	**5**

By collapsing the levels using a simple implicit-explicit dichotomy (with Abstraction nonverbal being classified as implicit, due to the fact that no explicit explanations are given) a further indication is given that children tend to have the same representational levels across both tasks (Table 3.7, χ^2 [1] = 15.5, p < .001, phi = .445). A majority of children showed the same basic type of representational level across both tasks (e.g. an implicit or an explicit level). However, a significant proportion of children were classified as implicit for the balance beam task were found to have explicit representational levels on the balance scale task. A large part of this finding seems to be due to the fact that so many children were classified at explicit transition level on the balance scale task. Is it simply due to differences in performance on the 2 tasks, or do children show different levels of verbal explanations on the 2 tasks?

Table 3.7: Crosstabulation of children's implicit and explicit RR levels on the balance scale and balance beam tasks

	Implicit Representational level for the balance scale task	Explicit representational level for the balance scale task
Implicit Representational Level for the balance beam task	**19**	21
Explicit Representational level for the balance beam task	4	**40**

2. Verbal explanation offered by children across both tasks

Children's verbal explanations, alongside their performance on the task, form the basis for coding children's representational levels. The successful application of the same coding scheme across both the balance scale and balance beam tasks provides initial evidence for the validity of the coding scheme, as children's verbal explanations were all coded across both tasks into one of the categories in the coding scheme (Table 3.2).

Of specific interest when comparing children's explanations across the 2 tasks is whether or not children's most complex explanations were the same across the two tasks. Following a prediction of the RR model, if children have access to explicit explanations (includes explicit weight and distance explanations) on one balance task, these same explicit explanations should also be available for the other balance task. Table 3.8 below shows the most complex explanations offered by children on both tasks. It is clear that children who offered weight explanations on one of the balance tasks were highly likely to also provide weight explanations on the other tasks, χ^2 (20) = 43.46, p = .002, Cramer's V = .319. For the implicit weight and implicit distance categories, it is notable that children who did offer this explanation on the scale task were more likely to show the same category as the most complex explanation on the balance beam task then the other category.

Table 3.8: Crosstabulation of the most complex explanations offered by children on the balance scale and the balance beam tasks

	Implicit (balance scale task)	Implicit Weight	Implicit Distance	Middle	Weight	Distance	Weight and distance
Implicit (balance beam task)	**0**	1	0	0	0	0	0
Implicit weight	0	**5**	4	0	4	1	0
Implicit distance	0	1	**0**	0	2	2	0
Middle	0	2	0	**1**	2	2	0
Weight	0	3	2	0	**27**	13	0
Distance	0	0	0	0	8	**4**	0
Weight and distance	0	0	0	0	0	0	**0**

By taking the same approach as looking at the representational levels in terms of implicit and explicit representational levels, it is clear from Table 3.9 that a majority of children who gave at least one explicit explanation on one task give at least one explicit type explanation on the other balance task (χ^2 [1] = 16.73, p <.001, phi = 0.457). Thus, the two balance tasks seem to elicit similar levels of verbal knowledge, along the implicit-explicit continuum. This leads to the conclusion that the two tasks elicited similar levels of verbal knowledge, in spite of the differences in the apparatus, and in terms of the task (e.g. because children had differing opportunities to provide explanations across the two tasks). Children who have access to explicit verbal knowledge on one task also have access to that explicit verbal knowledge on the other task, acting as a validation of the application of the coding scheme across both tasks, and acting as support for the prediction of the RR model that explicit knowledge, once achieved within a domain, should be available to other intra-domain tasks.

Table 3.9: Crosstabulation of implicit and explicit weight explanations offered as the most complex form of explanation on the balance scale and balance beam tasks

	Implicit explanations offered (balance scale task)	Explicit explanations offered (balance scale task)
Implicit explanations offered (balance beam task)	**11**	9
Explicit explanations offered (balance beam task)	8	**59**

Discussion

The discussion focuses on the following points: (1) representational levels as a measure of knowledge for a domain. (2) Generalisation of knowledge across tasks.

1. Representational levels as a measure of knowledge for a domain

The first aim of this experiment was to explore whether representational levels derived from one task could be used as a measure of knowledge for the domain on which a task is based – does a representational level derived from a balance beam task tell us about children's general level of knowledge for the domain of balance? It is important to note that the initial development of representations within the RR model is thought to be task-specific, as initial representations are "*encoded in procedural form*" (Karmiloff-Smith, 1992, p. 20). If these are the limits of knowledge for implicit representation, it follows that on different tasks for the concept of balance the same restrictions should apply, and they should similarly be implicit in their representational levels. Table 3.7 indicates that this seems to be the case. Simply using the implicit-explicit dichotomy of representational levels, children tended to show the same type of representational level on the same balance task. Table 3.6 showed a weaker association when focusing on specific representational levels – for example children might show abstraction verbal representational level on the balance beam task, and explicit transitional levels on the balance scale task. It seems therefore that task and or perceptual characteristics, as suggested by Messer et al.. (in print) may play some role in determining which specific representational level will show on a specific task. Regardless of the type of task though, children seem likely to show the same type of representational level in terms of the implicit-explicit dichotomy.

It follows therefore that this provides evidence that the levels of the RR model may be thought of as a measure of a child's knowledge for the domain of balance – and that by focusing on developing children's representations based on one task, children's general knowledge for the domain of balance is also being developed. This provides a view of how children's knowledge can develop from a simple task, to the point where they have generalisable knowledge, and the levels of the RR model begin to have the same type of utility as Piagetian stages, in terms of being able to predict children's likely level of knowledge for a task, though only with regard to a specific domain. This stands in contrast with other modern cognitive developmental models such as the Overlapping Waves model (Siegler, 1996) and the dynamic systems model (Thelen and Smith, 1994), which eschew stability in favour of looking at the importance of variability in development. In the current study children showed a high enough degree of similarity in RR levels, at the implicit-explicit level. Obviously if children showed the exact same representational level, this would be much stronger evidence of the children having access to the same representation to solve both balance tasks, and to access the same verbal knowledge. Certainly these findings indicate that children are not necessarily as variable as the overlapping waves model indicates. The current findings show a majority of children maintaining the same type of representational levels, in terms of whether they are implicit or explicit in nature, across 2 balance tasks within one session. There is therefore a need for a cognitive model which takes this stability in thinking across tasks within a domain into account – like the RR model does.

2. Generalisation of knowledge across tasks

The second, related issue for this experiment was the generalisation of knowledge. Pine et al.. (2003) noted that children were able to predict whether or not they would be able to balance the beams, prior to having explicit representations. This provides evidence that children have access to knowledge about the concept of balance, even prior to displaying explicit representations for the balance beam task. This study provides further evidence in support of the notion that children have access to knowledge which they can apply across different types of balance task. A majority of children provided explicit explanations on both tasks – thus showing that they could apply explicit knowledge to other tasks within the domain of balance (Table 3.9). The data seems less clear with regard to implicit explanations – it is not the case that a majority of children who used only implicit explanations on one task showed only implicit explanations on the other task. This finding may be explained by 2 factors – (1) An order effect, with children showing first use of explicit explanations on the second task they were exposed to, (2) task characteristics, as children were given 9 beams, and therefore gave 9 explanations for the balance beam task, whereas for the balance scale task, the number of verbal explanations was based on how many balance configurations the child produced. The evidence therefore seems to support Pine et al.'s (2003) findings that children can begin to generalise knowledge prior to having achieved fully explicit (e.g. E3) representational levels. In other words, the point at which children have generalisable knowledge, or knowledge that can be applied to different tasks within a domain is when they have explicit verbal knowledge – providing evidence that it may well be this form of knowledge which drives generalization. It is important to note that the "implicit weight" and "implicit distance" verbal explanation categories that emerged from the balance scale task were also applicable to the balance beam task. As Table 3.8 shows, though children may have given implicit weight explanations on both tasks, a majority of children gave explicit weight explanations on both balance tasks. Therefore, it is important to note that explicit verbal knowledge does not by any means indicate the end of a phase of development within a domain, as the verbal knowledge, though it may be accessible for other tasks within a domain, is not necessarily complete knowledge for that domain.

These findings also help address the studies involving the RR model and generalisability in relation to drawing (Picard and Vintner, 2006, Barlow et al.., 2003, Hollis and Low, 2005). The findings from the current study indicate that it is verbal knowledge which is generalisable, and not "procedures" – this can explain Hollis and Low's findings that children's ability to draw counterfactual humanoid objects did not generalise to being able to drawing other types of counterfactual objects (e.g. try drawing a house that differs in some way from what they should normally look like). It is simply the case that children have access to the same verbal knowledge for different tasks within a domain, but not necessarily the same procedures. This makes sense within the RR model, where the initial "*Level-I representations are bracketed, and hence no intra-domain or inter-domain representational links can yet be formed*" (Karmiloff-Smith, 1992, page 20). The clear implication to be drawn from this is that the procedural-type knowledge may stay in format which is not available for access to other tasks. This knowledge is rather redescribed into a verbal format, with the initial procedural data remaining in the same format and unaccessible.

In general, these findings showed that children could access the same knowledge across both balance tasks. It adds a note of caution to the work of Tolmie et al. (2005), or more precisely the original description of the RR model by Karmiloff-Smith (1992), as children can show general access to verbal knowledge prior to E3. Therefore, the finding that children can apply knowledge across tasks within a domain is not an indication that children have reached the final point in achieving fully explicit representations for that domain. This finding is also at odds with the findings mentioned by Siegler (2006) with regards to the overlapping waves model and generalisation. This may well be due to the fact that studies involving the overlapping waves model tend to focus on strategy use (e.g. Siegler, 2002). Strategies may often be task-specific – there are no clear examples of strategies which may be used on both the balance scale and balance beam tasks used in this experiment for example. Nevertheless, it is clear that children do have access to the same knowledge on both tasks – and the utility of the RR model is clear as it is the only one of the contemporary cognitive developmental models (e.g. overlapping waves and dynamic systems models) which addresses this issue

Summary

The aim of this study was to explore whether children's representations captured knowledge for a concept, and look at generalisation of knowledge across balance tasks. Using a simple implicit-explicit dichotomy, children tended to show the same types of representational level across the 2 tasks, and similarly showed the same types of verbal explanations across both balance tasks. This provides clear evidence that representational levels can be used as a general measure of knowledge for a domain, and that children can and do show generalisation of knowledge to other balance tasks.

Chapter 4: Using the RR model to describe the development of children's one-to-one principle for counting

Introduction

How do children learn to count? In this chapter, the focus shifts from the domain of balance to look at children's representations for counting principles. The current research on children's developing knowledge of number will briefly be recapped, and the advantages of applying the RR model will be outlined. Following this, 3 research questions are stated. (1) First, can the levels of the RR model be applied to describe the development of children's representations for the 1-to-1 principle for counting? (2) A large part of this question involves looking at whether children show an ability to detect and explain counting errors before being able to count accurately themselves? (3) Finally, what role does pointing play in the development of representations for the one-to-one principle for counting?

1. Children's developing concepts for counting

Gelman and Gallistel's (1976) work provides the main basis for current research into children's developing concepts for counting. They outlined 5 basic principles which are thought to underpin a valid counting system, which provides a logical underpinning for all more complex mathematical functions, as well as helping to provide a meaning for number. The first principle, which will be the focus of this chapter, is the one-to-one counting principle, which states that for every single object counted, a single counting term must be used. Gelman and Gallistel (1976) proposed that children had an innate concept of number. Further studies (e.g. Starkey and Cooper, 1980, Starkey et al, 1990, Wynne 1992, Clearfield and Mix, 1999) have focused on infants' numerical abilities, to see whether or not they are in fact innate. This has lead to neglect in researching the developing concepts of older children – e.g. when they begin to use verbal and notational counting systems. How do these principles develop – how do children develop an explicit representation for the one-to-one counting principle, for example? Sophian (1998) has stated that when studying children's developing concept of number, competence on a task should not be the sole criteria used to measure children's knowledge. Like the RR model, Sophian is interested in looking "beyond competence". Similarly, both Baroody (1992) and Rittle-Johnson and Siegler (1999) advocate models which focus both on children's understanding of a concept, alongside their ability to perform a task.

1.1 Why apply the RR model to children's representations for counting principles?

Why apply the RR model to this area? There are clear parallels between the type of model proposed by Sophian (1998), and the approach laid out by Rittle-Johnson and Siegler (1999), and the RR model, insofar as it looks beyond children's competence on a task, and looks at the development of children's explicit knowledge. Second, as Ginsburg (1975) notes, the early development of concepts of number is mainly verbal in nature, and it is exactly this type of development of verbalisable knowledge which the RR model focuses on. Furthermore, the levels of the RR model allow a possible framework within which to study children's developing representations for the counting principles, rather then simply focusing on whether or not they are innate.

This provides an opportunity to apply the RR model to a domain apart from the domain of Balance. As was discussed in chapter 2, there is a need to explore the types of tasks and domains to which the RR levels can be applied. So far, the levels have only been applied in tasks which show signs of a marked U-shaped curve (e.g. Pine et al's balance beam task, and the spelling task used by Critten et al, 2007). Do the levels, and indeed the process described generally by the RR model still apply when there is either a less prominent U-shaped curve (or no U-shaped curve at all) as children develop explicit verbal knowledge for the one-to-one principle? The current approach is also in keeping with Siegler's (1996) statement that the focus of developmental psychology should be in describing and explaining the development by looking at everyday tasks, such as counting. The RR model specifically states a course of development of a concept whereby an ability to perform a task precedes explicit understanding of a concept – is this the case for the one-to-one principle for counting?

2. Can children detect and explain counting errors prior to being able to count accurately?

Gelman and Gallistel (1976) have stated that children show competence in applying the one-to-one counting principle from an early age, though the “magic” experiment (where children had to guess the “winner” from a selection of sets, based on the numeric properties of the set, in comparison with the “winner” in a demonstration trial) gives an indirect view at best of children’s ability to employ counting principles accurately themselves. Following evidence that children were not in fact able to apply this concept to their own counting from an early age (e.g. Briars and Siegler, 1984, Baroody, 1984, Fuson and Hall, 1983), Gelman et al (1984) proposed a more complex model, whereby children have innate conceptual knowledge, which is masked by initial lack of procedural and utilization competence – e.g. they know the principle, but are unable to use it yet when they count themselves.

To provide evidence for this approach, Gelman et al (1983) conducted an error detection study – by their reasoning, children should be able to detect errors in others’ counting, because of their knowledge of the one-to-one counting principle. This does not require any utilization competence, as all the child has to do is to monitor another person counting. Gelman et al (1984) state simply that “implicit” knowledge of the one-to-one principle governs the detection of errors. The RR model on the other hand states that implicit representations are based on behavioural mastery – e.g. the ability to detect errors in other’s counting, even if they cannot provide an explicit explanation to explain the error made. Therefore, children should be able to apply a principal accurately in their own counting prior to being able to detect errors in others’ counting. Gelman et al (1983) performed a study with 24 three and four year olds, who were asked to watch puppets counting arrays of between six and twenty objects. Some puppets counted correctly, and some made errors. Objects were either skipped over or counted twice. Gelman et al (1983) found that a majority of these children could detect counting errors.

Briars and Siegler (1984) on the other hand found that four and five year olds were much more likely to detect errors in others' counting then three year olds. Younger children were much less likely to regularly reject real errors in counting than was found in Gelman and Merk's (1983) study, though Gelman et al (1984) attribute this to procedural differences between the two studies. Gelman and Merks' studies emphasised that the child knows more about counting than the puppets, which was not emphasised in Briars and Siegler's study. They also gave children more then one opportunity to indicate whether the count was right or wrong. Briars and Siegler (1984) also looked at the abilities of the children to count, and concluded that actual counting skills preceded knowledge of underlying principles. That is to say, the ability to detect errors does not seem to precede their own ability to count, and therefore innate conceptions for number are not necessarily needed. They did not provide any great details on children's ability to count however.

Gelman et al (e.g. Gallistel and Gelman, 1992) and Briars and Siegler (e.g. Rittle-Johnson, 1999) continue to assert the veracity of their viewpoints. Which one is in fact correct? The RR model includes implicit levels representations, whereby children should be able to detect errors, even though they cannot provide explicit explanations to describe the error made. This implicit representation involves behavioural mastery however. The implicit level involves "*information (which) is encoded in procedural form*" (Karmiloff-Smith, 1992, p.20). In this case, the ability to detect errors would not precede being able to count accurately. In order to find out whether this is in fact the case, an experiment will be carried out taking into account the methodological differences between the studies conducted by both groups. One important issue is that the sample sizes in the studies of Gelman et al (1983) and Briars and Siegler (1984) were relatively small – around 30 participants in each experiment. A larger sample is necessary to achieve significant outcomes either way. As was noted above, the experiments conducted by Gelman involved telling the child that the puppet that was being taught was learning to count, so made mistakes, whereas in Briars and Siegler's study, the children are told that the puppets know their numbers. In this study, the children will be told that the puppets are learning, so that they will point out errors if they think they are made, rather then thinking that the puppet knows how to count so it must be correct. Another point noted by Gelman et al (1986) is that their tests were done in a very interactive way, so that many trials were conducted more then once, allowing a greater possibility that children would detect errors. In this study, for the error detection task, 2 puppets will be used. For each trial, one of the puppets will count accurately and the other will make a mistake (the children are not told this!). Therefore, for each array of objects, children will see accurate and inaccurate counting. This may provide a basis for detecting errors, but it eliminates the possibility of errors not being detected because children have not seen an example of accurate counting for an array of objects. This experiment will also have a greater focus on children's own ability to count – do children truly detect errors in others' counting before being able to apply the one-to-one counting principle in their own counting?

3. What role does pointing play in the development of representations for the one-to-one principle for counting?

As well as looking at the development of children's verbal knowledge of the counting principles, children's hand movements whilst counting can also be analysed. In recent times, children's gestures have been viewed as a source of information of children's knowledge. Goldin-Meadow and Alibali (1993) have found that new and emerging concepts may be seen in children's gestures prior to children verbalising the particular concept. The role of pointing in counting has been explored by Alibali and DiRusso (1999), and Graham (1999). Graham asked whether children honoured the 1-to-1 principle for counting in their pointing prior to their verbal counting. The focus in Graham's study was on "gesture-speech mismatches", rather then focusing on whether or not children pointed accurately in accordance with the one-to-one counting principle, regardless of their verbal counting. Do children actually point accurately before they apply the one-to-one counting principle in speech, or do pointing and verbal errors coincide? A further question that has not been addressed is the continuing role of pointing in children's counting. Alibali and DiRusso (1999) note 2 possible roles played by pointing in learning to count: keeping track of the items counted, and co-ordinating between the counting words and the counted items. Graham (1999) raises the possibility that children may point whilst counting “to reduce the cognitive load in acquiring the counting principles” (p. 336). If pointing does play an important developmental role in initial learning to count, do children continue to rely on pointing once they have achieved fully explicit representations for the one-to-one principle?

Graham (1999) also conducted an error detection task to see if children were more likely to recognise errors in the 1-to-1 principle in pointing then in speech. This contrasts with the error detection task of Briars and Siegler (1984) which identified pointing errors as “pseudo errors”, which children tended to recognise – again this could be due to the procedural differences mentioned by Gelman and Merk (1986). Graham’s study used a sample of 2-4 year olds, and only a small set of objects (the biggest set was 6 items), so a replication would be desirable, with a wider range of sets. Furthermore, the error detection trials used by Graham (1999) were unnecessarily complicated, as there was no clear separation between trials where there are pointing errors, and trials where there are speech errors. They define the puppets’ counting in terms of the relation between the amount of number words and the amount of times children pointed, so that pointing errors coincide with speech errors. It is not clear therefore if errors detected are simply cases where the child has noticed the inappropriate pointing, or the inappropriate counting. The question to be asked here then is do children detect errors in pointing as “true” counting errors, even if they don’t impinge on the verbal counting being conducted?

Summary

The aim of this experiment is to begin to apply the RR model to children’s knowledge of the one-to-one counting principle number. Three sets of questions are asked to explore the beginnings of children’s representations for this principle: (1) Can the levels of the RR model be applied to children’s developing knowledge of the one-to-one principle for counting? (2) Do children learn to count accurately first, or can they detect and explicitly explain errors in others’ counting before they can count accurately themselves? (3) What role does pointing play in children’s developing representations for this principle?

Method

Participants

A convenience sample of 106 children was recruited from a nursery / junior school in Hertfordshire. This was a middle class school in a town near the University of Hertfordshire. Permission to conduct the experiment in the school was given by the head teacher, and consent letters were sent out to parents to sign. The sample comprised of 67 females and 39 males, with an age range of 3 years 10 months to 6 years and 10 months (mean age = 65.67 months, standard deviation = 9.44). Three separate age groups of children were tested: children in the nursery (n= 21, mean age = 51.76 months, standard deviation = 3.93), children in the reception year (n=46, mean age = 63.65 months, standard deviation = 3.88), and children in year one (n=39, mean age =75.54 months, standard deviation = 2.98).

Design

Table 4.1: Design for the counting task and the error detection task

	Counting task	Error Detection Task
Type of objects used	2D (e.g. an array of objects on a page) and 3D objects	2D (e.g. an array of objects on a page) and 3D objects
Numbers of objects used	4,6,8,10, and 12 objects – both 2D and 3D (10 trials)	4,6,8,10, and 12 objects – both 2D and 3D (10 sets of trials)
Number of counts per trial	One count by the child	The 2 puppets count each set of objects. One counts correctly, the other does not

Children were asked to perform two types of task, as described in Table 4.1 above. For the counting task, children were asked to count arrays of 2D or 3D objects. In the error detection task, they watched as two puppets each counted arrays of 2D and 3D objects. Each array was counted by both puppets, with one puppet counting correctly, and one puppet making a mistake for each array. The error detection task involved a total of 20 counts being made by the puppets, with half of the counts containing mistakes, and half of the counts being accurate. On half the trials, the mistake was made in the first count, and in the other half, the error was made when the second puppet counted. It is emphasised to the children that the puppets need to learn how to count, so errors must be highlighted. This acts as a staunch against criticism that children might be unlikely, due to politeness, or some other factor, to point out a mistake made by the puppets. Four types of counting error were made (see Table 4.2), two involved verbal errors, and two involved gesture errors. Each type of error was made twice by one of the puppets during their counting, with 2 further examples of counting where a puppet made both a speech and a gesture error in the course of one count. This gave a total of 4 counts involving speech errors, four counts involving gesture errors, and a further 2 counts involving both speech and gesture errors.

Table 4.2: Types of error made in the error detection task

	Speech Error	Gesture error
Skip over an object	Fail to count an object in speech	Fail to point at an object
Count an Object Twice	Count an object twice in speech	Point twice at an object

The order of the counting and error detection tasks was varied, so that half the children performed the error detection task first, and half the children performed the counting task first. This was done to ensure no systematic biases became apparent – e.g. all the children pointing whilst counting, after witnessing the puppets pointing while they counted in the error detection task.

Materials

A variety of 2D arrays and 3D objects were used as counting materials (see appendices for samples of the 2D arrays, and description of the 3D objects used). 2D arrays were comprised of laminated pages with homogeneous rows of common objects (fruits, vegetables, pets, circles, squares and triangles). 3D objects included toy soldiers, dominos, wooden blocks and small circular counters, all items that a child would be familiar with. Children were shown arrays of 4, 6, 8, 10, and 12 objects. Children were given 10 counting trials. These consisted of 5 sets of 2D arrays, and 5 sets of 3D arrays. Two hand puppets were also used in this experiment – a puppy and a kitten. Each of them performed in the error detection task, and the children were asked to explain to the puppet if they counted correctly, or if they made an error, what that error was.

Procedure

Children were taken from the classroom and told that they would be playing counting games with puppets. The child was sat down at a chair in front of a Table, facing a camera. The child was introduced to the two puppets (a dog and a cat puppet). The child was told that the 2 puppets wished to learn about counting, and wanted the child to help them. They were going to do some counting, and wanted the child to watch them and tell them if they were counting properly. They also wished to watch the child do some counting to see how he/she did it. The child was shown the camera, and told that the games were going to be recorded so that the puppets would be able to watch them again later. The child was asked if they were happy to participate in the experiment, and understood what had been told to them. The camera was then started and the experiment commenced.

Counting Task

Children were first presented with either the counting or the error detection task. For the counting task, children were presented with 2d and 3d arrays and were asked simply to count the objects. Children were not asked how many objects there were; cardinality was not addressed. The child was neither encouraged nor discouraged from counting verbally or from gesturing. If the child stopped before they had finished counting, or became distracted, they were asked "could you count them all for me please". The children's performance on the task was coded in terms of the number of trials where errors were made, and the types of error were noted. Errors in counting came in 3 categories – errors in verbal counting, errors in pointing, and errors involving both verbal and pointing errors in a single trial. Children's spontaneous use of pointing whilst counting was also coded.

Error detection task

For the error detection task, the child was told to watch the two puppets as they counted, to see if they counted "properly". The experimenter had the puppet on his hand and the experimenter counted aloud in a puppet's voice. The puppet pointed to each object as they counted. When a puppet had finished counting an array, the child was asked whether the puppet had counted properly. If they stated that the puppet had not counted properly, they were asked to explain to the puppet what they had done wrong. The puppet could either make an error in speech or in gesture. Speech errors involve the puppet skipping an object in their verbal counting or counting one object twice, with no accompanying gesture error. Gesture errors involve the puppet pointing twice at an object, or skipping an object in their pointing, without any errors in the verbal counting. Two of the 10 trials involve both speech and gesture errors, where the puppet skips an object in their count both verbally and in gesture, or counts an object twice, both in their gesture and their speech.

When both tasks were completed, the child was told that the experiment was finished. They were thanked for their participation, and asked if they had any questions, before being escorted back to their classroom.

Coding schemes

Coding verbal explanations offered on the error detection task

A set of categories was generated based on Karmiloff-Smith's R-R model (see Table 4.3), and on explanation categories previously used for tasks to which the RR model has been applied (e.g. Pine et al, 1999). This basically means that explanations were broken down into Implicit type explanations (where children could not state the nature of the error), and Explicit type explanations (where children could verbally state the nature of the error made, in relation to the concept being studies). Two specific subtypes of implicit explanations were identified during testing, and based upon initial viewing of the experiment tapes. One type involved the child repeating the count, including the error made. The second type involved simply counting, and demonstrating how it should be counted, omitting the error.

Table 4.3: Coding scheme for verbal explanations offered on the error detection task

Category	Example
None	No error detected
Implicit	No explanation offered, don't know, "it's 6"
Repetition	Repeats the puppet's counting, including gesturing, and repeating the error
Demonstration	Counts the set accurately, gesturing
Explicit	"he counted too many/too few", "he skipped that one" "he didn't point properly" "he pointed two times" "she counted on" "she counted that one twice

Coding children's RR levels for to one-one counting principle

Using children's verbal explanations on the counting errors, alongside their ability to perform the counting task, a set of criteria for coding children to RR levels can be set out. Table 4.4 below indicates four levels, ranging from children who have yet to achieve behavioural mastery, through to children who can count accurately, and explicitly explain errors made in others' counting.

Table 4.4: Coding scheme for RR levels for the one-to-one principle for counting

Representational Level	Performance on the Counting Task	Verbal explanations offered on the error detection task
Pre-implicit	1 or more counting errors made	No explicit verbal explanations offered
Implicit	No counting errors made	No explicit verbal explanations offered
Abstraction Verbal	1 or more counting errors made	At least one explicit explanation offered
E3	No counting errors made	At least one explicit explanation offered

Inter-rater reliability for verbal explanation codings and RR levels on the one-to-one counting principle

For the 1-to-1 counting principle tasks, a second coder viewed and coded 9 children's explanations for the error detection task, according to the verbal explanation coding scheme given. Using this data, and a measure of performance given to them, the second coder assigned each of the children a representational level(see Table 4.5). For the verbal explanations offered, there was a 92% level of agreement between the 2 (46 out of 50 verbal explanations agreed on). Similarly, for RR levels, there was a 100% level of agreement (9 out of 9). This finding helps to validate the verbal and representational level coding schemes for this particular task by affirming the reliability of the coding schemes used.

Table 4.5: reliability of coding of verbal explanations on the 3 balance tasks, and RR levels on the unconstrained balance task

Task	Level of agreement on verbal explanations (Kappa statistic in brackets)	Level of agreement on RR levels (Kappa statistic in brackets
	92% (Kappa = .9, p < .001)	100% (Kappa = 1, p < .001)

Results

The results are split into 2 sections, based on the 3 main questions broached in the introduction: (1) Children's ability to detect errors in others' counting in relation to their own ability to count (2). Application of the levels of the RR model to the one-to-one principle for counting. (3) Children's ability to honour the one-to-one principle in pointing and in counting, and their detection of others' pointing errors whilst counting. For the first 2 sections, only errors in speech within the error detection task are focused on, with pointing errors in the detection task being focused on in the third section.

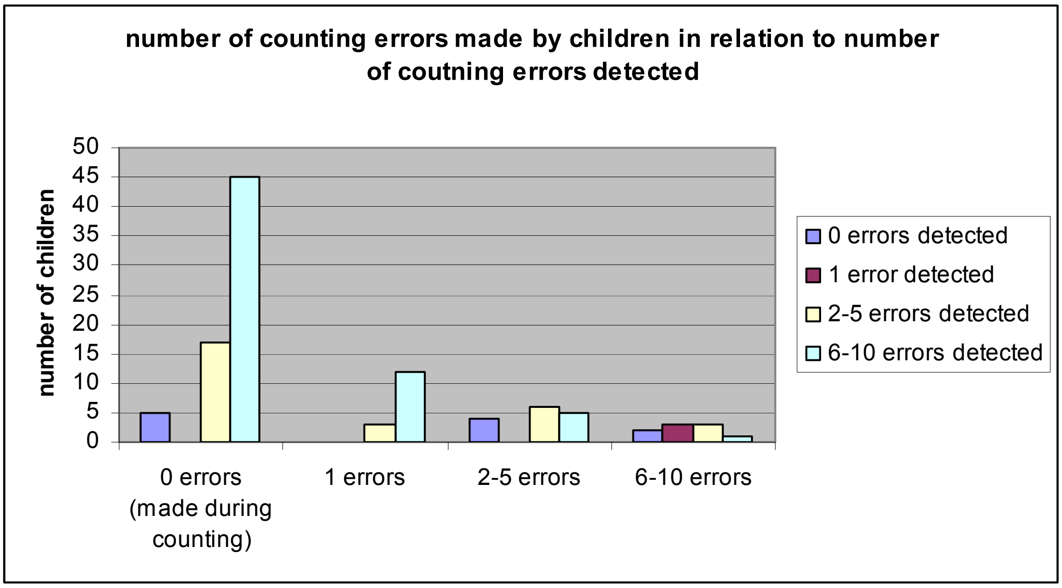
1. Children's ability to detect and explain other people's errors in relation to their own ability to count

This section will focus first on children's ability to count in relation to their ability to detect errors, and then focus on their ability to count in relation to the verbal explanations they give when explaining errors.

1.1 Children's ability to count in relation to their ability to detect errors in others' counting

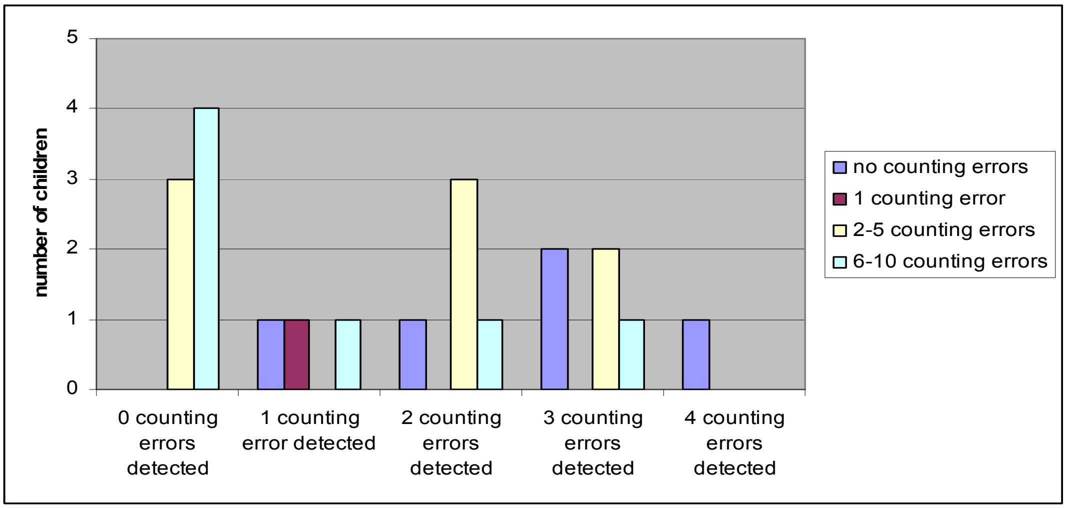
Figure 4.1 below shows children's ability to count in relation to their ability to detect errors in others' counting. Children who made one error or less in their own counting were highly likely to detect speech errors made during the error detection task (χ^2 [12] = 42.24, $P < .05$), whereas children who made many errors were unlikely to detect these errors.

Figure 4.1: number of counting errors made by children in the counting task, in relation to their ability to detect speech errors in the error detection task



To emphasise this point, the youngest age group, the nursery class children, are focused on. Figure 4.2 below shows that only one child at this age group detected four speech errors made by the puppet in the detection task. This child made no counting errors themselves. There were three children who detected three speech errors made by the puppets in the detection task, without being able to count accurately themselves. However, the main pattern emerging from this group is of children not being able to count accurately themselves, and not being able to detect errors.

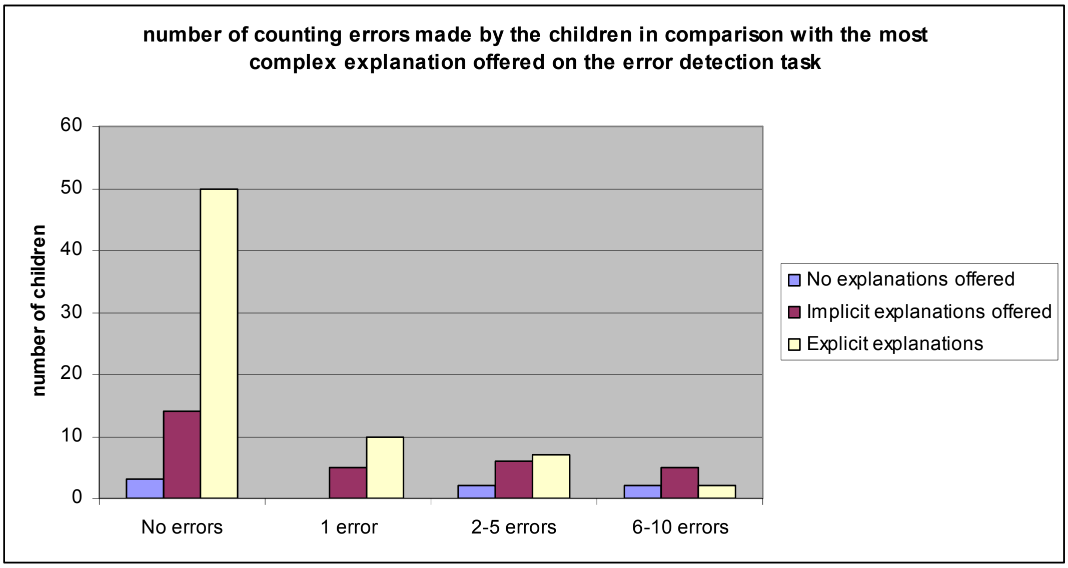
Figure 4.2: number of counting errors made by nursery class children in the counting task, in relation to their ability to detect speech errors in the error detection task



1.2 Children's ability to count in relation to their ability to explain errors in others' counting

Figure 4.3 below shows the verbal explanations provided by children in relation to their own ability to count accurately. This distinction is based on the most complex explanation offered by children (see Table 4.3). If children offered at least 1 explicit explanation, they were coded as having provided an explicit explanation as the most complex explanation, if not they were coded as having only given implicit explanations. The categories of "repetition" and "demonstration" are interpreted as implicit type explanations. Children who made no counting errors in their own counting were more likely to offer explicit type explanations in the error detection task, whereas children who made multiple counting errors tended not to offer explicit explanations (χ^2 [6] = 14.59, p < .05, Cramer's V = .393). This fits in with the data on children's ability to detect errors.

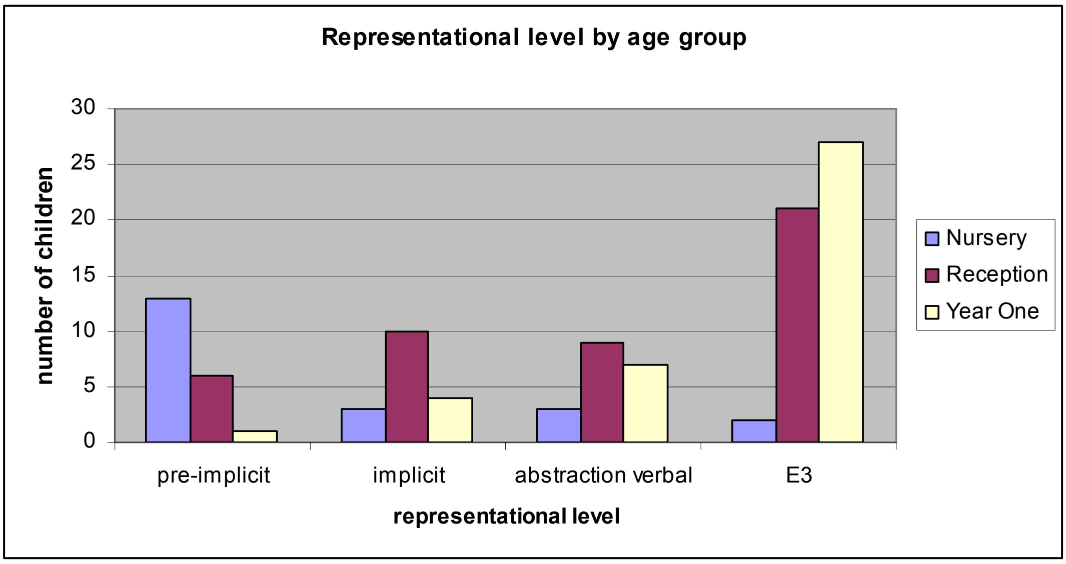
Figure 4.3 Number of counting errors made by children in the counting task by most complex explanation given in the error detection task



2. Applying the levels of the RR model to the one-to-one principle for counting

Table 4.4 (in methods section) shows the general patterns of performance on the counting task in relation to children's ability to detect and explain errors in other people's counting. The major patterns of the levels can be detected in the current study. First, there are pre-implicit children, who offer no explicit verbal explanations, and make multiple errors when they count. These errors involve either counting an object twice, or skipping an object whilst counting. The second group identified are implicit level children, who show behavioural mastery without explicit knowledge, which is represented by children who made no counting errors but gave only implicit type explanations in the error detection task. There were seventeen children who showed this pattern of performance (see Figure 4.4). Nineteen children showed a pattern of performance in keeping with the Abstraction Verbal level – explicit verbal explanations, though accompanied by counting errors. Finally, 50 children showed E3 levels of behaviour, representing a fully explicit knowledge of the principal, coupled with no counting errors made. The distribution of levels was tested using a chi square, and a significant association was found (χ^2 [n=106, df =6] = 39.3, p < .05, Cramer's V = .431). Younger children were more likely to be at lower levels in the model, whereas for the older children, a majority displayed fully explicit knowledge. This fits in with the developmental model set out by Karmiloff-Smith (1992), with a majority of year one children showing fully explicit representations for the one-to-one counting principle.

Figure 4.4: Distribution of RR levels by age group



3. The role pointing plays in children's developing representations for the one-to-one counting principle

This section will focus first on children's own pointing while they count – looking first at children's accuracy in speech and in pointing whilst counting. The focus will then shift onto children's ability to detect pointing errors in other people's counting.

3.1 Children's accuracy in speech and in pointing whilst counting

This section focuses on whether or not children's ability to apply the one-to-one principle in their pointing emerges prior to them applying it accurately in their verbal counting. Children's spontaneous pointing in the counting task was coded. Table 4.6 indicates that the nursery children made as many gesture errors as speech errors. For this group, many children made multiple mistakes in a single trial. There were 27 (out of a total of 51 instances where counts included an error for the nursery group) instances of nursery children making more then one error in a single count, as opposed to 8 instances for reception children, and no instances at all in year one children. It seems that this young group of children often make both counting and pointing errors in a single trial, indicating that children don't honour the one-to-one principle in their pointing or in their counting at this early point in development.

Table 4.6: Number of counting mistakes made by age group: (percentages by age of the total amount of errors made by that age group)

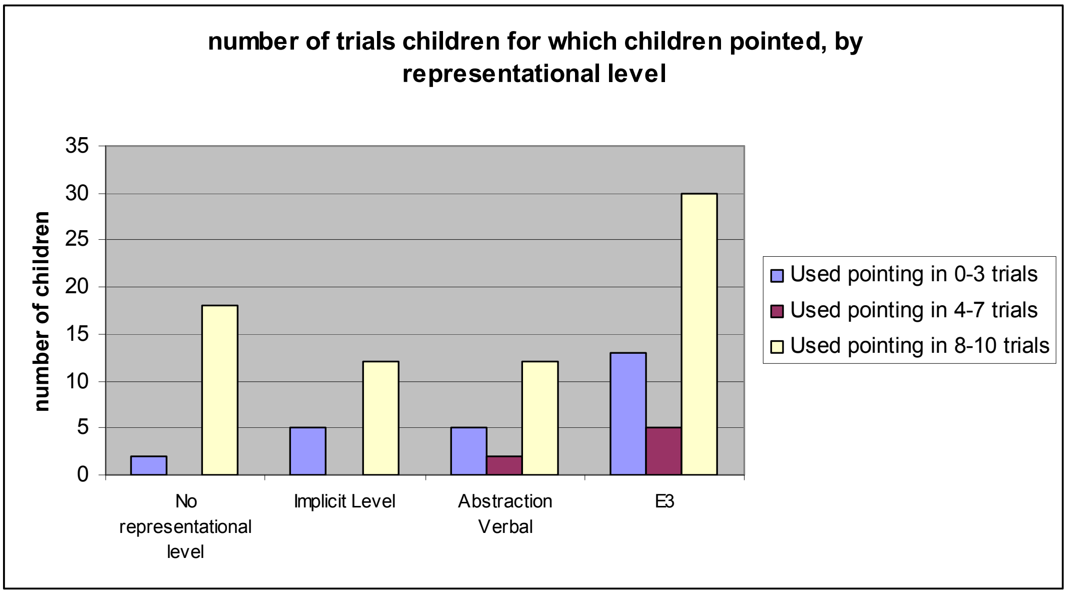
Age Group	Number of Mistakes Made	Number of participants who made counting errors	Gesture errors	Speech errors
Nursery	79	17	45.6%	54.4%
Reception	43	15	25.6%	74.4%
Year one	9	8	11.1%	88.9%
Total	131	40	37.4%	62.6%

Looking at the 2 older groups of children, gesture errors become much less prevalent, in proportion with speech errors. This coincides with the children making many fewer errors in general however. Therefore, though the older groups of children are less likely to make pointing then speech errors, there is no definitive proof that the youngest children are less likely to make pointing then speech errors in their counting. Figure 4.5 below shows the prevalence of pointing used by children in the counting task. It is clear that a majority of children point spontaneously in the counting task.

3.2 Children's pointing in counting in relation to R-R level

Figure 4.5 looks at children's use of pointing in relation to their representational levels. A majority of children at all levels point for a majority of trials, though there is some evidence that at E3, a certain group of children stop pointing whilst counting for a majority of trials. Figure 4.5 shows more than ten children who were at the E3 level who pointed for less than 3 of the trials (e.g. less than a third of the trials) in the counting task. This indicates that once children have explicit representations, they may be less dependent on pointing to aid them in their counting.

Figure 4.5 How often children pointed in the counting task, in relation to their RR level



3.3. Detection of errors in pointing made by others whilst counting

In the error detection task, children were unlikely to detect trials which included a pointing error on the part of the puppets as being wrong, with less then a quarter of the gesture errors being detected (see Figure 4.6 below). Children were more likely to detect speech errors then gesture errors. Children were also much more likely to detect errors in trials where there was both speech and pointing errors then trials where there are only pointing errors.

Figure 4.6: Frequency of Speech, Gesture and speech and gesture errors on the error detection task, and how often they were correctly identified

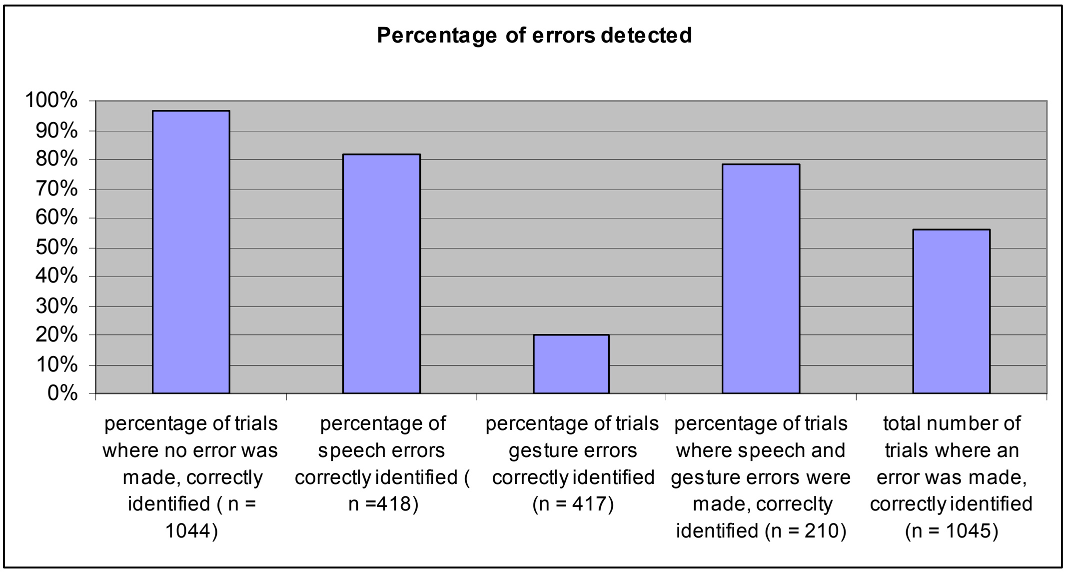
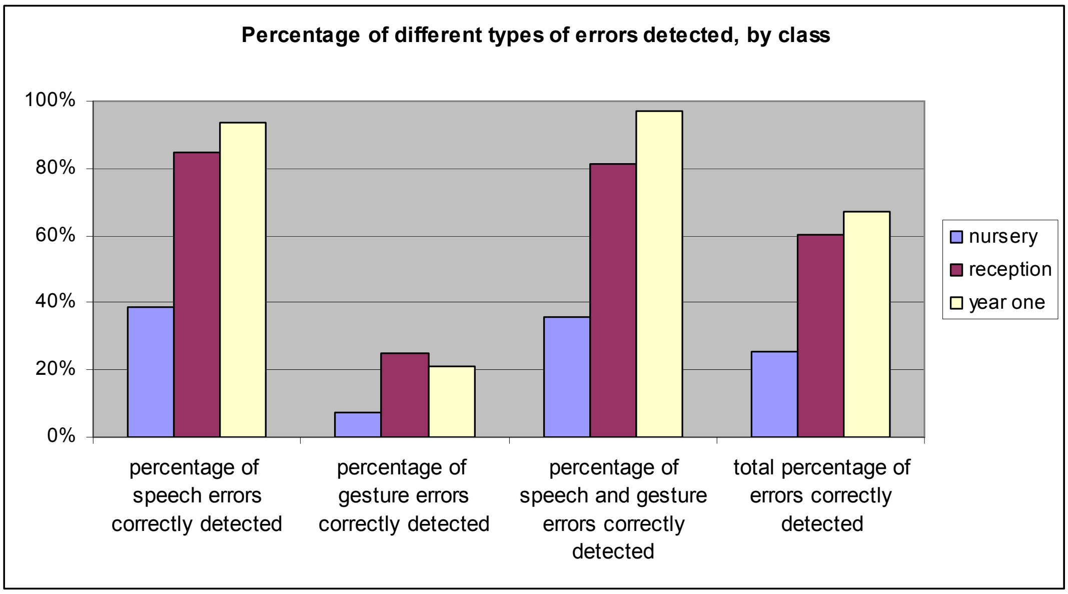


Figure 4.7 below further breaks down the data according to age. The youngest children failed to the greatest extent to detect gesture errors, and were also less likely to detect speech errors then the older children. The older groups were much more accurate in detecting speech errors, but not so much so in detecting gesture errors. In terms of absolute numbers of errors detected, the older two groups of children were found to be more likely to detect speech errors (χ^2 [n= 105, df = 8] = 56.11, p < .05, Cramer's V = .514) then the nursery children, but there was no significant association for gesture (χ^2 [8] = 12.43, p = .133, Cramer's V = .242), as they both seemed equally likely not to detect gesture errors as counting errors

Figure 4.7: Percentage of errors in the error detection task detected by children by class



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Discussion

This discussion focuses on 3 points – (1) the applicability of the levels of the RR model to the one-to-one counting principle. (2) Children's ability to detect and explain errors' in others' counting in relation to their own ability to count. (3) The role of pointing in children's developing representations of the one-to-one counting principle.

1. The applicability of the levels of the RR model to the one-to-one counting principle

This study was a first step in applying the levels of the RR model to the domain of mathematics. It shows that the levels of the RR model are not uniquely applicable to the domain of balance (Pine et al, 2001, see also Critten et al's, 2007 work applying the levels of the RR model to spelling tasks). Evidence that these levels can be applied to different types of tasks, in different domains continues the process of validating the RR model as a general model for cognitive development. This is important as, though Karmiloff-Smith (1992) discussed the applicability of the RR model across various important domains (e.g. chapters in beyond modularity focusing on language, physics, maths, and notation for example), most of the experiments discussed involved a post-hoc application of the RR model to explain the findings, rather than an experiment being set out to see whether or not the levels could actively be applied to a child's developing representation for a domain.

In the current study, two separate tasks were used to assess children's representational level for the one-to-one counting principle. A counting task was used to assess their ability to accurately apply this principle in their own counting. An error detection task was used to measure their ability to verbalise this concept by explaining errors made in relation to the one-to-one principle. These were straightforward tasks, following Siegler's (1996) dictum that simple, everyday tasks should be used to truly capture how children's thinking develops.

The levels of the RR model were clearly applicable to this concept. It is interesting to note that a "pre-implicit" representational level was codable in this study – which has not been possible with the balance tasks (though see Critten et al, 2007 – pre-implicit representational levels have also been applied for spelling and reading tasks). For this principle, children do not have innate "implicit level" representations. They cannot count accurately, and do not provide explicit verbal explanations in the error detection task, as they do not in fact detect any of the errors. This has clear implications for the next section, as it provides clear evidence that children do not have the ability to detect and explain errors in other people's counting prior to being able to count themselves. The levels depict the development of explicit verbal knowledge of the one-to-one principle, moving from a state where children cannot explain the errors made by the puppets in their counting, to a point where they can explicitly state the error made by the puppet, either by counting one object twice, or by skipping over an object. The verbal explanations provided by the children provide a clear signal of explicit verbal knowledge of this principle.

The "U-shaped curve" in terms of performance was less prominent in this particular study than in Pine et al's work – that is to say, Pre-implicit and E3 representational levels were clearly prominent for the youngest and oldest groups of children, but the abstraction verbal level was not as prominent as has been found in studies using the balance beam task (e.g. Pine et al, 1999, where in fact a majority of children are at the abstraction nonverbal and abstraction verbal levels). This is important insofar as the levels of the RR model are not limited in only being applicable to tasks which involve this type of pronounced U-shaped curve. The lack of a prominent U-shaped curve indicated that for a majority of children, the first abstract / explicit representation may not necessarily be inaccurate in the way that seems to be the case for the domain of balance. Another important aspect of the current experiment is that separate tasks were used to measure their ability to apply the one-to-one counting principle in their own counting, and their ability to verbalise this principle. This marks a different approach to those used by Pine et al (2001), where children's ability to balance the beams and their verbal knowledge was ascertained using one task. The possibility was raised in the first experimental chapter that perhaps the RR levels could only be applicable to tasks where there was a clear and appreciable link between performance on the task and levels of verbal knowledge. The approach of this particular experiment shows one way that the levels of the RR model can be applied to domains in which U-shaped curves are less predominant, by using different tasks to measure performance and verbal knowledge.

One final point is that by applying the levels of the RR model is the implication that the process of redescription described by Karmiloff-Smith, and the process of change of the RR model can occur within this domain. The next step obviously is to provide evidence that this process of redescription does actually occur as Karmiloff-Smith (1992) describes it.

2. Children's ability to detect and explain errors' in others' counting in relation to their own ability to count

The question of whether children could detect and explain errors in others' counting has clear theoretical implications. Gelman et al (e.g. 1982) stated that children's ability to detect errors arose from innate principles, which could be used to detect errors, though they could not yet be utilised properly by children. Briars and Siegler (1984) on the other hand found that children's ability to detect errors did not precede their own ability to count.

The current study took into account the methodological differences between Briars and Siegler's (1984) approach, and that of Gelman et al (1982). The approach of Gelman et al (1982) was taken, whereby children were told that the puppets were only learning to count, so would likely to make mistakes. Yet, the evidence of this study did not correspond with their findings. There was no evidence that children could consistently detect errors in others' counting prior to being able to count accurately themselves.

Gelman et al (1984) states that children's ability to detect errors stems from "implicit" knowledge of the one-to-one counting principle, which is not yet verbalisable. The comparison is made to grammar – children can recognise grammatical errors, even if they cannot describe the grammatical rule in question. This comparison is problematical as this ability to detect grammatical errors does not precede children's ability to speak, and employ grammar themselves. In the same way, the evidence from this study indicates that children do not begin to detect errors in others' counting prior to being able to count themselves. It is interesting to note that within the RR model, an Implicit representational level is found with regards to the one-to-one principle. At this stage, children can detect errors in others' counting, but cannot yet verbally explain the error, which is of course a complete contrast to the meaning of "Implicit" for Gelman. There is clear evidence of a pre-implicit representational level existing, which is particularly prevalent among the nursery school children, indicating strongly that the ability to detect errors does not precede the ability to count accurately.

The evidence of this study, though only cross-sectional indicates that children's developing representations of the one-to-one counting principle correspond to the path set out by the RR model (see Pine et al 2003). There is even evidence of an "abstraction verbal" representational level, e.g. a downturn in children's own ability to perform a count accurately, which is thought to accompany first use of explicit knowledge of the one-to-one counting principle. It is interesting to note that within this study, only 4 representational levels were necessary, with the most interesting inclusion being the pre-implicit level, which was most prominent amongst the nursery children.

What does this tell us about whether or not children have innate concepts of number? The RR model states that innate predispositions can have an influence on developing representations. The evidence from Starkey et al (1990) may indicate that infants are predisposed to certain numerical aspects of the world, but this does not go so far as providing an "implicit" set of counting principles for children to utilise in their own counting. Gelman et al (1986) move towards talking about innate abilities in terms of "constraints" on counting - which are more in line with the "innate predispositions" that

Karmiloff-Smith (1992) describes. Though children may be born with these innate predispositions, there is still an ongoing process of development that needs to be described, and the results of the current study show that the RR model may describe the development of these principles. That is to say, if there are innate components, they are more developmentally primitive than the implicit level of knowledge laid out within the RR model.

3. The role of pointing in children's developing representations of the one-to-one counting principle

Two sets of questions are asked in relation to children's to the role of pointing in counting. The first relates to children's own use of pointing – do they honour the one-to-one principle in their pointing prior to honouring it in their counting? Do children continue to use spontaneous gesturing once they have achieved fully explicit representations for the principle? The second set of questions relates to children's ability to detect gesture errors – do they detect them as errors, or are they only detected when accompanied by verbal errors?

3.1 Children's pointing whilst counting

In this experiment, children were not specifically asked to point as they counted, yet a majority did so spontaneously (see Fuson, 1988; Gelman, 1980). The nursery children are the most interesting group here, as a majority of them are yet to apply the one-to-one principle consistently in their own counting. A majority of this group made multiple errors within a single count. – i.e. they pointed at an object twice, and counted it twice. Therefore, it seems that young children make both pointing and counting errors, indicating that they do not initially honour the one-to-principle in either format. On the other hand, for the reception and year one children, counting errors are much more common then pointing errors, indicating that errors made by these children are purely related to their counting, not to their pointing. Therefore, there is some evidence to suggest that children may indeed honour the one-to-one counting principle in pointing prior to being able to honour it in speech. How do these relate to the findings of Graham (1999)? Graham focuses in her analysis on “gesture-speech mismatches”. Her findings were that younger children tended to make more “gesture-speech mismatches”, whereas this study simply reports that the youngest children tended to make more errors – both counting errors and pointing errors.

But what role does pointing play? Graham (1999) stated the possibility that pointing may help to reduce cognitive load as children acquire an explicit representation for the one-to-one counting principle. If this were the case, children who have achieved fully explicit representations for this principle might not use pointing so much. Figure 4.5 shows that a certain group of children who achieve E3 representational levels no longer predominantly point whilst counting. This provides some evidence to back-up Graham’s claim, though a majority of children who are shown to have fully explicit representations for the counting principle do still point whilst counting, indicating that pointing may also fulfill continuing roles such as keeping track of what has been counting and coordinating number words and the objects to be counted. In summary, it seems clear that as children develop more explicit representations for the one-to-one principle, they may become less dependent on pointing in order to count accurately.

3.2 Detecting pointing errors in others’ counting

The second question with to be asked regard to pointing is whether or not children detect errors in pointing in other peoples’ counting as errors. Graham’s (1999) error detection task failed to distinguish between speech errors and pointing errors. In this study, a majority of children failed to detect pointing errors as errors (see Figure 4.6). This is at odds with the findings of Briars and Siegler (1984) – where children were often found to detect these errors in pointing which they termed “pseudoerrors” (as children’s verbal counting was accurate in the trial, in spite of the pointing error). This difference in findings may arise due to the different methodologies used, though this seems unlikely as the children were told in this study that the puppets were just learning how to count, whereas in Briars and Siegler's study (1984) the children were told that the puppets knew their numbers. Another difference between the 2 studies is the larger sample size in this study, making the current set of data more reliable.

It is interesting to note that children were much more likely to detect errors in the trials with speech and pointing errors, than in trials which only contained pointing errors. This was true for all age groups – therefore, this inability to detect pointing errors may be due to 2 causes – (1) a lack of knowledge of the 1-to-1 principle early on, as children generally fail to detect errors, or (2) pointing errors are not viewed as "true errors" as such, unless accompanied by a speech error. The latter, which is the case in this study, implies that "pointing errors" on their own are indeed, as Briars and Siegler termed them "pseudoerrors". Therefore, though children's pointing may indeed play an important role in children's learning to count, there seems little utility in focusing on children's ability to detect pointing errors which do not coincide with speech errors.

Summary

This study showed that children do not detect errors in other people's counting prior to being able to count accurately themselves. The levels of the RR model were applied to children's ability to count and their ability to detect and explain errors in others counting. This provided a clear model of the developing one-to-one principle beyond competence in applying it in counting through to explicit verbal knowledge of the principle, in line with the recommendations of Sophian (1997) and Rittle-Johnson and Siegler (1999). The study provided some evidence to confirm that children did honour the one-to-one counting principle in pointing prior to in speech. Continuing use of pointing when children achieved fully explicit representations indicates that reducing cognitive load to aid in acquiring the principle is not the only role that pointing plays in the development of counting principles. Finally, this study found that children did not detect pointing errors in others' counting as errors, when there was no accompanying verbal counting error.

Chapter 5: Using the RR model to predict the effect of different types of intervention on children's representations for the cardinality principles for counting

Introduction

What can the RR model contribute towards the teaching of numerical knowledge?
In this chapter, the focus will shift to the cardinality principle for counting. A number of contemporary models which have been applied to children's developing mathematical knowledge which have implications for teaching will be described, and the implications for the RR model in particular will be contrasted with these model. The research looking specifically at the concept of cardinality will also be focused on, in order to address 2 key issues – (1) can the levels of the RR model be applied to children's developing representations for the cardinality principle for counting, and (2) Does the efficacy of different types of teaching intervention depend upon the child's representational level?

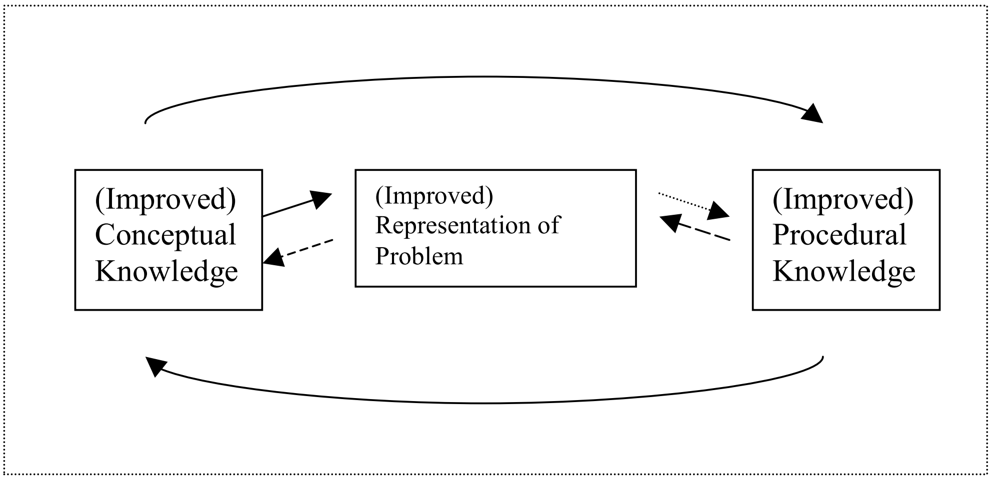
1. Models for the development of mathematical knowledge

In the previous chapter, it was noted that few if any models have been put forward which attempt to describe the ongoing development of children's numerical knowledge. This current study will focus on two models which have more recently been applied to specific mathematical tasks – Rittle-Johnson et al's (2001) iterative model, and Bermejo's (1996) model for the development of knowledge for cardinality. This model will be contrasted with the RR model in terms of the implications for teaching.

1.1. Rittle-Johnson et al's (2001) iterative model

Rittle-Johnson et al (2001) propose an iterative model whereby children's conceptual and procedural knowledge develop in a cyclical manner (see Figure 1). Procedural knowledge is based on children's ability to perform a task. Conceptual knowledge is defined as *"implicit or explicit understanding of the principles that govern a domain*" (Rittle-Johnson et al, 1999, p 346). They sidestep the question of whether procedural or conceptual knowledge comes first by stating that either may develop first, depending on how common the concept or task is within the everyday environment (initial development is thought to be conceptual if the child does not engage in concept specific tasks in the everyday environment, whereas if the child does do so, then initial development should be procedurally based – for the concept of number, as it occurs in the everyday environment, initial development should presumably be procedurally based). Their experiment related to the development of knowledge about decimal fractions. The study used a complicated design and methodology which cannot be gone into in any great detail here, but the main findings stated that not only does initial conceptual knowledge predict later improvement in procedural knowledge, but also improvements in procedural knowledge predict gains in conceptual knowledge (see Figure 5.1). This relation is moderated by a variable called "representation of the problem" which arises from verbal explanations offered by children. The main conclusion to be drawn here is that there is claimed to be an open-ended relationship between ability to perform a task and explicit verbal knowledge. Therefore, at any point during development, it would appear from this open-ended model, interventions aimed at improving conceptual and procedural knowledge would be equally effective in improving children's knowledge.

Figure 5.1. Rittle-Johnson et al's (2001) Iterative model for the development of conceptual and procedural knowledge



1.2. Bermejo's (1996) model for the development of knowledge for cardinality

Bermejo's (1996) work has focused specifically on the cardinality principle. Gelman and Gallistel (1978) state that the cardinality principle entails knowing that the last counted word in a set represents the numeric value of that set. Though some understanding of cardinality can be attributed to children who can state the value of a single set, Bermejo's (1996, 2004) model states that this is not the endpoint in terms of children's understanding of cardinality. Cardinality is thought to be a key concept as a fully developed concept of cardinality allows children to move from simply being able to count, to begin to apply this counting to mathematical problems. Bermejo's model involves a number of levels of understanding based on children's ability to perform a counting task. It is important to note that no clear attempt is made within the model to describe how children move through these levels. The model only focuses on cardinality in relation to task involving one set of objects. A more goal-focused approach is taken by Sophian (1993, see also Muldoon (2003). The most important application of the cardinality principle is that it allows one to compare groups of objects. Thus, the important aspect of cardinality is based on an ability to compare two sets, rather than simply being able to ascribe a value to a single set. This rules out for the present study the possibility of using tasks where a single set is used, such as tasks involving simply asking "how many objects are there" (Fuson et al, 1982, Frye et al, 1989), and "give me x" objects tasks (Wynn, 1990, Fluck et al 2005). Comparative tasks such as those used by Sophian (1993) provide a basis for examining children's knowledge using this working definition of cardinality. An example of a task used was placing a series of complementary objects beside each other, for example bottles and bottle tops, and asking the child if there are enough bottle tops to go on each bottle. It is important to note for these studies that children were NOT asked to count the objects, but simply to state whether or not there were enough tops to go on all the bottles.

Bermejo has described a series of levels which children pass through in developing knowledge about cardinality in a single-set test. There is also a need to describe how children's knowledge, and their ability to use the cardinality principle to compare 2 sets, develops across time. Having already applied the RR model to the 1-to-1 counting principle, it stands to reason that the levels of the RR model should also be applicable to the cardinality principle. With regard to the RR model, measures of verbal explanation and performance are necessary. The comparison test used by Sophian (1992) provides a good opportunity to elicit verbalisations, as the child justifies why they think the two sets are, or are not, numerically equivalent. A second experiment described by Sophian (1992) involves the production of a set of objects which is numerically equivalent to a second set. Two different types of set-up were used – children were asked either to make the new set by placing the new objects beside the original set in such a way as to make them symmetrical, or asking children to make a new set with a different spatial configuration. As has been noted (Fuson, 1988), children find it easier to deal with objects which are organised in straight lines, as opposed to sets of objects which are not arranged in straight lines. This may give rise to a U-shaped curve in performance, in keeping with the RR model (Figure 2).

1.3 The RR model

The RR model presents a different description of the developing relation between procedural and conceptual knowledge than Rittle-Johnson et al's (2001) model. In this model, task performance and verbal knowledge do not simply mutually progress. The RR model allows for a U-shaped curve in children's ability to perform a task in relation to their verbal knowledge. This means that the RR model allows both for increases and decreases in performance on the task. Initial behavioural success can be followed by a downturn in performance which arises from the development of a verbally accessible but oversimplified conception for a domain (e.g. for the concept of balance described already in this thesis, there is a simplified notion that weight is the only aspect of interest, so that if there are more weights on one side of the beam, it can not be balanced). In this case, increases in verbal knowledge are not always linked to improvements in performance on a task. Pine et al (2000) have shown that children's sensitivity to teaching interventions is likely to depend on their representational level, indicating that specific types of intervention are more likely to be certain points in development, rather then the open-ended model specified by Rittle-Johnson et al (2001).

2. Teaching interventions to improve mathematical knowledge

One key recent study which has focused on children's developing understanding for the concept of cardinality was conducted by Muldoon et al (2003). Muldoon et al (2003) looked at the relation between children's ability to use counting to compare sets and their ability to detect and reason about other people's miscounts. Muldoon et al (2003) argue that a focus on developing explicit representations, and being given an opportunity to conceptualise the rules surrounding a particular concept is necessary for the development of these concepts. The findings were that children's ability to detect other people's miscounts and reason about why these miscounts were inaccurate was the best predictor of performance on numeric comparison tasks. Again, the value of looking at children's verbalisations are emphasised, and given a central role in the development of mathematical abilities. Rittle-Johnson et al's (2001) study similarly focused on "*improving representations for problems*" implying a focus on conceptual knowledge. Muldoon et al (2003) concluded from their research that giving children an opportunity to reason about others' miscounts may provide an important opportunity for children to develop their understanding of cardinality.

How can this hypothesis (and with regard to testing within an intervention-based methodology, it so far remains only a hypothesis) be interpreted in relation to the RR model? As has already been pointed out, Pine et al (2000) have shown that initial representational level may play an important role in terms of the types of teaching they may be receptive to. One could predict from the RR model for example that children who have yet to achieve an explicit level of representation would not benefit from conceptually derived teaching methods, such as those espoused by Muldoon et al (2003) and by Rittle-Johnson et al (2001). Without behavioural mastery, underpinned by initial procedurally based implicit representations for the task or domain, there is no solid framework within which the child may begin to verbalise a concept. Therefore it is hypothesised that a procedurally-focused teaching method aimed at fostering behavioural mastery would be more efficacious for children who have yet to achieve implicit representations – with a focus on improved performance for cardinality tasks. Similarly, it is hypothesised here that conceptually-based teaching methods may be most efficacious for children who have yet to achieve explicit verbal knowledge (see Table 5.1), because as has been noted, they have already got adequate frameworks to allow them to solve tasks, and conceptual interventions afford them an opportunity to verbalise the principles arising from these initially implicit frameworks, leading to more complex and varied types of explanations being offered.

Table 5.1: Predicted effects of the procedural and conceptual interventions, in relation to initial representational level

Initial Representational Level	Predicted effect of Procedural Intervention on representational level at post-test	Predicted effect of Conceptual Intervention on representational level at post-test
Pre-Implicit	Should lead to an improvement in performance leading to a higher representational level	*No representational change at post-test*
Implicit	Should lead to an improvement in performance leading to a higher representational level	*The child should begin to produce explicit verbal explanations, leading to a higher representational level*
Abstraction Nonverbal / Abstraction Verbal / Explicit transition	*No representational change at post-test*	The child should produce more explicit verbalisations at post-test, leading to a higher representational level
E3	*No representational change at post-test*	No representational change at post-test

Rittle-Johnson et al's (2001) iterative model on the other hand would predict that both types of intervention should be equally effective at any point in development (see Table 5.2), both in terms of improvement on the measure for which the intervention is focused (e.g. a procedural intervention improving performance on tasks at post-test) but also on the other measure (e.g. a procedural intervention bringing about improvement in procedural knowledge which in turn brings about an improvement in conceptual knowledge).

Table 5.2: Predicted effects of the procedural and conceptual interventions according to Rittle-Johnson et al's (2001) model

	Effect on Conceptual knowledge	Effect on Procedural Knowledge
Conceptual Intervention	Will bring about an increase in conceptual knowledge	Increase in conceptual knowledge will bring about increase in procedural knowledge
Procedural Intervention	Increase in procedural knowledge will bring about an increase n conceptual knowledge	Will bring about an increase in procedural knowledge

Summary

The aim of this experiment is to apply the RR levels to a set of tasks designed to measure children's representations of cardinality. It is predicted that the effects of conceptual (derived from Muldoon et al 2003) and procedural interventions will be dependent on initial representational level (see Table 1). This is contrasted with Rittle-Johnson's iterative model, which proposes a more open-ended relationship between conceptual and procedural knowledge.

Method

Subjects

94 children were recruited from 3 primary schools in Hertfordshire. The children from these schools came from a mainly middleclass background. They were randomly assigned to two experimental groups (conceptual and procedural intervention groups), and a control condition. Because this was the first time an attempt was being made to apply the RR levels to the cardinality task, children could not be assigned to the different conditions based on initial RR level. Furthermore, there was no significant age difference between the three groups (F [2,91] = .293, p = .75). Thirty-two children were assigned to the conceptual intervention experimental group, twenty-eight children were assigned to the procedural intervention experimental group, and thirty-four children were assigned to the control group. Children were recruited from nursery, reception, and year one classes, to capture the full range of children's competence for the tasks. The sample had a mean age of 66.5 months (Standard Deviation = 9 months). Children's ages ranged from 48 to 81 months. There were 58 boys and 36 girls.

Materials

A variety of small toys were used in this experiment as counting items. Children were given crayons, checkers counters, plastic bottle tops, dominoes (with the blank sides face up, to avoid potential confusion if the children counted the dots), toy soldiers, and large coloured blocks for the matching and comparison cardinality tasks. A set of large Lego blocks were used for the two interventions for the two experimental groups. Two hand puppets (a dog and a cat) were also used. For both the cardinality tasks and the interventions, the tasks involved both puppets having the same amount of objects.

Methodology

The current experiment used a pre-test – intervention – post-test design (see Table 5.3). Children were given matching and comparison tasks, followed by an intervention, after which they were retested on the matching and comparison task. Children were randomly assigned to each of the 3 conditions. Because of the way the experiment was conducted, with children being drawn from different classes depending on the availability of a class on a particular day, it was not possible to attempt to ensure that children from the different age groups were equally represented in the three conditions. Similarly, it was not possible to ensure representational levels were equally presented across the three conditions. The order of the matching and comparison task was counter-balanced to ensure there were no possible order effects in performance on these tasks.

Table 5.3: Layout of the methodology of the current study

	Pre-test	Intervention	Post-test
Conceptual intervention group	Matching Task, Comparison task	Conceptual Intervention	Matching task, Comparison task
Procedural intervention group	Matching Task, Comparison Task	Procedural Intervention	Matching task, Comparison Task
Control group	Matching Task, Comparison Task	Unrelated drawing task	Matching Task, Comparison Task

Cardinality Tasks

The children were told that they would be performing a series of tasks in which they must ensure that the 2 puppets have the same number of toys. The two puppets were introduced to the child before the experiment commenced.

Matching Task

For the matching task, the experimenter gave one of the puppets some toys and the child was asked to give the second puppet "*exactly the same amount of toys to play with*". The toys given to the first puppet could be arranged in straight lines, or in a non-symmetrical arrangement. Arrays of between 2 and 11 objects were given to the first puppet, and placed in front of them on the Table. For each trial, the child was asked to give the second puppet exactly the same number of toys to play with as the first puppet. Children had 5 matching problems at pre-test, and another 5 matching problems at post-test (see appendices for full details). No feedback was given during the experiment; children were not told whether they had accurately matched the two sets.

Comparison Task

In the comparison task the two puppets were both given a number of objects by the experimenter, and children were asked whether or not the puppets had the same amount of objects each. Objects were placed in front of the two puppets on the Table, and children were asked – *"do the two puppets have the same amount of toys each to play with, or does one of them have more?"* Both possibilities were explicitly stated by the experimenter to eliminate the risk of instructional bias affecting children's responses. After giving a response, children were asked to explain their answer – *"how did you know that?"* or *"how did you Figure that out?"* The objects could be arranged in straight lines, or in a non-symmetrical manner, as with the matching task, to make the task more difficult. The puppets had arrays of between 2 and 11 objects, across 5 comparison problems at pre-test and 5 comparison problems at post-test. For both pre-test and post-test, in one of the five comparison problems, the two puppets had the same number of objects, in two of the problems a puppet had one object more then the other, and in the other two comparison problems a puppet had 2 objects more then the other (see appendices for experimental sheets). Children were not given any feedback or evaluation of their performance on the comparison tasks at pre or at post-test.

Interventions

Procedural Intervention

For the procedural intervention, children were shown an example of puppets performing a matching ask, using a "counting to compare" strategy across five trials. The first puppet was given a series of 3, 5, 7, 9, and 11 objects. The second puppet (manipulated by the experimenter) counted aloud the number of toys the first puppet had, and then counted out aloud the same number of objects for itself. The puppet physically picked out their own toys from a container. The second puppet copied the configuration of the other puppet's toys – for 2 trials the toys were arranged in straight lines, and for 3 trials the toys were arranged in non-symmetrical formations. The child was then asked to repeat the strategy, to ensure that the puppet had been accurate in their performance. This procedure was repeated across 5 trials, giving children the opportunity to practice the "counting to compare" strategy used by the puppet.

Conceptual Intervention

The conceptual intervention involved giving children an opportunity to detect and conceptualise other people's miscounts. Children were asked to watch a puppet performing a matching task, and check to see whether the puppet's performance had been accurate. Five examples were shown, with one puppet being given a number of objects (3, 5, 7, 9, and 11 objects respectively). The second puppet was manipulated by the experimenter, to take exactly the same amount of toys as the other puppet. The puppet used a "counting to compare strategy"; counting aloud the number of objects that the other puppet had, and then taking the same number of objects for itself. The puppet, controlled by the experimenter, picked out and counted toys from a container. There were 2 accurate trials, and 3 inaccurate trials, with the puppets taking the wrong number of objects. The puppet used a counting-to-match strategy, but made a one-to-one counting error, miscounting the number of objects that the other puppet had, so that they took the wrong number of objects for themselves. The second puppet matched the configuration in which the other puppet's toys were laid out – the toys were arranged in straight lines for 2 trials (where discrepancies would be more visually apparent), and in a non-symmetrical manner for 3 trials. Children were asked whether the puppet had performed the task accurately. The child was asked *"do the two puppets have the same amount of toys each, or does one of them have more?"* If the child did not recognise the error, they were encouraged to copy the puppet's strategy, to see if they had counted accurately.

When errors were pointed out, children were asked to try and explain to the puppet what they did wrong, or why they do not have the same amount of toys as the other puppet. This gives them the opportunity to conceptualise the error made by another in counting which Muldoon et al (2004) consider so influential. In all cases where an error was made, the second puppet ended up having less objects then the first puppet.

Control group

This group engaged in a non-numeric maze-based task, which took up a similar amount of time to the two intervention tasks (2-3 minutes).

Procedure

Children were taken from their class to the room where the experiment was being conducted. They were told that they would be playing a few games to try to teach puppets about counting and number. The child was sat down in front of the camera, and introduced to the puppets involved in the study. The basic nature of the tasks was explained to the children – that they would be playing games where they had to make sure that the two puppets had the same number of toys to play with. The Child's assent was confirmed, and the camera was switched on. Children first performed the pre-test comparison and matching tasks as outlined above. The order of the matching and comparison tasks was counterbalanced, so that half the children performed the matching task first, and the other half performed the comparison task first, to minimise the possibility of order effects occurring. Children were randomly allocated into one of the three intervention conditions, the conceptual intervention condition, the control condition, or the procedural intervention. Following this, they performed the post-test matching and comparison tasks, which were again counterbalanced, to avoid the possibility of systematic order effects occurring. When the experiment was finished, the children were thanked for their time, asked if they had any questions and were then brought back to their class room.

Coding Schemes

Two main variables are of interest for coding children's representational level for the concept of cardinality. Children's performance on the matching task is of importance, alongside the verbal explanations offered in the comparison task. The video recordings of the experiment were used to code these variables using a computer based coding system (the Noldus Observer System).

For the verbal explanations offered in the comparison task a basic coding scheme with 3 categories initially was envisaged, with a number of subcategories emerging following initial viewing of the video tapes of the experiment. The initial categories focused on simple, number-based explanations, visual-based explanations, and "other" or implicit explanations. As Table 5.4 below shows, a number of different subcategories of explanations were coded, to more fully elucidate the range of verbal explanations offered, with a particular emphasis on the variety of number-based explanations offered. Children could give number-based explanations where they mention the numeric value of both sets, and compare them. They could also simply mention the numeric value of one of the sets, or they could say that they counted the two sets, without further elaboration. Finally, children could also give irrelevant numeric explanations, focusing for example on another mathematical function – e.g. "*because 2 + 2 is 4.*" Visual explanations on the other hand eschew numeracy, and focus on the visual aspects of the two sets – whether or not one set just looks bigger then the other. Implicit explanations cover other non-specific explanations offered by children, ranging from *"I don't know",* through to the primitive numeric aspect which may be inferred from phrases such as "*that one's got more"*.

Table 5.4 Coding scheme for the verbal explanations offered on the comparison task

Category	Category definition and examples
Implicit	*explanations without mention of number or any other relevant factor.* *"don't know" "I just know"*
Implicit - More	*Explanations where the children simply state that one set has more objects, without further elaboration, or use of number to compare the two sets* *"because that one's got more"*
Counting – gives non-relevant mathematical explanation	The child gives a numeric-based explanation, but one which is not relevant to the current task, such as giving addition based answers, without giving any mention of comparison between the two sets *"Because 2 and 2 makes 4"*
Counting – I counted	Explanations where the child says simply that they counted the objects, without any further elucidation, or mention of what they counted, or how many objects there were in either set *"because I counted them"*
Counting – Mentions 1 Value	Explanations where the child states the numeric value of only one of the two sets, without mentioning the other, or any relation between the two sets *"Because that one's got 5"*
Counting – Mentions 2 values (one for each set)	Explanations where the child specifically states the numeric values of the two sets, and uses these values to justify their answer with regard to whether or not the two sets were equal *"Because that one's got 4 and this one's got 5"*
Visual	Explanations which are based on children's visual assessments of the two sets, and their relative sizes where counting is not mentioned *"that one looks longer" "I can see that one's bigger"*

In terms of applying the RR model, the counting explanation where 2 values are mentioned is taken as an explicit type of explanation, as there is recognition of the relative importance of the values of both sets. This most closely matches the concept of cardinality, and children were required to offer this type of explanations in order to be coded to an explicit level of representation. Children were expected to give this type of explanation at least once, rather then using the most common form of explanation offered as the basis for coding children's representational level. This approach was chosen due to the relatively small number of trials in which children could provide an explanation (five at pre-test and five at post-test). It is also chosen as a measure which shows the competence of the child, while recognizing, as per the overlapping waves model, that children may show some variability in the strategies they use or in the verbal explanations they use for a task.

Children's performance on the matching task was evaluated in terms of the number of correct matches they made for trials where objects were arranged in straight lines and trials where objects were arranged in a non-symmetrical manner. This division was made based on the findings that children have more difficulty with sets which are not arranged in straight lines (Fuson, 1988). Children at all levels should be able to perform matching tasks involving straight lines, whereas children have more difficulty with sets of objects which are not arranged in a linear manner. Children performing at the implicit level for example would accurately match both types of trial (e.g. trials involving sets of straight lines, and trials involving non-symmetrical sets, see Table 5.5), but would offer no explicit explanations on the comparison task. Children at the abstraction level on the other hand are able to offer explicit explanations on the comparison task, and are able to perform accurately on the matching task where the objects are organised in lines, but are not successful when the objects are not organised in lines. This distinction between objects arranged in straight lines or objects arranged in non-straight lines is broadly analogous to the distinction in performance made by Pine et al with regard to balance beams, where children's ability to balance symmetrical and asymmetrical beams is juxtaposed.

Table 5.5: Coding scheme for applying the RR levels

RR level	Performance on the matching task	Verbal explanations offered on the comparison task
Pre Implicit	0 out of 2 correct for straight line problems, 0 out of 3 correct for non-symmetrical problems	No explicit explanations offered (e.g. counting explanation where two numeric values are compared)
Implicit	2 out of 2 correct for straight line problems, 2-3 out of 3 correct for non-symmetrical matching problems	No explicit explanations offered (e.g. counting explanation where two numeric values are compared)
Implicit Transition	1-2 out of 2 correct for straight line problems, 1-2 out of 3 correct for non-symmetrical problems	No explicit explanations offered (e.g. counting explanation where two numeric values are compared)
Abstraction Non Verbal	2 out of 2 correct for straight line problems, 0-1 out of 3 correct for non-symmetrical problems	No explicit explanations offered (e.g. counting explanation where two numeric values are compared)
Abstraction Verbal	2 out of 2 correct for straight line problems, 0-1 out of 3 correct for non-symmetrical problems	At least one counting explanation where two numeric values are compared
Explicit Transition	2 out of 2 correct for straight line problems, 1-2 out of 3 for non-symmetrical problems.	At least one counting explanation where two numeric values are compared
E3	2 out of 2 correct for straight line problems, 3 out of 3 for non-symmetrical problems	At least one counting explanation where two numeric values are compared

Inter-rater reliability for verbal explanation codings and RR levels on the cardinality principle experiment

For the cardinality task, a second coder viewed and coded 10 children's explanations offered for the error detection task (5 children at pre-test and 5 at post-test). Looking at the verbal explanations, there was a 93% level of agreement (43 of 46 explanations offered). For representational levels, there was a 90% level of agreement (9 out of 10). This helps again to demonstrate the reliability of the coding schemes used here to assign representational levels to children following the error detection task. Table 5.6 shows the high levels of agreement using Cohen's Kappa, indicating the reliability of the coding of these

Table 5.6. Reliability of the coding schemes for the verbal explanations and RR levels for the cardinality principal

Level of agreement on verbal explanations (Kappa statistic in brackets)	Level of agreement on RR levels (Kappa statistic in brackets
93% agreement (Kappa = .924, p < .05)	90% (Kappa = .881, p < .001)

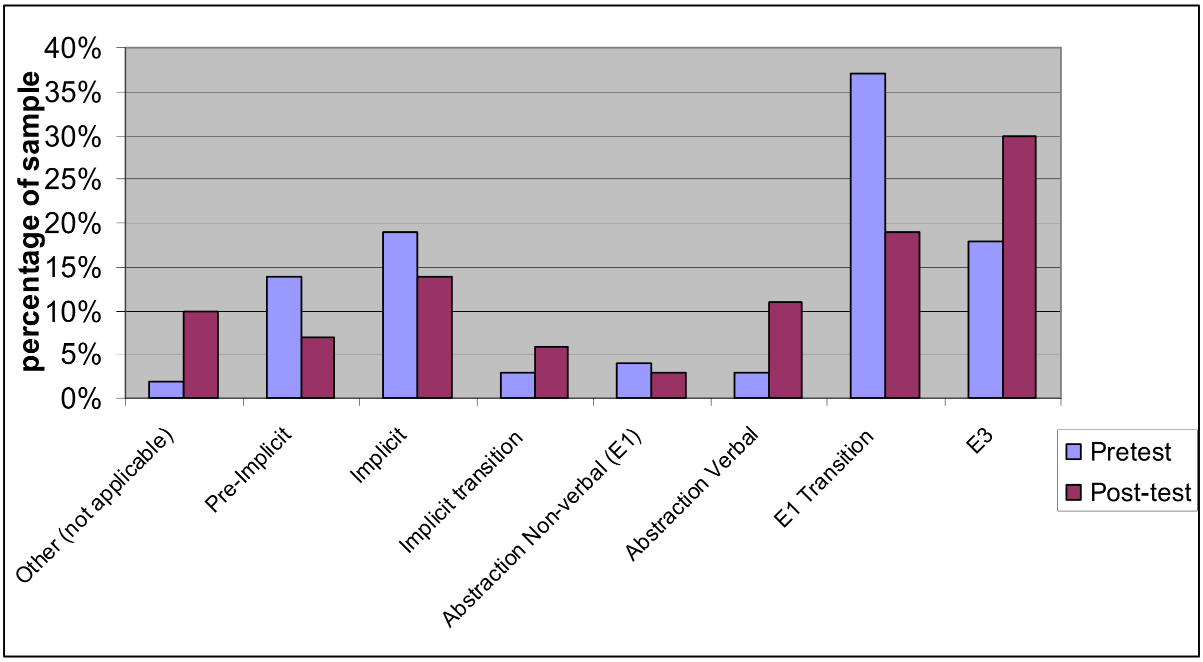
Results

The results will be split into the following sections: (1) Application of the RR levels to the cardinality tasks. (2) Effects of intervention on performance on the matching task, and on explanations offered in the comparison task. (3) Effects of intervention on RR levels

1. Application of the levels of the RR model to the cardinality tasks

Table 5.5 in the methods section was used to apply R-R levels based on performance on the matching task, and verbal explanations offered on the comparison task (Figure 5.2). An immediately arresting fact is the polarity of the distribution – at pre-test a third of the sample are at pre-implicit or implicit levels of performance (and 20% at post-test), with a further 56 % (49% at post-test) at Explicit transition level of performance or above. Very few children seem to perform at the abstraction nonverbal, and abstraction verbal levels. The pre-implicit level of performance noted here gives a strong indication that children do have difficulties with the matching task, and that high levels of performance on this task are not simply a product of the task being too easy.

Figure 5.2: RR levels for the cardinality principle at pre and post-test



There is an interesting increase in children who are not classifiable into RR levels at post-test. There are also increases in children classified into the highest E3 level of performance at post-test, alongside decreases in the percentage of children being classified into the pre-implicit and implicit transition levels at post-test.

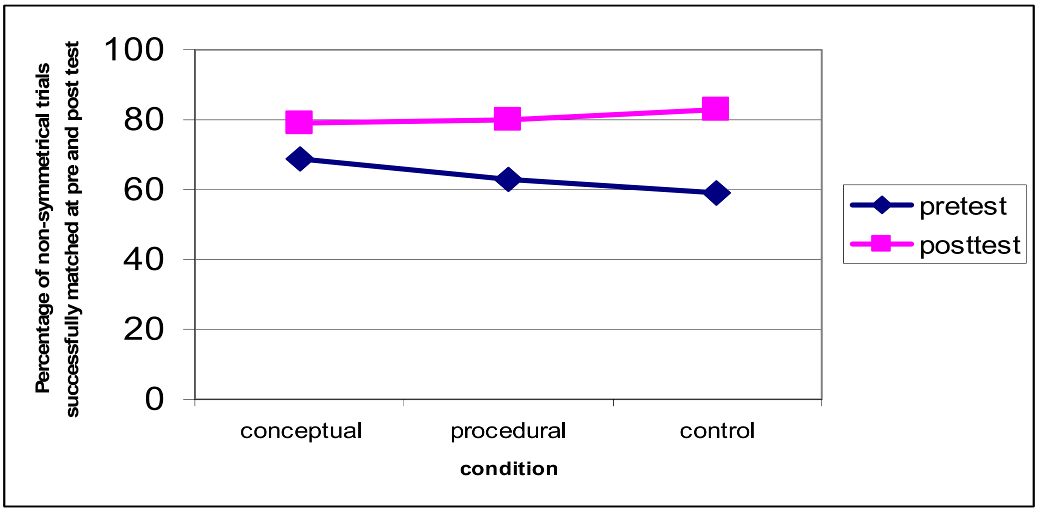
2. Effects of the interventions on performance on the matching and comparison tasks

In this section, the effects of the interventions on the individual tasks will be assessed. The effects of interventions on both tasks will be assessed. Following this, 2 predictions from Rittle-Johnson et al's model will be tested – increases in conceptual knowledge lead to increases in procedural knowledge, and increases in procedural knowledge lead to increases in conceptual knowledge.

2.1 The effect of the interventions on performance on the matching task at post-test

Children's performance on the matching task at post-test was tested by condition. A repeated measures ANOVA was carried out, with performance on non-symmetrical trials at pre-test and at post-test on the matching task as the repeated measures dependent variable, and condition as a factor. The non-symmetrical trials were focused on in the same way that the asymmetrical beams are focused on in the previous chapters, as children were likely to show ceiling levels of performance on the symmetrical trials. There was no significant interaction (F, [2,88] = .5, p = .6). As Figure 5.3 below shows, no appreciable interaction is notable, beyond a simple, non-significant mean improvement in performance from pre to post-test, across all 3 conditions.

Figure 5.3: Performance on the non-symmetrical trials for the matching task at pre- and post-test, by condition



2.2. The effect of the interventions on the verbal explanations offered for the comparison task

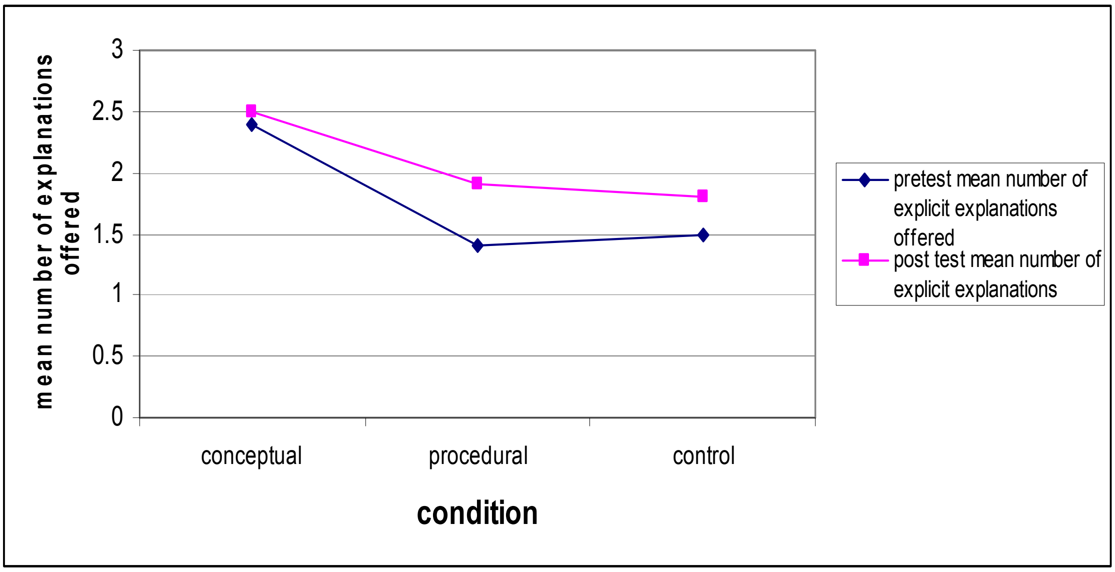
For the verbal explanations offered in the comparison task, it is of interest not only to test whether there has been an increase in specific types of explanations from pre to post-test (as already has been described in the initial results), but whether in particular the conceptual condition led to children offering explicit type explanations involving mentioning the values of both sets. Whether there was an increase in the variety of explanations offered at post-test was also of interest. Table 5.7 below indicates that in general there were no significant mean increases in the use of explanation categories from pre to post-test, with the exception of a significant mean increase in the number of "I counted" explanations, which can be interpreted as a fatigue effect – children were more likely to provide less detailed verbal explanations at post-test.

Table 5.7: Increases in use of explanation categories on the comparison task from pre- to post- test

Verbal explanation category	Mean number of times offered at pretest (+ Standard deviation)	Mean number of times offered at post-test (+ standard deviation)	df	t	p-value
Other	.5 (.96)	.77 (1.07)	93	-1.83	.07
Other – more	.47 (.88)	.59 (.9)	93	-1.15	.25
Counting – “I counted”	.47 (.9)	.83 (1.27)	93	-3.29	.001
Counting Irrelevant maths explanations	.36 (.89)	.3 (.64)	93	.65	.52
Counting mentioning 1 value	.07 (.3)	.16 (.68)	93	-1.09	.28
Counting mentioning 2 values	2.06 (1.74)	1.78 (1.72)	93	1.75	.08
Visual explanations	.29 (.73)	.4 (.75)	93	-1.21	.23

Looking specifically at explicit type explanations (e.g. counting explanations where two values are mentioned and compared) in Figure 5.4 below, it is clear that there is no strong interaction. This finding was confirmed by a repeated measures factorial ANOVA, with the number of explicit explanations offered at pre and post-test as the repeated measures dependent variables, and condition as the factor (F [2,91] = 3.22, p = .08). A further question to be asked was whether children were more likely to introduce explicit explanations at post-test following the conceptual intervention. Of the 9 children who gave explicit explanations at post-test, but not at pre-test, only 2 underwent the conceptual intervention, with 3 children in the procedural condition and 4 in the control condition also showing first use of explicit explanations at post-test.

Figure 5.4: Mean number of times verbal counting explanations involving the comparison of 2 numbers are offered on the comparison task, at pre and post-test, by condition



2.3. Testing the predictions of Rittle-Johnson et al's (2001) iterative model – The relation between gains in procedural knowledge and gains in conceptual knowledge

One prediction derived from the work of Rittle-Johnson et al (2001) was that gains in procedural knowledge predicted gains in conceptual knowledge. Gains in procedural knowledge are measured by improvement in performance on the matching task from pre- to post-test. Gains in conceptual knowledge are measured by whether or not children first show use of explicit explanations on the comparison task at post-test. As Table 5.8 below shows, of the 8 children who did show first use of an explicit explanation at post-test, only 2 showed an improvement in performance at post-test on the matching task (χ^2 [1] =

0.21, p = 0.65, phi = -0.48). Similarly, first use of explicit explanations at post-test was not significantly associated with improvements in performance from pre- to post-test.

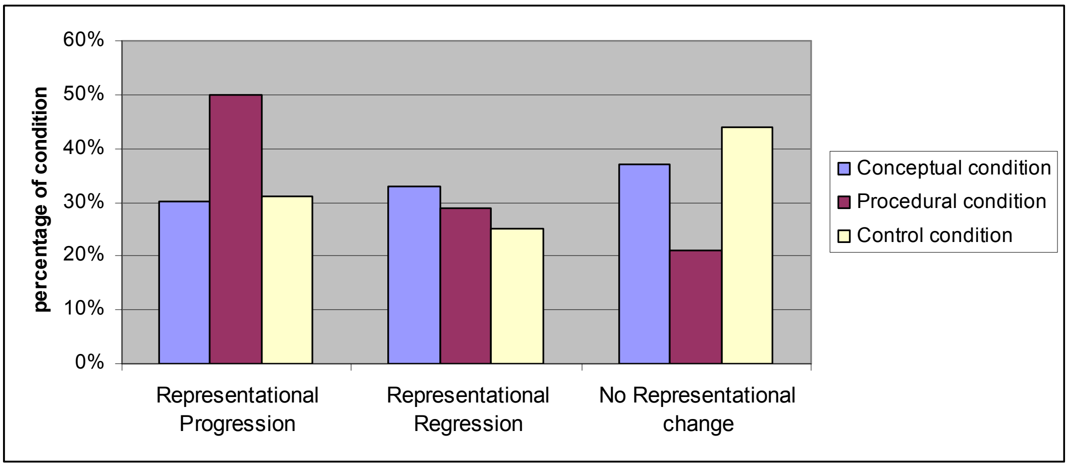
Table 5.8: Crosstable indicating the occurrence of improvements in performance on the matching task from pre- to post-test in relation to first use of explicit verbal explanations at post-test

	Children who first used an explicit explanations at post-test on the comparison task	Children who did not show first use of explicit explanations at post- test on the comparison task
Children who showed improvement in performance on the matching task from pre- to post-test	2	27
Children who did not show improvement in performance from pre to post-test	6	55

3. Effects of the interventions on RR level at post-test

In order to look at the effect of the interventions on RR level, the change in RR levels between pre and post-test was coded. Children could show representational progression (to a later representational level), representational regression (to an earlier representational level), or no representational change. Figure 5.5 below shows representational change across the three conditions. A large proportion of children in the control group showed the same representational level at post-test as at pre-test. Representational progression occurred for 50% of the children in the procedural condition, whereas for the conceptual condition there was almost an equipotential for progression, regression, or no representational change. The chi square Figure from this crosstable did not prove significant (χ^2 [4] = 4.57, p = .33, Cramer's V = .159), indicating that neither the conceptual nor the procedural condition showed any trend towards representational change greater then chance, at the representational level. It has however been predicted that Initial representational level should significantly affect the efficacy of the interventions however, which we will look at next.

Figure 5.5: Representational change from pre to post-test, by condition (%ages by condition)

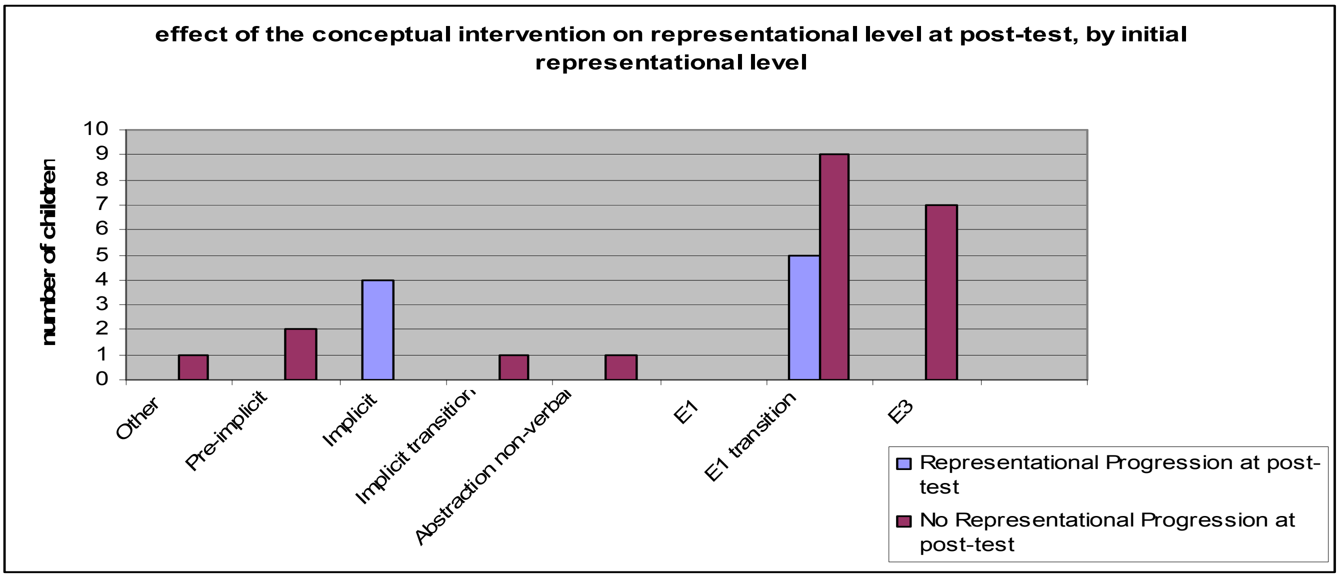


In the introduction (Table 5.1), a specific set of predictions was made with regard to the effect of the interventions, given initial representational level. It was predicted that children who show pre-implicit levels of performance would benefit more from a procedural intervention, which would help to improve performance on the matching task, whereas children who have achieved abstraction non-verbal levels of representation and higher would benefit more from the conceptual intervention, as it would give them a further opportunity to conceptualise errors, and therefore gain a clearer explicit knowledge of the cardinality principle.

3.1. The effect of the conceptual intervention on Representational change, in relation to initial RR level

Figure 5.6 below shows how many children in the conceptual condition showed representational progression at post-test, by representational level (χ^2 [6] = 14.7, p = 0.02, Cramer's V = 0.7). Due to the random nature by which children were allocated to conditions, and the limited number of children who were coded as showing abstraction non-verbal and abstraction verbal levels of representation, only 1 subject who showed one of these two particular levels which are of particular interest for this intervention, were allocated to the conceptual condition. For children showing Explicit transition levels of representation, the conceptual condition did not have any strong effect on whether or not children showed representational progression at post-test. It is interesting to note however that all children who were at the implicit level showed representational progression following the conceptual intervention.

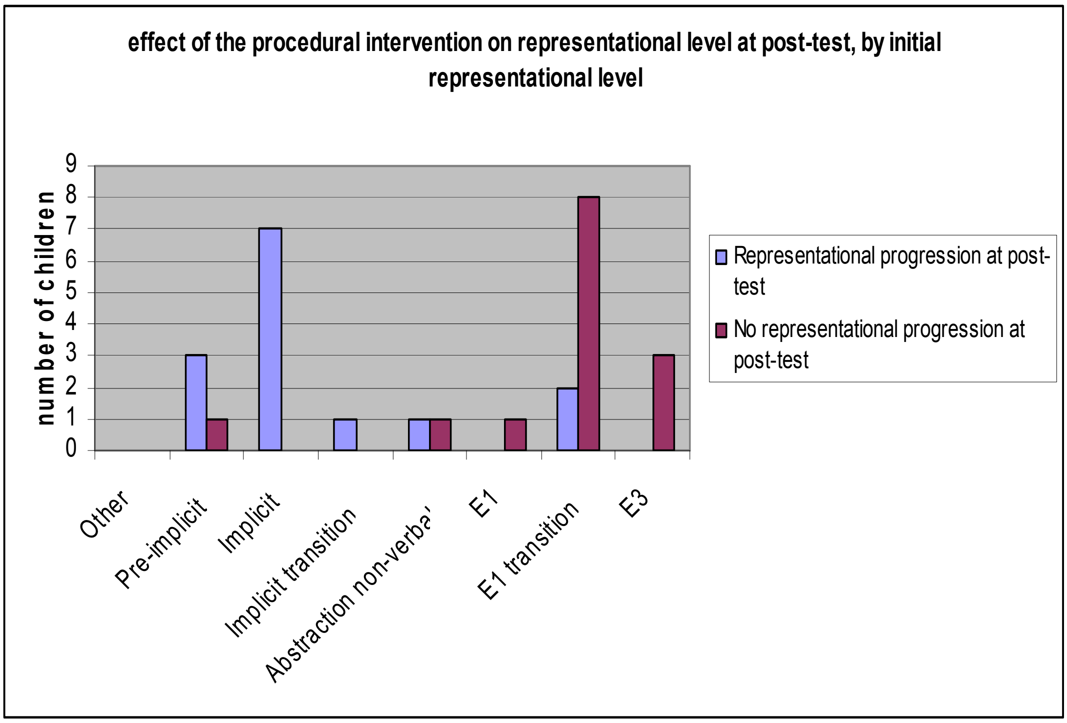
Figure 5.6: effect of the conceptual intervention on RR level at post-test, in relation to initial RR level



3.2. The effect of the procedural intervention on Representational change, in relation to initial RR level

Figure 5.7 below shows whether or not children in the procedural condition showed representational progression at post-test, by initial representational level (χ^2 [6] = 16.6, p = 0.01, Cramer's V = .77). Children at pre-implicit and implicit levels of representation showed representational progression at post-test following the procedural intervention, whereas by Explicit transition level (the next level where there was any appreciable number of children), the effect had disappeared.

Figure 5.7 effect of the procedural intervention on RR level at post-test, in relation to initial RR level



Discussion

This discussion will focus on the following aspects: (1) The application of the levels of the RR model for the concept of cardinality. (2) The efficacy of different types of teaching interventions in relation to initial representational level.

1. The application of the levels of the RR model for the concept of cardinality

This chapter followed on from the previous chapter which applied the levels of the RR model to the one-to-one principle for counting. The levels are in this case applied to the cardinality principle, and carry the same implications in terms of validating the RR model as indicated in the previous chapter. The specific interest in applying the RR model to the concept of cardinality lies in the importance of this concept for children's ability to apply counting as a true mathematical tool. Within Gelman and Gallistel's (1976) definition, the cardinality principle allows a person to know the value of a set. Using the definition derived from Sophian (1993), cardinality allows one to compare sets, and thus begin to apply counting to truly mathematical tasks involving multiple sets.

The application of the levels of the RR model to this task again provide evidence that children do not have an innate implicit representation for this principle, as young children clearly have pre-implicit representations for the cardinality principle. As with the one-to-one counting principle, by looking at children's verbal explanations for how they knew if the 2 puppets had the same number of objects in the comparison task, a range of implicit type explanations (i.e. " I just know", "that one has more"), to the beginnings of understanding of the importance of number (e.g. a verbal explanation such as "because I counted", or giving non-relevant mathematical explanations such as "because 2 and 2 makes 4), through to a fully explicit verbal explanation mentioning the amount of objects both puppets have (i.e. "because that one has 4, and that one has 5"). Again, the RR model provides a description of the development of knowledge beyond simple ability to perform a task (Sophian, 1998). The verbal explanations also show a rich understanding of the importance of cardinality for determining relative value, which is missing in much other work in this area, such as Piaget's number conservation task, whose findings in fact indicate a very late onset understanding of the concept of cardinality.

With regard to the issue of innate knowledge of the counting principles, if children do have "innate" knowledge for these counting principles, they are more primitive than implicit representations within the RR model. What is important is the ability to describe a series of levels through which a child passes as they develop explicit knowledge of the cardinality principle. Bermejo (1996) has already described a series of levels which children pass through in their understanding of the cardinality task. That model however focused on children's performance on a task involving only a single set (which involved children counting in reverse, and then giving the value for a set of items), rather than focusing on children's use of cardinality to compare sets, as this experiment does. Neither does Bermejo's (1996) model focus on children's ability to verbalise knowledge for the cardinality principle. It is also important to note that there has been an argument over when a child has a full knowledge of the cardinality principle. On one side, children may be thought to have some knowledge of cardinality if children use the "last word rule" (e.g. they answer with the last counted number to state how many items are in a set), which occurs early on (Fuson and Hall, 1983). On the other hand, Piaget would state that children do not have a full understanding of cardinality until they can correctly answer number conservation tasks. The RR model offers a description of how explicit knowledge of this principle emerges, which allows for early ability to perform tasks, and give accurate answers to questions, without explicit knowledge of the principle. It is also important to state that a final advantage of the RR model in comparison with Bermejo's model is that it is not a model specific to the development of knowledge of cardinality. Furthermore, as will be seen, it has general implications for how the concept of cardinality should be taught.

2. The efficacy of different types of teaching interventions in relation to initial representational level

In the introduction, studies which focus on the use of interventions to improve "conceptual knowledge", or children's ability to verbalise mathematical concepts were emphasized (e.g. the studies of Rittle-Johnson et al, 2001, Muldoon et al, 2003). While the RR model emphasizes this aspect, it also recognizes the need for a base of behavioural mastery to allow for explicit, verbal knowledge to emerge. This is contrasted with Rittle-Johnson's iterative model, which is open-ended. This model, because it lacks levels, cannot make the type of predictions that the RR model makes – instead it would seem that interventions based either on improving performance (e.g. procedural knowledge) or the ability to verbalise the cardinality principle (e.g. conceptual knowledge) would be equally effective at either point in development. These interventions could also potentially bring about improvements on both measures (e.g. verbal knowledge and performance on the matching task).

The current study tested both these positions, and found no significant main effects for the interventions used. There was no simple effect on representational level at post-test for the interventions. Neither was there a simple effect on the individual measures – the procedural intervention did not bring about a significant increase in performance on the matching task, nor did the conceptual intervention bring about discovery and first use of explicit verbal explanations on the comparison tasks. The specific predictions of Rittle-Johnson's model was also not supported – mutual increases in both measures (e.g. "procedural knowledge" and "conceptual knowledge") were not observed, though there is a possibility that a multi-session experiment might be necessary to observe these kinds of mutual increases.

On the other hand the effects of the interventions seem dependent on the child's initial RR level. Children who were at implicit or pre-implicit representational levels benefited from the procedural intervention. In the case of pre-implicit children, this likely gave them a strategy with which they could use to achieve mastery, and achieve implicit representational levels. For the implicit level children, it seems likely that children used the strategy to consolidate their behavioural mastery, and begin to be able to verbalise the concept of cardinality. Conclusions for the conceptual intervention are more difficult to make. The conceptual intervention, was predicted as being helpful for children who had implicit or more advanced representational levels. The conceptual intervention gave children the chance to verbalise others' miscounts, which Muldoon et al (2003) suggested may help improve children's knowledge of cardinality. This provides an important theoretical basis for Muldoon et al's (2003) findings by explaining how at a certain developmental level, this type of conceptual input may help children to develop more advanced representations for the cardinality principle. There were not enough children allocated to abstraction nonverbal and abstraction verbal representational levels to look at the efficacy of this intervention for children at those levels, though there is some evidence that children at explicit transition still find this particular intervention helpful in improving representational level. This does not warrant a strong conclusion on this particular intervention however, particularly given the nature of the experiment, being done within a single session. A larger study, either with a greater number of students, or over a longer time frame, is necessary to draw clearer conclusions

Taking this caveat into account, it is worth noting that the current findings provides some support for the work of Pine et al (2000) who also stated that initial representational is likely to play an important role in determining the efficacy of teaching interventions. While it is important to note that the focus on conceptual knowledge in the studies of Muldoon et al (2003) and Rittle-Johnson et al (2001) is in agreement with a certain part of the RR model, there is a need to focus on both the child's ability to perform a concept-specific task, alongside their ability to verbalise this concept. This study did show some indication of support for Muldoon et al's (2003) prediction that giving children the opportunity to conceptualise other people's errors can help children improve their representations for the concept of cardinality, though this seems to depend on children's initial representational level. It seems that for this task anyway (see Rittle-Johnson et al, 2001), the ability to perform the task emerges prior to explicit verbalisable knowledge of the task. There is an emphasis therefore on children gradually developing explicit representations for the cardinality principle, with different types of intervention potentially being effective at different points in development. It is important to note that the endpoint of explicit knowledge of the concept of cardinality is highly likely to be important for more complex mathematical tasks. The performance of simple written sums for example rests upon the concept of cardinality; the knowledge that the 2 sets of figures on either side of the equals sign in a simple sum rests upon knowledge of cardinality. The 5 principles for counting outlined by Gelman and Gallistel (1976) are fundamental for more complex mathematical functions, and it is therefore essential to ensure that children have explicit representations for these counting principles, in order to allow them to perform more complex mathematical tasks.

Therefore, children's representational levels not only tell us about the development of children's knowledge for the concept of cardinality, but they can also provide information about which type of intervention children are most likely to find useful. This provides further validation of the RR model

Summary

This experiment applied the levels of the RR model to tasks based on the cardinality principle for counting. The use of different types of "procedural" and "conceptual" interventions were analysed, and it was determined that their utility in improving representational level was dependent on children's initial representational level. This finding supports the prediction of Muldoon et al (2003) that giving children the opportunity to conceptualise others' errors may help children's developing representations for cardinality, though only if they have already achieved an implicit representational levels for this principle. This utility is not open-ended as Rittle-Johnson et al's (2001) model would predict however. Therefore, the RR model was shown to be applicable to this principle, and predictions with regard to the efficacy of different types of teaching intervention in relation to initial RR level were supported.

Chapter 6: A Microgenetic investigation of the stability of children's representations for the domain of balance

Introduction

The aim of the current chapter is to compare the RR model with another contemporary model of cognitive development; the Overlapping Waves model (Siegler, 1996). These models are compared by analysing stability in representational levels across time, and by analysing representational levels at different degrees of magnification to see if children are variable or stable in their representational levels across time.

1. Are children's representational levels stable across time?

A striking feature of the RR model is Karmiloff-Smith's (1992) claim that when children apply their existing representations to a problem in an efficient and consistent manner, thereby achieving a degree of mastery, then progress to a higher level of representation is likely to occur. This is believed to be relevant for progression at each of the levels. An important first step in investigating this claim is to look at whether or not children's representational levels are stable across time. Assessing the stability in representational levels is also an important part of the process of validating the RR model. Pine et al's (2003) work has already shown that changes in representational levels usually involve progress to a higher level. However, the assessments were conducted on 5 separate days so that only limited information was available about the stability of children's representational levels. To investigate stability, a calculation will be made of the percentage of occasions that children showed the same representational level at different time points in comparison with the percentage of time that they show movement to more or less advanced representational levels.

1.1. RR and the Overlapping Waves Model: Stability or Variation?

The question of whether children show stability in representational levels also has a wider theoretical importance because of differences between Karmiloff-Smith's RR model and the Overlapping Waves Model of Siegler (1996). The Overlapping Waves model focuses on variability in children's thinking, and states that most stage-based models underestimate the range of strategies children employ when they are considered to be at one particular stage. Instead, Siegler suggests that children have a variety of "ways of thinking" which they apply to a task. For example, Siegler and Jenkins (1989) have shown that children have a number of different strategies which they use with a simple addition task, rather then simply having only one way of solving addition problems. Siegler has employed formal models such as ASCM (Siegler and Shipley, 1995) to explain how children choose between strategies. These types of models state that children try to make optimal choices in their strategy use, based on the needs of the task – whether for example accuracy or speed are the most important factor. Siegler (1995) notes that children may show variability in strategy use both within and across sessions – children may employ different strategies for similar trials within a session, and may employ different strategies for the same trial at different sessions. Although the term 'level' is used rather than stages in the RR model, it is similar to many stage models, because it is supposed that children's thinking and performance will be similar until they progress to a higher level. To address issues about stability and to compare these two models, two types of analysis will be employed. The first analysis will involve an examination of the stability of representational levels on different days, and also during multiple attempts at the balance beam task on the same day – this analysis looks at stability across time. The second set of analysis will focus on stability of representational levels within a session.

1.2. Magnification and the stability of representational levels

Thelen and Smith (1994) have noted that by analysing data at greater levels of magnification, greater amounts of variability are noted. They state "*development is messy. As we turn up the magnification of our microscope, we see that our visions of linearity.. break down. What looks like a cohesive, orchestrated process from afar takes on the flavour of a more exploratory… function-driven process*" (p. xvi). The basis for this statement was an investigation the emergence of motor skills in infants. Thelen and Smith (1994) observed that by following the individual trajectory of children, various different types of strategies were employed by children to traverse inclined planes (e.g. they could slide, crawl, go down headfirst, or bottom first). On the other hand, at low levels of magnification – e.g. in single session, cross-sectional studies, less variability was apparent. Thelen's work suggests that when investigating variability in behaviour it is important to carry out analyses at different levels of magnification. Moreover, if stability is present at the most detailed level of magnification, in relation to the RR model, then this will provide the strongest form of evidence that stability is in fact an important aspect of child's thinking.

One way to test this is to break down a session into a series of smaller, comparable units for analysis. This approach is drawn from the "microgenetic method" (Siegler and Crowley, 1991) for analysing change. The microgenetic method involves in-depth analysis of change as it occurs. It also involves a high density of testing around a period of change, and a trial-by-trial analysis to infer the processes that give rise to change. This trial-by-trial type approach to analysis can be used to address the question of variability in representational levels. To obtain these detailed data it was decided to analyse children's performance in relation to *subsets* of 3 beams; this was the minimum number of beams needed to assign children to a representational level. A less detailed level of magnification was obtained by looking at children's representational level within a *set* which consisted of a sequence of three of the subsets, and these were considered to make up a *set.* The least detailed level of magnification involved *sessions,* these were based on the coding of the two sets which were administered on the same day. Analyses were conducted to compare the stability of representational levels at these three different levels of magnification.

2. Stability in representational levels within a session

The most direct way to investigate the stability of levels is to analyse the stability of behaviour at the different degrees of magnification across time as discussed in the previous section. A further way to investigate this question is to determine whether coding at different degrees of magnification identifies the same level of representation. In particular, whether coding the levels at different degrees of magnification results in similar or different assessments of children's cognitive functioning. This is relevant to Siegler's (Siegler, 1996) assertion that children are variable both across time (e.g. using different strategies for the same item across sessions), and within sessions (e.g. using different strategies for similar items within a session). The current approach will allow an opportunity to compare RR levels at different degrees of magnification in order to see if children's behaviour remains codable into the same RR level at different degrees of magnification.

3. Do children show variability in the most complex explanations they offer across sessions?

The examination of the stability of children's levels of representation can provide valuable information relevant to validating the RR model and comparing it to the overlapping waves model. It is worth noting that by using Representational levels as a measure to analyse stability, some of the variability in verbal explanations offered by children, particularly within sessions, may not be accounted for. This may arise because representational levels act as summaries, rather then capturing the variety of different types of verbal explanations offered per session. For this reason, a decision was taken to also look in more detail at the stability in children's cognitions, by analysing children's explanations, as these are one of the important constituent behaviours of the levels (Pine et al., 1999). Furthermore, children's verbal explanations have been used by Siegler

(1987) as the basis for coding children's strategy use on addition tasks. Thus, it might be expected according to the RR model that children should either maintain a certain level of verbal explanation, or show improvement across sessions. On the other hand, one would expect from the overlapping waves model that children should show a certain amount of "*ebbing and flowing of the frequencies of alternative approaches*" (Siegler, 1996, p 86). By looking directly at trajectory of the most complex explanations offered by individual children across sessions, a further comparison between the RR model and the Overlapping Waves model can be made.

Summary

In this experiment, the focus concerns whether or not children are stable in their representational levels across time. Three specific questions are addressed: (1) Are children stable in representational levels across time? (2) Do children show stability in their representational levels within sessions? And (3) Are children equally stable in maintaining the most complex explanations offered across time at different degrees of magnification?

Method

Participants

65 children from reception and year one classes from 2 Hertfordshire schools were recruited for this study. The mean age was 66.6 months (standard deviation = 7.4 months, age range: 4 years 6 months, to 6 years 9 months). There were 41 boys and 24 girls.

Design

Table 6.1 below outlines the design for this experiment. Children were tested on a balance beam production task at 4 different time points across a 2 week period. At each of these *sessions* children were given two *sets* of 9 beams to balance. Each set of beams was made up of three *sub-sets.* The same two sets of beams were used in each of the 4 sessions. The order of the sets and beams varied across the 4 sessions to ensure there were no systematic biases. The number of beams used (a total of 18 per session) also helped to minimise the possibility of practice effects on specific beams.

Table 6.1: Experimental Design

Number of sessions	4
Number of beams per session	2 sets of 9 beams (3 symmetrical, 6 asymmetrical for each set)
Order of the sets of beams	Partially counterbalanced, so that children were not always given the same set of beams first
Order of the beams	Within each set the beams were quasi-randomly arranged. Children never received the beams in a set in the same order twice. Each set of beams consisted of 3 subsets of beams, each consisting of 2 asymmetrical and 1 symmetrical beams

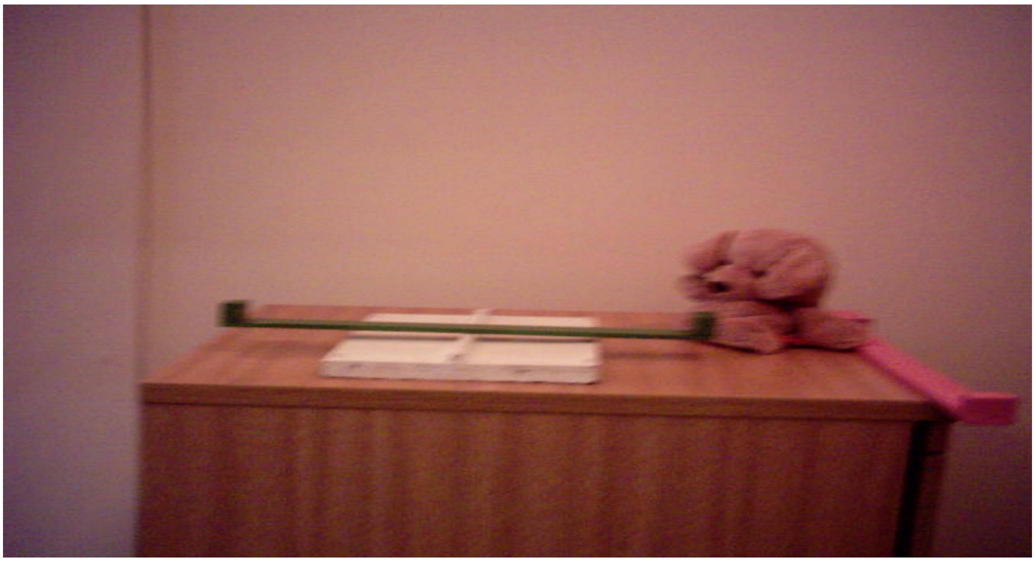
Materials

The balance beam task used was the same as described in chapter 3. In each set of beams there were 6 asymmetrical (for example, see Figure 6.1) and 3 symmetrical beams (for example see Figure 6.2), which children were asked to try and balance on a fulcrum. Symmetrical beams have the same number of weights on both ends. Asymmetrical beams have more weights on one end of the beam then on the other. The 2 sets of beams were carefully chosen so that both sets contained beams with similar characteristics (e.g. number of weights on the beams, length of the beams), to ensure similarity from one set to another. Because of the variety of the beams being used, it was not possible to counterbalance the beams within a subset, other than to ensure each subset contained 2 asymmetrical and 1 symmetrical beam, and vary the presentation of the subsets, so that children weren't always given the subsets in the same order within a set.

The balance beams were wooden and ranged in length between 25cm and 45 cm. Wooden and metallic blocks were placed at either end of the beams acting as weights. These beams could be placed on a fulcrum which consisted of a raised plane of wood, 1cm above a wooden board, and 1 cm in width. Children were asked to try and place the beams across this fulcrum in such a way as to make the beam balance and stay straight.

Figure 6.1: example of an asymmetrical balance beam

Figure 6.2: example of a symmetrical balance beam



Procedure

The first three sessions took place within a week, with a day between each session (the children were tested on a Monday, a Wednesday, and a Friday). The final session was to take place a week after the third session (the following Friday). A similar procedure was used in each session.

The child was brought from the classroom, and told they were going to be playing a few games, and learning about how to make things balance. The child was sat at a Table, and introduced to the apparatus to be used (e.g. the balance beams, and the puppet, to whom they would be explaining why the beams did or did not balance). A different puppet was used at each of the 4 sessions, in order to encourage children to keep giving explanations to these ‘new’ characters.

When they were introduced to the balance beam task the child was asked to try and make each beam balance on the fulcrum. For each beam they were told: *“I would like you to try and make this beam balance and stay straight on top of this piece of wood here (*the fulcrum*)".* The child attempted to make the beam balance and stay straight, and either succeeded, or stated that the beam would not balance. If the child could not balance the beam after 1 minute, they were prompted with the question: *"Do you think you can make this beam balance and stay straight?"* When they managed to make the beam stay straight and balance, the child was asked *"Can you explain to the puppet why this beam is balancing?"* If the child did not succeed in balancing the beam, they were asked *"Can you explain to the puppet why this beam is not balancing?"* The same procedure was used for each of the 18 beams given to the child per session, across the 4 sessions.

Coding schemes

The 2 measures of interest were the children's verbal explanations offered, and their performance. Together these were used to identify the level of representation shown by a child. Table 6.2 below shows the coding scheme applied to children's verbal explanations. Children can offer various types of Implicit or Explicit explanations as to why a beam does or does not balance. A key difference between Implicit and Explicit weight explanations is the use of weight and weight-based terms (e.g. "weight", "heavy", "bigger"), whereas children who are coded as having given implicit weight explanations focus only on the number of blocks on each side of the beam, without explaining that more blocks on one side means that there is a larger weight on that side.

Table 6.2: Coding Scheme for Verbal explanations on the balance beam task:

Coding Category	Examples of explanations
Other	Explanations which do not show any signs of explicit knowledge of the variables of weight and distance, or which are not relevant, or audible
Implicit	Explanations which do not show any signs of explicit knowledge of the variables of weight or distance. Examples: *"I thought really hard" "I just tried" "it's the same as the others"*
Implicit (weight)	Explanations which enumerate the set of weights placed on the scale, or the number of blocks on the beam, without any use of explicit, weight-based words. Examples: *"there's more on that side" "there are 2" "that one's got two blocks, this has got one"*
Implicit (distance)	Explanations which only use vague distance-based explanations which lack any explicit mention of distance based terms (e.g. centre, far, or near). Examples*: "I put it there"*
Same/ Middle	Explanations which focus on how the two sides are the symmetrical, or how the geometric centre of the beam was placed on the fulcrum. Examples: *"in the middle" "they're symmetrical"*
Weight	Explanations which give explicit mention of the weight of the blocks, or the weights placed on the pegs, or which give some mention of weight-related terms. Examples: *"It's a bit heavy" "there aren't the same amount of weights" "It's stronger" "It's bigger" "It doesn't have a weight at the other end"*
Distance	Explanations which focus on the relative placement of the weights in relation to each other, or in relation to the fulcrum. Examples: *"It's closer on this side"" "It's not in the middle" "It's further out"*

Assigning RR levels per session

Table 6.3 below contains the criteria used for assigning a child a representational level in each session. These criteria are derived from the work of Pine et al (2003). For the balance beam task, the important criteria include the number of symmetrical and asymmetrical beams successfully balanced (e.g. the beams stay balanced around the fulcrum after the child has finished manipulating them), the initial placement of the beams (e.g. whether the child placed beams at their geometric centre to start with, or whether they initially placed them off-centre), and the explanations offered.

Table 6.3: Criteria for coding the Balance Beam production task into RR levels, per session

RR Level	Number of beams balanced	Initial Placement of beam	Verbal explanations offered
Implicit	Successfully balances 4 of 6 symmetrical beams, and at least 8 of 12 asymmetrical beams	No bias in initial placement across the beams (e.g. initially places the beams in the geometric centre for less then 9 of the 18 beams)	Less then 4 implicit/explicit weight explanations offered
Implicit Transition	Successfully balances 3 of 6 symmetrical beams at most, and at best 4 of 12 asymmetrical beams	Initially places the beams in the geometric centre for at least 12 of the 18 beams	Less then 4 Implicit Weight explanations offered
Abstraction Non-Verbal (E1)	Successfully balances at least 4 of 6 symmetrical beams, and at best 4 of 12 asymmetrical beams	Initially places the beam in the geometric centre for at least 12 of the 18 beams	Four or more implicit weight explanations offered, less then 2 explicit weight explanations
Abstraction Verbal	Successfully balance all symmetrical beams, and 4 or less asymmetrical beams	Initially places the beam in the geometric centre for at least 12 of the 18 beams	At least four explicit Weight / centre based explanations offered
E1 transition	Successfully balances all symmetrical beams, and 6 or more asymmetrical beams	Initially places the beam in the centre, with adjustment to place heavier side closer to the fulcrum, for at least 6 of the 12 asymmetrical beams	At least four explicit Weight/centre based explanations offered
E3	Successfully balances all symmetrical beams, and 8 or more asymmetrical beams	Initially places the beam in the centre, with adjustment to place heavier side closer to the fulcrum, for at least 6 of the 12 asymmetrical beams	At least four explicit explanations involving both weight and two explicit explanations involving distance

Assigning RR levels per set

Table 6.4 below shows the criteria for coding children in to representational levels per set of 9 beams. The criteria are very similar to those in Table 6.2 above, and maintain the same format of representational levels.

Table 6.4: Criteria for coding the Balance Beam production task into RR levels, per set

RR Level	Number of beams balanced	Initial Placement of beam	Verbal explanations offered
Implicit	Successfully balances 2 of 3 symmetrical beams, and at least 4 of 6 asymmetrical beams	No bias in initial placement across the beams (e.g. initially places the beams in the geometric centre for less then 6 of the 9 beams)	Less then 2 implicit/explicit weight explanations offered
Implicit Transition	Successfully balances 2 of 3 symmetrical beams at most, and at best 2 of 6 asymmetrical beams	Initially places the beams in the geometric centre for at least 6 of the 9 beams	Less then 2 Implicit Weight explanations offered
Abstraction Non-Verbal (E1)	Successfully balances at least 2of 3 symmetrical beams, and at best 4 of 6 asymmetrical beams	Initially places the beam in the geometric centre for at least 6 of the 9 beams	Two or more implicit weight explanations offered, less then 2 explicit weight explanations
Abstraction Verbal	Successfully balance all symmetrical beams, and 2 or less asymmetrical beams	Initially places the beam in the geometric centre for at least 6 of the 9 beams	At least two explicit Weight / centre based explanations offered
E1 transition	Successfully balances all symmetrical beams, and 3 or more asymmetrical beams	Initially places the beam in the centre, with adjustment to place heavier side closer to the fulcrum, for at least 3 of the 6 asymmetrical beams	At least two explicit Weight/centre based explanations offered
E3	Successfully balances all symmetrical beams, and 5 or more asymmetrical beams	Initially places the beam in the centre, with adjustment to place heavier side closer to the fulcrum, for at least 3 of the 6 asymmetrical beams	At least two explicit explanations involving both weight and two explicit explanations involving distance

Assigning RR levels per subset

The representational levels were also applied to subsets of 3 beams within each session. A different coding scheme was required, as with only 3 beams, transitional levels were not readily codable. The criteria also had to be more stringent with a smaller set of beams, with an "all or nothing" criteria in terms of the number of beams balanced. Table 6.5 below shows the criteria used. An extra "no level coded" category was created to categorise performance that could not be coded into any of the levels set out below.

Table 6.5: Criteria for coding the Balance Beam production task into RR levels, per set

	Number of Asymmetrical Beams Balanced	Number of Symmetrical Beams Balanced	Explanations offered	Initial placement of the beam
No level coded	–	–	–	–
Implicit	2	1	No explicit explanations offered, no implicit weight or distance explanations offered	No pattern
Abstraction Nonverbal	0	1	No explicit explanations offered, at least one implicit weight explanation offered	Middle placement for at least 2 beams
Abstraction Verbal	0	1	At least 1 explicit weight explanation	Middle placement for at least 2 beams
E3	2	1	At least 1 weight and distance explanation	No pattern

Because the balance beam task has been applied to tasks before, and has been second-coded already in this PhD (see chapter 3), it was felt that there was no need to second code any of the data from this experiment.

Results

The results are divided into the following sections:

(1) Representational stability and representational change across time
(2) Comparing representational levels at different degrees of magnification
(3) Stability in the most complex verbal explanations offered across time

1. Representational stability and representational change across time

This section will comprise an examination of the percentage of occasions children showed the same representational levels across sessions. The first set of analyses focuses on children's representational levels from session to session, at the lowest degree of magnification. This is followed by a graphical analysis of the data from session to session to give a clear overview of how often child showed the same representational levels or showed change in representational levels from session to session. Following this, the focus will shift to analysis at higher degrees of magnification; representational levels from set to set and from subset to subset. The three degrees of magnification will then be compared to see how often children show stability, and movement to more or less advanced representational levels at these different degrees of magnification.

1.1 Representational stability and representational change across the 4 sessions

For sessions 2, 3, and 4, the coding of a child's representational level was compared with that of the previous session. Each session was coded as being a more advanced representational level, a less advanced representational level, or the same representational level as the previous session. Table 6.6 below shows that for a majority of the time, children show stability in representational levels from session to session. A One-Way ANOVA, with the number of times children showed progression to a different RR level, regression to a less advanced level, and the type of change (or stability) as the independent variable indicated there was a significant difference in the frequencies of the three different types of transitions in levels between sessions (i.e. same, more advanced, less advanced) F [2,192] = 77.39, $p < .001$. Post-hoc comparisons using Tukey's HSD revealed significant differences between all 3 measures. The frequency of transitions involving the same representational level were significantly higher than those involving a transition to a less advanced ($p <. 001$) and more advanced representational level ($p < .001$) and transitions to a less advanced representational level were significantly more frequent to those to a more advanced representational level ($p = 0.022$).

Table 6.6: The percentage of times children showed stability, progression to a more advanced RR level, or movement to a less advanced RR level from session to session

	Percentage
Percentage of time children maintained the same representational level from session to session	61.3%
Percentage of time children showed movement to a more advanced representational level from session to session	14.3%
Percentage of time children showed movement to a less advanced representational level from session to session	24.4%

A closer view of these data show that (Table 6.7) the majority of advances to higher levels of representations were to the next representational level in the sequence (e.g. Abstraction Nonverbal to Abstraction Verbal, or Abstraction Verbal to Explicit transition, were the 2 most common changes occurring). The majority of movements to less advanced levels involved Abstraction Verbal to Abstraction Nonverbal representational levels (22/51 transitions).

Table 6.7: Frequency of occurrence representational changes involving movement to a hierarchically adjacent RR level from session to session

	Percentage of movement to a hierarchically adjacent representational level
Movement to a more advanced representational level	88.5%
Movement to a less advanced representational level	67.3%

1.2. Graphical Analysis of children's representational levels from session to session

The trajectory of children's representational levels across the four sessions are shown in Figs 6.3-6.5. Due to the relatively large sample it was decided to carry out separate analyses for the following groups of children: (1) children showing a decline in their representational level from the first to the last session; (2) children showing the same level at the first and last sessions, and (3) children showing a more advanced representational level at the last session than at the first session. Analyses based on these groups only occur in this section.

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1.2.1 Children who show movement to a less advanced representational level from the first to the last session.

Figure 6.4 charts the trajectories of children (N=30) who showed a less advanced representational level at the last session than at the first session. This group are most likely to provide support for the notion that children show variability, rather than stability in their representational levels across time. In figures 6.3 through 6.5 , thicker dark lines indicate a high frequency of children followed a particular trajectory. Several features of the graph deserve comment.

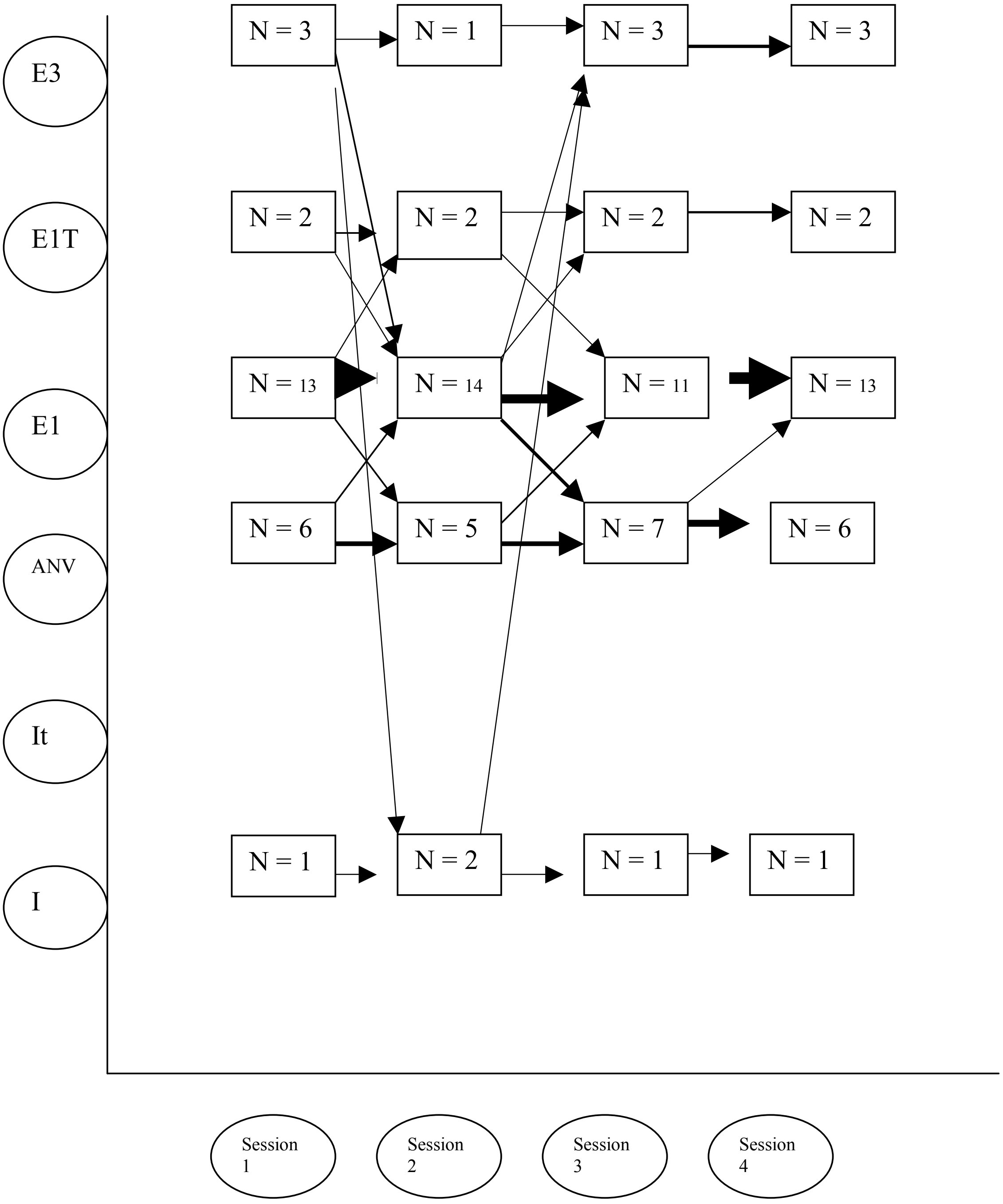


Figure 6.3: trajectory for children showing same representational level at first and last session

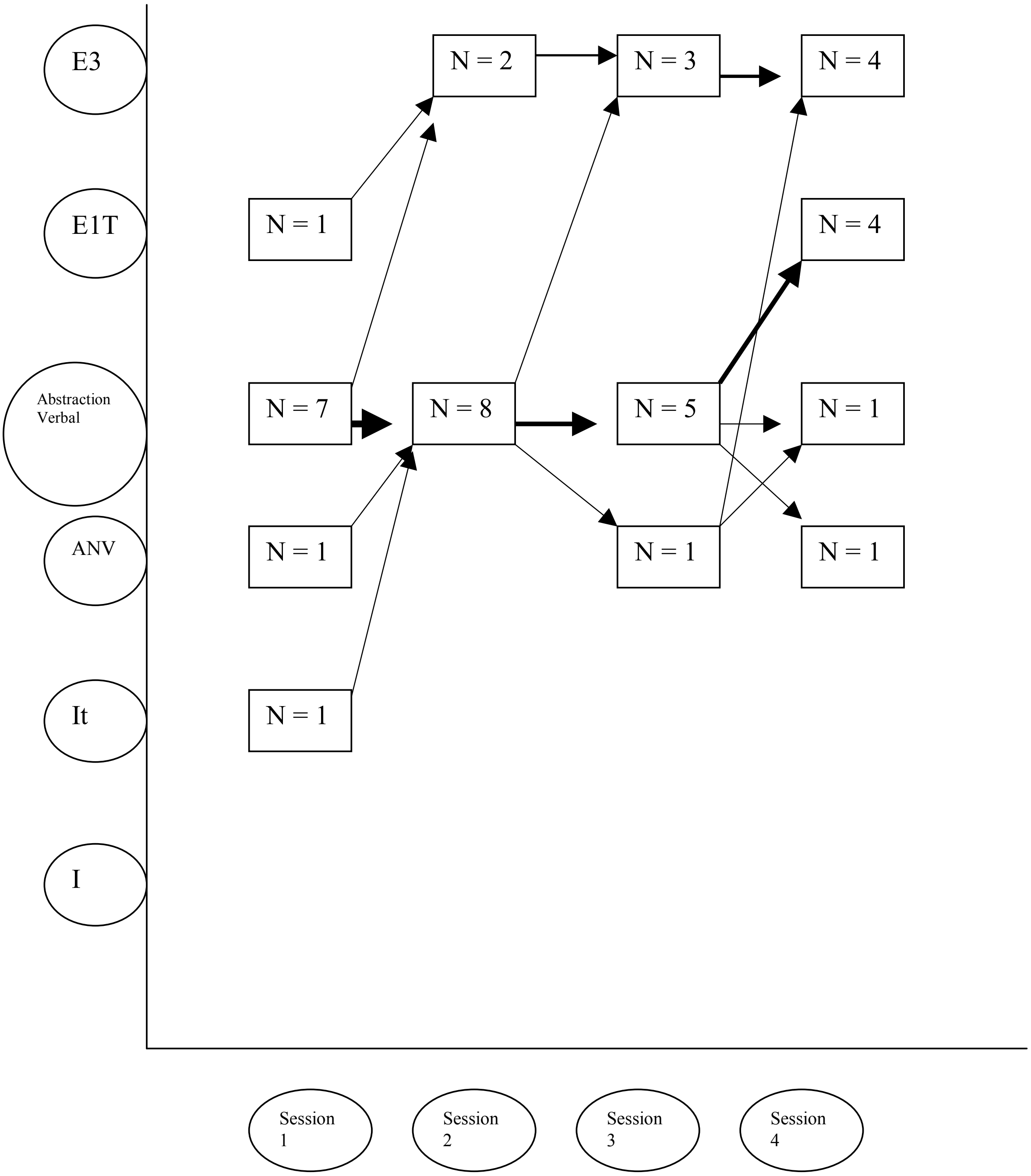


Figure 6.4: graphing children who showed representational progression from the first to the last session

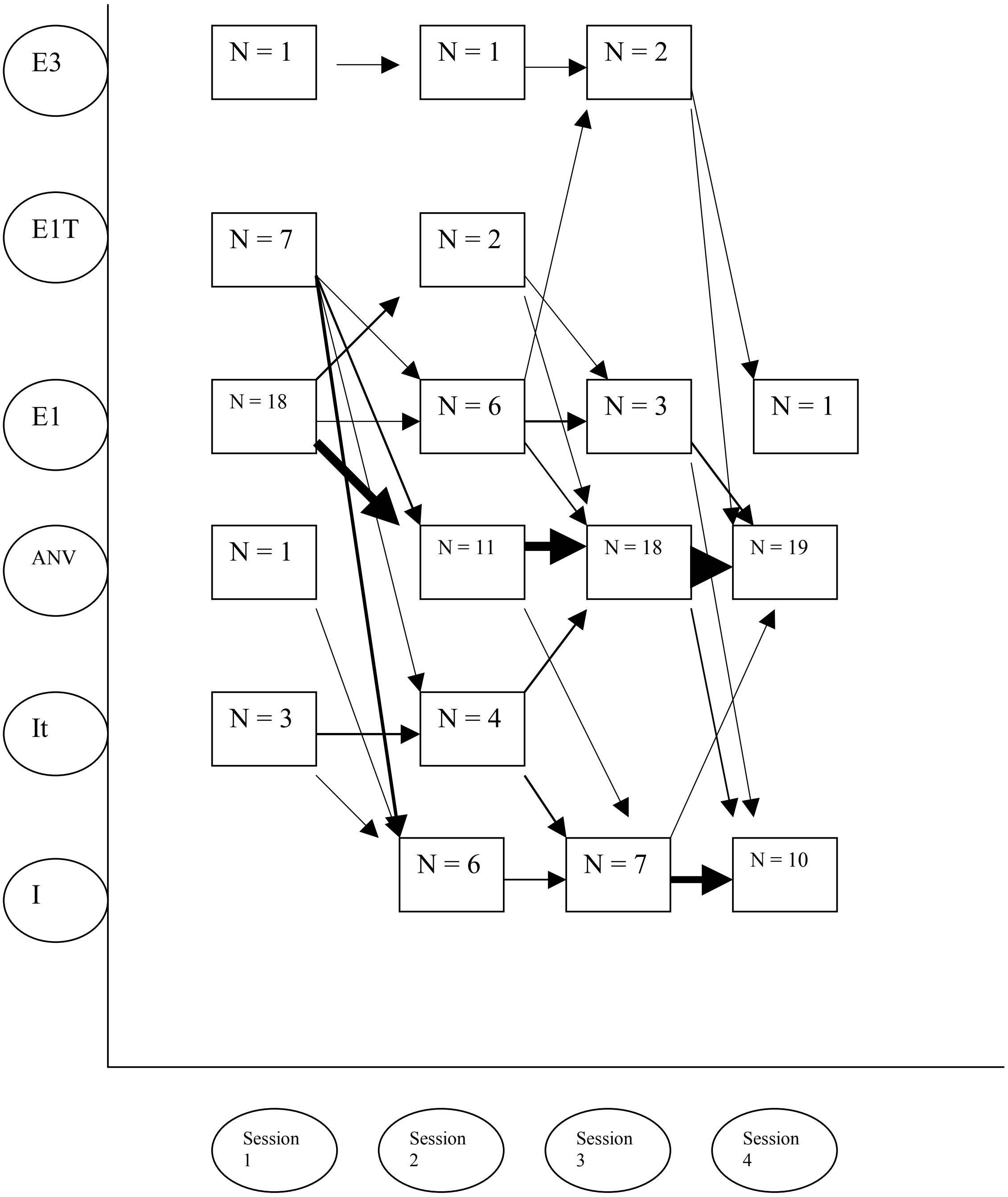


Figure 6.5: graphing children who showed a lower RR level on the final session in comparison with the first session

The change involving the largest number of children was from Abstraction Verbal to Abstraction Nonverbal, from the first to the second session. Once children achieve this level, most of them stayed at this level for the remaining sessions. Thus, it would appear that the greatest variation in this group involved two levels which are closely related and only differ according to whether or not children explicitly mention weight in their explanations.

It also is apparent that the majority of children who moved to implicit representational levels from a higher level, then maintained this representational level. Thus, although a decline in the level of representation occurred, there was not a great deal of variation afterwards. Other changes to lower levels, involved movement from transitional levels to the levels believed to precede them (e.g. implicit transition to implicit, and from explicit transition to abstraction verbal representational levels).

1.2.2 Children who show the same representational level at the first and last sessions

Figure 6.3 gives the trajectories of the children (n=24) who show the same representational levels at the first and last sessions. Of these children, 15 showed the same level across each session (Implicit = 1, Abstraction Nonverbal = 4, Abstraction Verbal = 7, Explicit transition = 1, E3 = 2). The other 9 children showed some inconsistency from the first to the second session, but returned to the original level by the third session and maintain it at the fourth session. Thus, a significant proportion of the sample showing stability in their representational level across the 4 sessions.

1.2.3 Children who show movement to a more advanced representational level from the first to the final session

Figure 6.5 shows the trajectories for children who had a higher representational level at the final session than at the first (N= 10). The majority of children showed a change from

Abstraction Verbal to explicit transition or E3, with one child progressing from Abstraction Nonverbal to abstraction verbal representational levels, and one child progressing from Implicit transition to Abstraction Nonverbal representational levels.

1.3. Representational stability and representational change across sets

Each session contained 2 sets, so data at this level of magnification was obtained about 8 representational levels across the course of the experiment. Graphical descriptions of 8 sets were too complicated to display and inspect. As a result, an analysis was carried out similar to that carried out in section 1.1 – for the second through to the eighth set, children's representational levels were coded, in relation to the representational level in the previous set. Children could show the same representational level, a more advanced representational level, or a less advanced representational level. On 65% of occasions, children's representational were the same as in the previous set (see Table 6.8). As with children's representational levels from session to session, movements to a lower representational level from set to set occurred more often then advances. A One-Way ANOVA, with the number of times children showed the three types of movement from set to set as the dependent variable(e.g. maintaining the same representational level per set, showing a more advanced representational then in the previous set, or showing a less advanced representational level then in the previous set) and the type of change as the independent variable showed that the frequency of stability between sets was significantly higher than the frequency of decreases or increases between sets F [2,192] = 172.77, $p < .001$. Post-hoc comparisons using Tukey's HSD revealed 2 sets of significant differences between measures the frequency of stability was significantly higher than the frequency of either decreases ($p < .001$)or increases ($p < .001$), but there was no significant difference between the frequency of advances and decreases in level ($p = 0.156$).

Table 6.8: Frequency of stability, advance, and decline in representational level from set to set

	Percentage
Percentage of time children maintained the same representational level from set to set	65.1%
Percentage of time children showed an advancement in representational level from set to set	14.5%
Percentage of time children showed movement to a less advanced representational level from set to set	20.4%

A more detailed analysis of these movements is shown in Table 6.9. This shows that a majority of these movements to less advanced representational level involved movements to the level immediately prior in the sequence (e.g. from abstraction verbal to abstraction nonverbal, and from explicit transition to abstraction verbal,), rather then movement to levels that are at a much earlier point in the sequence of representational levels.

Table 6.9: Frequency of occurrences of representational changes involving movement to a hierarchically adjacent representational level from set to set

	Percentage of movement to a hierarchically adjacent representational level
Movement to a more advanced representational level	77%
Movement to a less advanced representational level	79%

1.4 Representational stability and representational change across subsets

At this level of magnification there were 24 subsets (as there were 3 subsets in each set). Again, from the second subset onwards, children's representational levels were compared with the representational level from the previous subset – children could show the same representational level, a more advanced representational level, or a less advanced representational level. When subsets could not be coded into representational levels, it was not possible to say whether there was movement to a more or less advanced representational state. Table 6.10 shows that 35.3% of the time, children maintained the same representational level from subset to subset. This percentage is lower than the rates for stability from session to session (Table 6.6) and from set to set (Table 6.8) –and is in large part due to the incidence of "noncodable" subsets, which does not happen at session and set levels of magnification. The second column of figures in Table 6.7 shows the percentages for children maintaining the same representational level, movement to more advanced representational level, or movement to a less advanced representational when the uncodable levels are not taken into account. Uncodable subsets were subsets where the patterns of performance and verbal explanations did not fit in with the criteria set out in Table 6.5. Out of 1572 codable subsets, 33.5% (527 of 1572) were uncodable. When an uncodable subset occurred, at least 2 points of potential change are lost – from the subset prior to the uncodable subset, and then from the uncodable subset to the following subset. Due to the unknown nature of the uncodable subset, movement from a codable subset to "uncodable" subset is not coded as change – indeed it is not coded as "not measurable". Therefore, though uncodable subsets occurred just over a third of the time, this had a much higher impact in measuring changes in RR level and stability in RR level from subset to subset.

A One-Way ANOVA, was conducted on the number of times children showed movement from subset to subset as the dependent variable, and the type of movement as the independent variable (e.g. maintaining the same representational level between subsets, showing a more advanced representational than in the previous subset, or showing a less advanced representational level than in the previous subset, discounting occasions where children's representational level was uncodable). This showed that there was a higher frequency of occasions where children showed stability from session to session than showing an increase or decrease in representational level from session to session, F [2,192] = 253.63, p < .001. Post-hoc comparisons using Tukey's HSD revealed 2 sets of significant differences between measures. Children showed stability on a significantly higher number of occasions than showing a more advanced representational level (p<. 001) or showing a less advanced representational level (p <. 001). There was no significant difference between the frequency of movements to more advanced representational levels and less advanced representational levels from subset to subset (p = 0.918).

Table 6.10: Frequency of occurrence of stability, movement to a more advanced RR level, and movement to a less advanced RR level from subset to subset

	Percentage	Percentage (not taking uncodable subsets into account)
How often children showed representational stability from subset to subset	35.3%	71.7%
How often children showed an advancement in representational level from subset to subset	5.6%	12.7%
How often children showed movement to a less advanced representational level from subset to subset	6.1%	15.6%

Table 6.11 below shows a break down the types of movement from representational level to level from subset to subset. The majority of increases and decreases in representational level involved movement to an adjacent representational level (e.g. movement from abstraction nonverbal to abstraction verbal representational level which occurred 49 times, or movement from abstraction verbal to abstraction nonverbal, which occured 54 times – the 2 most common forms of movement between representational levels which was codable here).

Table 6.11: Frequency of occurrence of representational changes involving movement to a hierarchically adjacent representational level from subset to subset

	Percentage of movement to a hierarchically adjacent representational level
Movement to a more advanced representational level	71.7%
Movement to a less advanced representational level	83.7%

1.5. Comparing stability in representational levels at different degrees of magnification

The next step in the analyses was to compare the different degrees of magnification in terms of how often children showed the same representational levels across time, how often they showed movement to more advanced representational levels across time, and how often they showed movement to less advanced representational levels across time. Table 6.12 below shows the relative frequencies of these 3 measures at the different degrees of magnification. A series of One-Way ANOVAs, using degree of magnification as the independent variable (e.g. session, set, and subset), and percentage frequency for maintaining the representational level, movement to a more advanced representational level, and movements to a less advanced representational level as dependent variables were conducted. There was no significant difference between the different degrees of magnification in terms of the percentage of times children maintained the same representational level across time (F, [2,192] = 2.87, p = 0.6), nor in terms of how often children showed movement to a more advanced representational level (F, [2,192] = 0.291, p = .748). There was a significant difference for movement to a less advanced representational level, F [2,192] = 7.35, p = 0.001, with a post-hoc Tukey's HSD revealing a significant difference between session and subset (p = .001). Movement to less advanced representational levels occurs significantly less frequently at the subset degree of magnification then at the session degree of magnification.

Table 6.12: Percentage of times children the same representational, showed movement to more advanced representational levels, and movement to less advanced representational levels at different degrees of magnification

	Mean %age of times children maintained the same representational level	Mean %age of times children showed a more advanced representational level then in the previous session/set/subset	Mean %age of times children showed a less advanced representational level then in the previous session/set/subset
From session to session	61.3% (standard deviation = 29.04)	14.3% (standard deviation =17.47)	24.4% (standard deviation = 20.44)
From set to set	65.1% (standard deviation =23.25)	14.5% (standard deviation =12.5)	20.4% (standard deviation =12.75)
From subset to subset	71.7% (standard deviation =23.54)	12.7% (standard deviation = 12.91)	15.6%(standard deviation = 12.43)

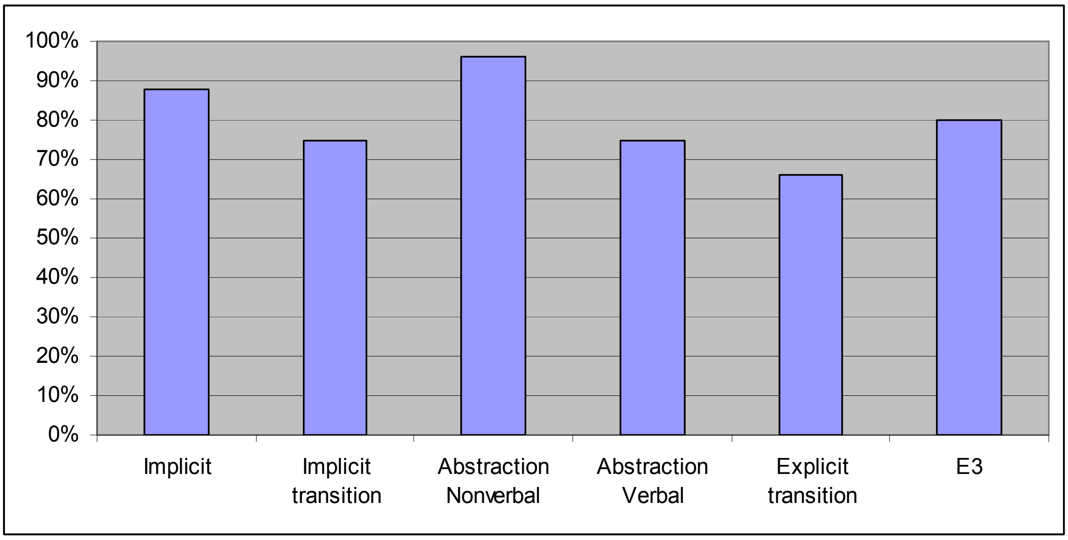
2. Comparing representational levels at different degrees of magnification

The next step in analysis was to investigate the relation between the different levels of magnification. This involved examining the relation between the coding of representational levels for each session and the corresponding sets, and between each set and the corresponding subsets.

2.1. Comparing representational levels coded per sets and per session

A calculation was made of the percentage agreement between the level of representation identified for a session and the levels of representation identified for the two corresponding sets. Figure 6.6 shows how often a child's representational level for a set matched the representational session assigned to the session of which that particular set was a part. A very high degree of agreement between the representational levels was identified for the sessions and the representational levels identified for the sets within these session (χ^2 [25] = 1473, p < .001, Cramer's V = .87).

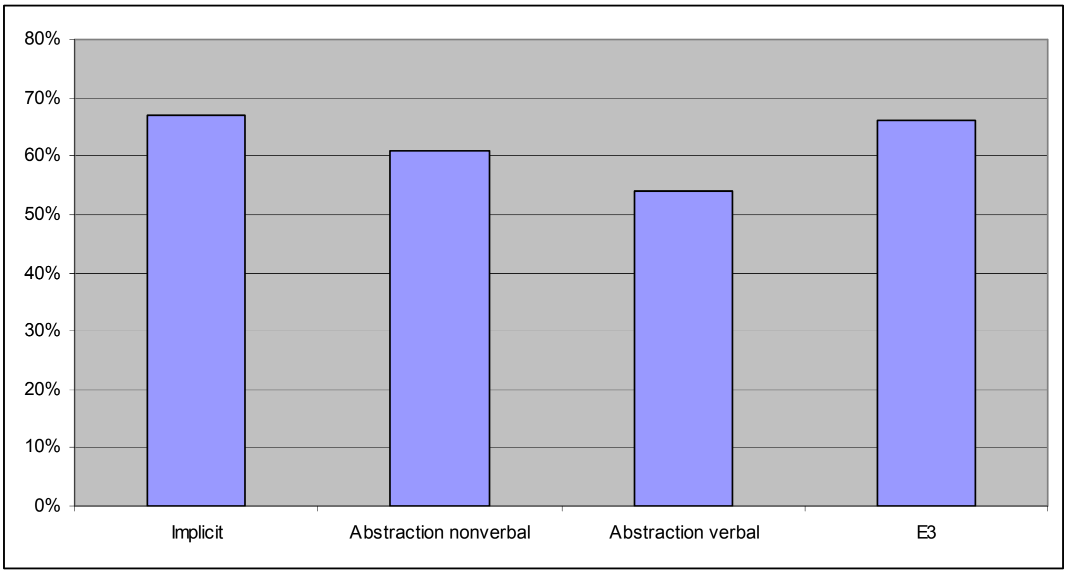
Figure 6.6: Percentage agreement between RR level per session and RR level per set



2.2 Comparing representational levels coded per set and per subset

A calculation was made of the percentage agreement between the level of representation identified for a set and the levels of representation identified for the three corresponding sets. Figure 6.7 shows how often children's representational level for a subset were in agreement with the representational level for the set of which the subset was a part. There were lower levels of agreement between set and subset then between session and set. The results still indicate relatively high levels of reliability though (χ^2 [20] = 2276, p < .001, Cramer's = .89). That is to say roughly two thirds of the representational levels in a subset matched those of the set from whence they came time (these data do not include occasions where representational level in a set was a transitional level, as there is no subset equivalent).

Figure 6.7: Percentage agreement between RR level per set and RR level per subset



3. Stability in the most complex verbal explanations offered across time

3.1 Stability in most complex verbal explanations offered across sessions

The most complex explanations children offered in a session, set, and subset were analysed. The sequence laid out in Table 6.2 (see methods section – verbal explanation coding scheme) was used as the basis to code the most complex verbal explanations – it is worth noting that technically, distance will be coded as a more complex verbal explanation than weight explanations, as a majority of studies in the area of balance show that weight explanations emerge prior to distance explanations.

The same method as in section 1.1 was used to analyse the changes in the most complex explanations between the different levels of magnification. Three possibilities were coded – children showing a more complex explanation than the previous session, children showing the same type of verbal explanation as the previous session, or children showing a less complex verbal explanation than the previous session. Table 6.13 shows that for a majority of the time, children show stability in the most complex explanation they offered from session to session. The percentages produced in this analysis, do not differ greatly from those of representational levels from session to session (see Table 6.1). A One-Way ANOVA was conducted with the frequency of the types of changes (e.g. same, increase or decrease in most complex verbal explanations offered) as the independent variable and the type of change as the dependent variable. This revealed a significant difference, F (2,192) = 68.11, $p < .001$. A post-hoc analysis using Tukey's HSD revealed significant differences showed significant differences between children showing the same explanation from session to session and children showing less complex explanations from session to session ($p<.001$), and between children showing the same explanation from session to session and increases in most complex explanations from session to session ($p < .001$). No significant difference is found between the mean amount of times movement to more complex and movement to less complex explanations from session to session occurs ($p = 0.22$).

Table 6.13: Frequency of occurrence of stability, advance, and decline in most complex explanations offered from session to session

	Number of occurrences	Percentage
How often children showed stability in most complex explanation offered from session to session	122	62.5%
How often children showed an advancement in most complex explanation offered from session to session	29	14.9%
How often children showed movement to a less advanced verbal explanation from session to session	44	22.6%

3.2 Stability in most complex explanations offered across sets

In order to look at stability in most complex verbal explanations offered from set to set, the most complex verbal explanations offered from the second set onwards were compared with the previous set. Again, children could be coded as showing a more complex explanation than the previous set, the same explanation as the previous set, or a less advanced verbal explanation then the previous set. Table 6.14 below shows that for a majority of sets, children showed stability in the most complex explanations offered from set to set. Though children showed more movement to less advanced verbal explanations, again 39 of 84 of these incidences involve movements from explicit weight explanations in one set to implicit weight in the following set.

Table 6.14: Frequency of occurrence of stability, advance, and decline in most complex explanations offered from set to set

	Number of occurrences	Percentage
How often children showed stability in most complex explanation offered from set to set	299	65.7%
How often children showed an advancement in most complex explanation offered from set to set	72	15.8%
How often children showed movement to a less advanced verbal explanation from set to set	84	18.5%

3.3. Stability in most complex explanations offered across subsets

In order to look at stability in the most complex verbal explanation from subset to subset, the most complex verbal explanations offered from the second subset onwards were compared with the previous subset. Again, children could coded as showing a more complex explanation then the previous set, the same explanation as the previous set, or a less advanced verbal explanation than the previous set. Table 6.15 shows similar figures to tables 6.13 and 6.14, and again is comparable with the data for representational levels as similar levels of magnification. Children showed high levels of stability in the most complex explanations offered from subset to subset. Movements to less complex verbal explanations from subset to subset were again slightly more common then movements to more advanced verbal explanations. Again however, 122 of the 259 of these incidences of movement to a less advanced verbal explanation involve movements from offering explicit weight explanations in one subset to implicit weight explanations in the next.

Table 6.15: Frequency of occurrence of stability, advance, and decline in most complex explanations offered from subset to subset

	Number of occurrences	Percentage
How often children showed stability in most complex explanation offered from subset to subset	980	65.5%
How often children showed an advancement in most complex explanation offered from subset to subset	256	17.1%
How often children showed movement to a less advanced verbal explanation from subset to subset	259	17.4%

3.4 Comparing stability in most complex explanations offered at different degrees of magnification

The final step in this analysis is to compare the stability in verbal explanations across the different levels of magnification. Because of the different scales used for the different degrees of magnification, for each child the data was recoded into the percentage of times the children stayed stable from session to session, from set to set, and from subset to subset (e.g. if a child stayed stable across the 4 sessions, they were coded as 100% stable). The same was done for increases and decreases in terms of the most complex verbal explanation offered per session, per set, and per subset. Table 6.16 below shows that children were similar in terms of percentages of stability, increase and decrease across these sessions. A series of One-Way ANOVAs with percentage of time children were stable as the dependent variable and the degree of magnification as the factor (e.g. session, set or subset) was not significant, F (2,191) = 0.33, p = 0.71). Similar findings arise for increases in most complex verbal explanation at different degrees of magnification (F [2,192] = 0.25, p = 0.78), and decreases in most complex verbal explanation at different degrees of magnification (F, [2,192] = 1.58, p = 0.2).

Table 6.16: Frequency of occurrences of stability, advance and decline in most complex explanation offered at different degrees of magnification

	Mean %age of times children maintained the same level of verbal explanation	Mean %age of times children showed a more complex verbal explanation then in the previous session/set/subset	Mean %age of times children showed a less complex verbal explanation then in the previous session/set/subset
From session to session	62.2% (standard deviation = 31.78)	15.23% (standard deviation = 18.5)	21.42% (standard deviation = 20.5)
From set to set	65.63% (standard deviation = 24.4)	15.58% (standard deviation = 13.9)	18.11% (standard deviation = 12.2)
From subset to subset	65.18% (standard deviation = 20.13)	16.97 (standard deviation = 10.8)	16.94 (standard deviation = 9.7)

Discussion

The main aim of this experiment was to compare the RR and Overlapping Waves models. An important difference between the 2 models is that Karmiloff-Smith believes that within a level, children are generally stable in their thinking, whereas Siegler believes that children are varied in their strategies. To address this issue, children were tested across 4 sessions on a balance beam task over a period of 2 weeks. They were asked to try and balance symmetrical and asymmetrical beams along a fulcrum, and explain to a puppet why the beam did or did not balance.

The different degrees of magnification played a key role in the analyses. Thelen and Smith (1994) have stated that by looking at low levels of magnification – e.g. by using a summary variable such as a representational level to summarise a whole session, variability in children's development may not be detected. Consequently, in the current study, analyses were conducted at three different levels of magnification. At the lowest degree of magnification were the 4 sessions, each of which involved the presentation of 18 beams to the children. The sessions were subdivided into 2 sets and these provided a higher degree of magnification, with the highest degree of magnification being provided by the subsets of 3 beams within each set.

Using these data 3 inter-related questions could be addressed. First, were children stable in their representational levels across time? The second question concerned the different degrees of magnification; were children equally stable at these different degrees of magnification, or were children more variable when studied at higher degrees of magnification? The third question concerned whether or not there was consistency between different the levels identified at different degrees of magnification.

Because the analysis of children's representational levels may favour the RR model it also was decided to analyse children's explanations. This had the advantage of capturing aspects of the children's strategies which are often used in microgenetic studies to provide information abut the different types of strategies used during and across sessions

(Adolph, 1999, Coyle and Bjorklund, 1997, Siegler and Svetina, 2002). Therefore, the same analyses were also conducted on measures of the most complex explanations offered by the children.

Stability in representational levels across time

The first data to be discussed are shown in figures 6.3-6.5. These figures depict the trajectory of children's representational levels across all 4 sessions. For clarity of presentation and analysis, children were divided into 3 groups – children who showed the same representational level at the first and last sessions, children who showed a less advanced representational at the first session than the last session, and children who showed a more advanced representational level at the last session than at the first session. The children who showed the same representational level (Figure 6.4) at the first and last sessions showed clear evidence of stability and this provides support for Karmiloff-Smith's claims. Indeed 15 of the 24 children in this group (Figure 6.4) showed stability in maintaining the same representational level across all 4 sessions. The other 2 groups (figures 6.3 and 6.5) showed changes in representational levels from the first to last sessions, however, these generally involved a change of only one representational level, and this was usually followed by a period of stability, rather than there being a generally variable developmental trajectory. The clearest example of this phenomenon involved children who moved from abstraction verbal to abstraction nonverbal representational levels (Figure 6.3) – these were the majority of the children who showed a movement to a less advanced representational level. Following movement to Abstraction Nonverbal representational levels, children then remain at this level for the following sessions, rather then showing further movement in representational levels. The implication from these data about sessions is that children are generally stable in their representational levels, rather then being more variable in their thinking, as Siegler asserts. These data also help to further validate the findings of Pine and Messer (2003) about the RR model. Pine and Messer (2003) showed that children move through the RR levels in the sequence laid out in Table 6.3; the present findings indicate that not only do children follow the sequence of levels suggested by Messer & Pine, but also that they maintain these representational levels for a period of time. The occurrence of decline from Abstraction Verbal to Abstraction Nonverbal was more pronounced then in Pine and Messer's (2003) findings. This is in large part due to the inclusion of the "implicit weight" verbal explanation category. The fact that a number of children stopped giving explicit weight explanations, and instead gave implicit explanations, which focused on the number of blocks on either end of the beam, instead of their physical properties may play a role in the decline in performance defined in the u-shaped curve. That is to say, the children may revert to implicit type explanations as part of the "*temporary disregard for features of the environment*" described by Karmiloff-Smith (1992, p 19) in relation to how representational change is thought to occur.

Stability in representational levels at different degrees of magnification

The stability of children's representational levels were analysed at different degrees of magnification, to see if children were more variable in their representational levels at higher degrees of magnification, as Thelen and Smith (1994) have suggested. At the lowest level of magnification, that of the 4 sessions, children showed a high degree of stability from session to session, with children being significantly more likely to maintain the same representational level than to show change to a different representational level, as discussed above. When stability was analysed at the level of sets, a very similar pattern of behaviour was identified, with a high degree of stability in representational levels being present. Children were significantly more likely to remain at the same representational level from set to set than show movement to a more or to a less advanced representational level.

At the subset level there was a slightly different coding scheme for representational levels and children were not found to be as stable as at higher degrees of magnification. This may partly be because of the occurrence of "uncodable subsets". Where these uncodable subsets are not included in the calculations (Table 6.10), it is clear that children were equally stable in maintaining their representational levels across time, even at the highest degree of magnification. This provides clear evidence contrary to the position of Thelen and Smith (1994), that children can in fact be equally stable in their thinking when analysed at very high degrees of magnification.

With regards to the overlapping waves model, one defence that could be made against the current data is that it aggregates data, rather than trying to look at individual variability. This argument would suggest that a great deal of variability has been glossed over or overlooked. However, the microgenetic method employed did in fact aim to look in very fine grained detail at individual's performance. While there was still a need to summarise data from a group of children, the data that was being analysed basically followed the type of data that the overlapping waves model would want to look at – children's performance on tasks at a degree of magnification very close to trial-by-trial analysis, which displayed a much greater regularity in thinking in terms of continuity in RR levels (and therefore in performance and verbal explanations offered) at this degree of magnification.

Consistency in representational levels within sessions

Having looked at stability in representational levels across time, the next step was to look at consistency in representational levels within a session, to determine whether the same level of representation was identified at different degrees of magnification. Addressing this issue was especially important when levels of representation were coded in subsets as the percentage of occasions where children showed stability in representational level from subset to subset was lower then at the other degrees of magnification.

Figures 6.6 and 6.7 indicate a high percentage of consistency in terms of children's representational levels at different degrees of magnification. Children were consistent within sessions (Figure 6.6) – when children were coded as showing Abstraction Nonverbal Representational levels within a session, the sets within this session were highly likely to also be coded as being at the Abstraction Nonverbal representational levels. Given the similar amounts of stability in representational level from session to session and from set to set, this is not surprising. There also was a high level of agreement between the representational levels identified in a subset and the representational level for the set from which the subsets came (Figure 6.7). The representational levels in a subset were significantly more likely to match the representational level for the corresponding set than show a different representational level. This finding is contrary to the position of Thelen and Smith (1994), that at greater degrees of magnification, greater variability will be apparent. This further validates the use of representational levels, which do not seems to be artifacts based on the degree of magnification used in analysis. Another important conclusion to be drawn from this it that the R-R model is not incompatible with modern methodological approaches such as the microgenetic approach, whose analytical focus is at a high degree of magnification in order to track developmental changes.

Variability in verbal explanations offered – implications for the Overlapping Waves Model

Children were stable in their representational levels across time. Tables 6.3-6.5, show that children were also stable in the most complex verbal explanations they offered from session to session, from set to set, and from subset to subset. Indeed, Table 6.16 indicates that children were equally stable in terms of the most complex explanation they offered across time at all degrees of magnification – providing further support for the stability in children's performance. Using the highest degree of magnification offered, which approaches trial-by-trial type analysis, children were stable in the most complex explanations offered across time. The conclusion to be drawn here is that children's thinking for the balance beam task, as measured by looking at the most complex explanation they provide with regard to the balance beam task is not as generally variable as the Overlapping Waves model would predict. The RR model does not imply that children have only one way of thinking about a task at any given time. Nevertheless, once children do achieve a certain level of knowledge (e.g. a more advanced representational level, or a more complex form of verbal explanations), they will show stability in maintaining this level of knowledge across time, rather than showing variability involving using a variety of different types of explanations to explain why a beam would or would not balance.

The RR levels focus both on the type of verbal explanation children offer and on their ability to perform a related task – in this case balancing beams. It is important to note that stability in representational levels across time not only means stability in terms of the most complex verbal explanation shown, but also stability in terms of the number of symmetrical and asymmetrical beams balanced across time. The overlapping waves model can sometimes overlook this level of analysis – for example children may use multiple strategies in addition (Siegler and Shrager, 1989) and multiplication (Siegler and Jenkins, 1989) tasks – yet the analysis rarely covers children's accuracy in these tasks – it is likely that variability in strategy use is not matched by variability in accuracy on these addition and multiplication tasks. Therefore, children are not necessarily characterisable as being variable across all measures one can derive from a task; it is important to state that children are not totally variable in their thinking. This is not to say that children did not show any variability at all in their representational levels– which helps to introduce a key question for the next chapter. Children may not be characterisable as being varied in their thinking in general, but does variability play a role in representational change, and if so, what role? The implication here is that the overlapping waves model may best be thought of as a model which can help to us understand how change occurs, rather then how children behave during periods when change is not occurring. It is worth noting here that development does not follow a continuously progressive curve (as the U-shaped curve clearly shows, Strauss, 1982).

It is also important to note that Siegler (1996) has criticised his balance scale task, which obviously shares the same domain characteristics as the balance beam task used here, as not being compatible with the approach of the overlapping waves model. This allows for the possibility of dismissing the current findings as critiques of the overlapping waves approach. Siegler and Chen (1998) have however used the balance scale task within a microgenetic overlapping waves based study. Therefore, the possibility of dismissing the findings of this study with regard to the overlapping waves model due to the balance beam task being unsuited for use with the overlapping waves model are limited. Furthermore, as stated at the outset, an important aspect of a general model for cognitive development is that it should be applicable to a variety of tasks within a domain. The lack of applicability of the overlapping waves model in this case provides evidence that it may not best be thought of as forming the basis for a general model of cognitive development. While it is the case that children do provide different explanations for why the beams did or did not balance, it was not the case that this variability leads to children showing significant variability either in terms of their representational levels, or in terms of the most complex verbal explanations offered, across time.

Summary

Children's representational levels on a balance beam task were stable across time, at 3 different degrees of magnification. An analysis of children's most complex explanation offered for the balance beam task produced similar findings. Both these sets of findings support the position of the RR model that children are generally stable in their thinking across time, rather then being variable, as Siegler (1996) suggests in his overlapping waves model. These findings also show that children do not always show greater levels of variability when analysed at higher degrees of magnification. It is likely therefore that variability in representational levels is restricted to periods of representational change. Representational change and the possible role of variability in relation to this will be covered in the next chapter.

Chapter 7: A microgenetic investigation of representational change on the balance task

Introduction

The previous chapter addressed issues with regard to stability in the RR model. In this chapter, the focus switches to the process of representational change. Two questions are addressed; first is representational change driven by increasing verbal access to knowledge? Second, what role does stability play in the process of representational change?

1. Is representational change driven by increasing verbal access to knowledge?

The focus of this chapter is on whether or not the RR model can explain changes that occur on the balance beam task across time. Within the model, representational redescription underpins the change from one level to another, but Karmiloff-Smith (1992) fails to elucidate on where the immediate impetus for representational redescription comes from. The main criticism arising from the RR model as noted in the literature review was that it was underdescribed (Campbell, 1994, Freeman, 1994, Scholnick 1994). The levels of the RR model for the balance beam task are based on children's ability to balance symmetrical and asymmetrical beams, and by their ability to provide verbal explanations for each trial. It is therefore plausible that micro-changes in performance or verbal explanations may provide the first signs that representational change is imminent.

Some theorists argue that performance drives cognitive change. Piaget states that cognitive change occurs using a process of "accommodation", "assimilation", and "equilibration" – knowledge is constructed and reconstructed based on interaction with the environment, and cognitive changes occur as a reaction to the environment. Our knowledge of balance for instance would change as we perform a task Though “equilibration” may involve some internal abstraction process, Bryant (2001) notes that there isn’t any testable model of equilibration. Within the RR model, cognitive change is not always driven by changes in performance. Karmiloff-Smith sets out the possibility of representational redescription, which involves the redescription of knowledge, which was initially encoded in a procedural format, being redescribed into verbally accessible formats. This process is thought to occur through an “*internally driven phase during which the child no longer focuses on the external data*” (Karmiloff-Smith, 1992, p. 19). Therefore, the process of abstraction is given a key or potentially driving role in cognitive change, rather then being what may be thought of as a secondary role (e.g. equilibration is not a primary process, but more of a reaction to the processes of assimilation and accommodation). Broadly speaking, if this is the case, then one prediction that can be made is that changes in children’s verbal explanations would precede changes in performance on the balance beam task. It is possible that representational change results from the redescription of knowledge into a consciously accessible format. However, if the Piagetian approach is correct, then changes in performance on the balance beam task would precede the introduction of new and more complex verbal explanations.

In order to test these 2 possibilities, a microgenetic approach is necessary. This allows the opportunity to capture children’s performance on the balance beam task immediately prior to, and following, first use of new and more explicit verbal explanations. Four types of verbal explanation will be focused on to investigate whether change in the type of explanation precedes changes in performance on the balance beam tasks. These 4 types of verbal explanation are set out in Table 7.1, alongside the levels of performance on the balance beam task which the levels of the RR model would predict to accompany the specific level of verbal knowledge.

It is important to emphasise that a U-shaped curve is expected to occur in performance on the balance beam task in terms of the number of asymmetrical beams balanced. Children who give implicit explanations are thought to show a high level of performance in terms of asymmetrical beams balanced. As weight-based explanations arise, the number of asymmetrical beams balanced drops. As children begin to give distance based, and weight and distance based verbal explanations however, an increase in the number of asymmetrical beams is predicted.

Table 7.1: Predicted Relations between Verbal explanations and performance on the balance beam task

Most Complex level of explanation offered	Predicted level of performance in terms of Asymmetrical beams balanced
Implicit Weight	Fails to balance a majority of asymmetrical beams
Weight	Fails to balance a majority of asymmetrical beams
Distance	Begins to show higher levels of performance in balancing asymmetrical beams
Weight and distance	Balances a majority of asymmetrical beams

1.1 First use of Implicit Weight and explicit weight verbal explanations

According to the predictions derived from the RR model, the introduction of new and more complex explanations should precede changes in performance. Therefore, there should be a downturn in performance on the balance scale task following the first use of implicit weight explanations. The easiest way to test this is by comparing performance in terms of number of asymmetrical beams balanced in the subset before first use of implicit weight explanations, and the subset following first use of implicit weight explanations. The RR model would predict a lower level of performance in the subset following first use of implicit weight explanations compared to the subset prior to first use. The Piagetian approach however would predict no significant differences between these 2 subsets, as changes in performance are thought to precede changes in cognition.

The same type of predictions can apply for explicit weight explanations – the first use of explicit weight explanations would be predicted to lead to a downturn in performance in the subset following first use of explicit weight explanations in comparison with the subset prior to first use. However, there is also the possibility that children should already be at the abstraction nonverbal level prior to this, and therefore would already show low levels performance – if this is the case, decreases in performance may not be detectable. The prediction arising from the Piagetian view overlaps with this possibility – e.g. it also predicts no significant difference between the subsets prior to, and following first use of explicit weight explanations. It is simply important to note at this stage that there is more then one reason why the results for this particular prediction may not reach significance.

1.2. First use of distance and weight and distance explanations

In the case of more advanced verbal explanations (those that focus on the role of distance), the first use of these explanations may reflect the child’s more advanced representation and lead to upturns in performance. Once children begin to use distance explanations, an upturn in performance should occur (on the presumption that the first use of explicit distance explanations should not precede first explicit use of weight-based explanations. As children begin to notice that weight is not the only factor in determining how to balance an asymmetrical beam, their ability to balance asymmetrical beams should improve. This should form a part of an upturn on the u-shaped curve. Children should be able to balance more beams, but may not achieve ceiling levels of performance until they use weight and distance explanations. The same would apply to the introduction of weight and distance verbal explanations. In this case, the prediction derived from the RR model is that higher levels of performance on the balance beam task in the subsets should follow first use of weight & distance explanations, in comparison to the subset prior to first use of these explanations. Piaget on the other hand would predict no significant differences to arise, as changes in performance would precede changes in cognitions.

2. What role does stability play in representational change?

In this section the focus is on the impetus for change, at the level of representations. The RR model takes a different stance to change than most modern models of cognitive development. Karmiloff-Smith (1992) clearly states that "*it is representations that have achieved a stable state that are redescribed*" (p. 25). Stability forms the basis for movement "*between phases*" – e.g. movement from Implicit to Abstraction Nonverbal representational levels requires stable representational levels prior to this change occuring. However, there is room for some variability "*within phases*" – e.g. a child may show some variability in their representational levels between first showing signs of achieving abstraction nonverbal prior to maintaining this representational level on a stable basis. This stability is important as it plays a role in the process of redescription described above – e.g. it is this period of stability within a phase which allows the opportunity for knowledge to be redescribed into more consciously accessible formats, bringing about change across phases.

Two predictions arise from this theory. First, children should show stability in representational level in the sets prior to showing representational change. Second, if variability does occur, it is most likely to occur "within phases" – e.g. a period of variability in representational levels following first showing a new and more advanced representational, prior to maintaining the more advanced representational level on a stable basis across sets.

In contrast, Dynamic System theory (Thelen & Smith (1994) states that instability is a prerequisite for cognitive change, a position that also accords with Piaget's. As Karmiloff-Smith (1992) notes "*For Piaget, a system in a state of stability could not spontaneously improve upon itself. Rather, the Piagetian process of equilibration takes place when the system is in a state of disequilibrium*" (p.25). For Piaget, change was brought about by a conflict between children's performance on a task and their knowledge of the concept underpinning that task – as has already been described, it is the sudden inability to perform a task that is said to lead to cognitive change. Microgenetic

studies have also shown that variability may play a role prior to change Chen and Siegler (2000) states that within the overlapping waves model children may initially show variability in their thinking, before becoming more consistent in their strategy use once they are able to adaptively apply the right strategy to the right task.
These approaches state the importance of variability prior to children advancing to more advanced forms of thinking.

This reveals an outstanding question regarding the role of variability in representational change. According to microgenetic studies and dynamic systems theory, variability may be crucial to bringing about representational change. The RR model on the other hand only mentions the role of stability in representational levels prior to change. One aim of the current study is to empirically compare the predictions arising from these two views within a microgenetic context where change is under close scrutiny.

2.1. What role do transitional representational levels play?

Pine et al (2003) have added "transitional" levels to the RR model. These transitional levels imply a period of instability prior to representational change occuring. It is important to note that these "transitional levels" are discussed by Pine et al (2003) in the context of research on children's gestures (see Alibali and Goldin-Meadow, 2003). In this context, transitional levels are seen as periods where "*related forms of knowledge can co-exist but may be isolated from one another*" (Pine et al, 2003, p 300) – gesture-speech mismatches for example provide an example of this (Perry et al, 1992). These transitional levels may capture children who are beginning to show signs of knowledge of more complex knowledge via gesture for example, but are not yet able to verbalise this knowledge. If this is the case, then the prediction of stability preceding representational change can still apply, but rather than immediately preceding representational change, it simply occurs at some point prior to representational change. Therefore, 2 questions are asked with regard to stability: (1) does it occur immediately prior to representational change, and (2) does it occur at any stage prior to representational change?

A further 2 questions are asked with regard to the transitional levels, based on a comparison between the view of transitional levels set out above, and Karmiloff-Smith's (1992) views on when variability may occur in relation to representational change – (1) how often do transitional levels precede representational change, and (2) how often do they follow representational change? This will allow a distinction between the position put forth by Pine et al (2003) that these transitional levels may capture children who are beginning to gain access to knowledge, but are not yet able to verbalise it, and the position set forth by Karmiloff-Smith (1992), that variability in representational levels is more likely to occur "within phases" (e.g. following representational change).

Summary

In this chapter representational change is being investigated. Two issues are addressed – whether representational change is driven by the introduction of new and more complex verbal explanations or by changes in performance on the balance task, and the role of stability in representational levels in the process of representational change. It is important to note that these are 2 very complimentary research questions – according to the RR model, representational change occurs through a process of redescription of knowledge into a more consciously accessible format. For this process to occur however, a period of stability is thought to be necessary. These 2 issues are fundamentally intertwined.

Method

See previous chapter for a full description of the method used in this experiment.

Results

The results are split into 2 sections: (1) Performance on the balance beam task in the subsets before, and following first use of new and more complex verbal explanations. (2) Stability in representational levels in the sets immediately prior to, and following representational change.

1. Performance on the balance beam task in the subsets before, and after first use of more complex verbal explanations

In the introduction, predictions were derived from the RR model, which stated that changes in ability to balance asymmetrical beams would not precede the first use of more complex verbal explanations. These predictions will be tested for each of the 4 following types of verbal explanations: Implicit weight, Explicit Weight, Distance, Weight and Distance.

1.1. First use of implicit weight explanations

Table 7.2 shows mean levels of performance in the subsets prior to, during, and following first use of implicit weight explanations. There is a decrease in the mean number of asymmetrical beams balanced across the 2 subsets, though a One-Way ANOVA, with subset (e.g. whether the subset was prior to, or following first use of implicit weight explanations) as the factor, and mean number of asymmetrical beams balanced in the subsets as the dependent variable, was not significant, F (1,85) = 1.16, p = 0.28. There is more data for the subset after first use of implicit weight explanations, due to the number of children whose first use of implicit weight explanations occurred in the very first subset of the experiment, with the same being the case for all the subsequent ANOVAs. A crosstable did not provide a clearer or significant finding. Although failing to reach significance, a downturn in performance follows in the subset after first use of implicit weight explanations, in comparison with the subset prior to first use of implicit weight explanations.

Table 7.2: Mean number of asymmetrical beams balanced in subset prior to, and following first use of implicit weight explanations

Subset	Mean Number of Asymmetrical Beams Balanced (standard deviation in brackets)	Number of Subjects
Subset prior to first use of implicit weight explanations	0.78 (0.79)	23
Subset following first use of implicit weight explanations	0.58 (0.77)	64

1.2. First use of explicit weight explanations

Similar findings emerge for first use of explicit weight explanations, (see Table 7.3 below). A one way ANOVA using subset as factor, and performance in terms of mean number of asymmetrical beams balanced per subset as the dependent variable showed a significant finding, F (1,82) = 2.47, p = 0.048. This indicates a significant mean downturn in performance on the subset following first use of explicit explanations, in comparison with the subset prior to first use.

Table 7.3: Mean number of asymmetrical beams balanced in the subset prior to, and following first use of explicit weight explanations

Subset	Mean Number of Asymmetrical Beams Balanced (standard deviations in brackets)	Number of Subjects
Subset prior to first use of explicit weight explanations	0.92 (0.84)	26
Subset following first use of explicit weight explanations	0.55 (0.75)	58

1.3 First use of distance explanations

The third prediction states that children's performance should begin to increase with the introduction of distance based explanations. Table 7.4 below shows an increase in the mean number of asymmetrical beams balanced from the subset prior to the subset of following first use of distance explanations. A One-Way ANOVA, with subset as factor, and performance in terms of asymmetrical beams balanced per subset as the dependent variable, fails to reach significance, F (1,44) = 0.652, p = .36. Again, though not statistically significant, there is an increase in performance in the subset following first use of distance explanations in comparison with the subset prior to first use.

Table 7.4: Mean number of asymmetrical beams balanced in the subset prior to and following first use of distance explanations

Session	Mean Number of Asymmetrical Beams Balanced (standard deviation in brackets)	Number of Subjects
Subset prior to first use of distance explanations	0.68 (0.82)	19
Subset following first use of explicit weight explanations	0.93 (0.92)	27

1.4. First use of Weight and Distance explanations

Table 7.5 below shows the changes in performance in the subsets prior to, during, and following first use of weight and distance explanations. Again, clear increases in the mean number of asymmetrical beams balanced per subset are seen, though a one-way ANOVA conducted with subset as the factor and performance as the dependent variable, is not statistically significant, F (1,31) = 0.46, p = 0.5. Again, though not significant, the clear indication is that changes in performance are not apparent in the subset immediately prior to the first use of weight and distance explanations.

Table 7.5: Mean number of asymmetrical beams balanced in the subset prior to and following first use of weight and distance explanations

Subset	Mean Number of Asymmetrical Beams Balanced (standard deviations in brackets)	Number of Subjects
Subset prior to first use of weight and distance explanations	1.00 (0.96)	14
Subset following first use of explicit weight and distance explanations	1.22 (0.88)	18

2. Stability in representational levels prior to and following representational change

This section looks at representational change, and how often children show stability in representational levels prior to, and following representational change. Representational change is looked at from set (of 9 beams) to set – giving 8 measures of representational level across the experiment. This gives the maximum level of magnification without the loss of transitional representational levels, whose incidence will also be analysed. Change is coded as having occurred when a child shows a more complex level in one set then in the previous set. Do children maintain stability in representational in the sets immediately prior to representational change, indicating that this is an important precursor for change? The other possibility, arising from the work of Pine et al (2003) is that children may show transitional levels as part of a gradual representational change. In this case children must still show some representational stability across sets, at some stage prior to representational change. The same question applies to the period following change – do children immediately show stability in representational levels following this change, or is there a period of time between first use of a more advanced representational level and stable use of this more advanced representational level?

The final part of this section focuses on the role of transitional levels, and whether or not they always indicate transitions occurring prior to children first achieving a more advanced representational level.

2.1. Stability in representational levels prior to representational change

Table 7.6 below shows how often children changed to each of the representational levels. Implicit representational levels were omitted as children cannot move from a lower representational level to implicit levels, due to the fact that pre-implicit representational levels were not codable for this task. The implicit transitional level was also omitted, as change to this level occurred infrequently. The key data on stability in representational levels is laid out in Table 5 below. Stability in representational levels is regularly seen prior to all forms of representational change laid out below, with the exception of changes to abstraction nonverbal representational levels. This may be due to the fact that children who achieved implicit representational levels did not show much further progress during the course of this experiment, so it is impossible to look in depth at this particular type of change. For changes to abstraction verbal, explicit transition, and E3, children frequently show consistency in maintaining a lower representational level prior to representational change – this consistency is not always present immediately prior to change however. In all cases however, stability at some stage prior to representational change occurs more frequently then stability in representational level in the 2 sets immediately prior to change.

Table 7.6: Number of times representational change occurs, and how often children are stable in maintaining representational levels prior to this change

	Abstraction Nonverbal	Abstraction Verbal	Explicit Transition	E3
Number of times children showed representational change	15	22	14	11
Percentage of times children showed stability in representational level in the 2 sets immediately prior to representational change	7%	42%	62%	27%
Percentage of times children showed stability in representational levels for 2 or more sets prior to change	15%	56%	100%	60%
Mean number of sets where children showed the same representational level prior to change	3 sets	2.78 sets	3.4 sets	2.5 sets

The mean number of sets children maintained a lower representational level consistently prior to representational level is laid out in Table 7.6. For changes to all the representational levels above, the mean number of sets is over 2, lending credence to the notion that a period of sustained consistency is an important prerequisite for representational change. This seems especially to be the case for change to explicit transitional levels.

2.2. Stability in maintaining the new representational level across sets following change

The next question to be asked is how often children show stability in maintaining new representational levels, once they achieve them. Table 7.7 shows that children are especially stable in maintaining Abstraction Nonverbal representational levels, once they achieve them. They show stability in representational level in the set following this change, and seem to maintain this level thereafter (as seen in the second and third rows of Table 7.7). The same cannot be said for the other representational levels, where stability in maintaining the new level does not occur a majority of time following the initial change or discovery of a new and more advanced representational level. Change to the abstraction verbal RR level does not often lead to children immediately maintaining this representational level in the following set(s) [only 16% of the 22 cases of children showing representational change to Abstraction verbal]. For both abstraction verbal and explicit transition RR levels, stability is not likely to occur in the set immediately following change, if at all. Stability following change is slightly more common once children achieve E3 representational levels, though again not as high as for the children who achieve abstraction nonverbal level. Overall, stability in representational levels following change seems less likely to occur in the set immediately following change. Indeed a large percentage of the children who showed change do not proceed to show stability thereafter for the course of this experiment.

Table 7.7: Number of times representational change occurs from set to set, and how often children are consistent in their representational levels following this change

	Abstraction Nonverbal	Abstraction Verbal	Explicit Transition	E3
Number of times children showed representational change	15	22	14	11
Percentage of times children showed stability in representational level in the sets following representational change	71%	10%	8%	40%
Percentage of times children showed stability in maintaining the new representational levels following representational change	71%	16%	12%	60%
Mean number of sets where children showed the same representational level following change	3.7 sets	2.6 sets	3 sets	3.3 sets

2.3 The role of transitional levels

The final question to be asked is what role transitional levels play – do they capture children who are in transition from one representational level to another, or do they capture children showing variability in representational level following first use of the more advanced representational level? In contrast to the previous sections, as will be seen, this section will deal with "regressions" to less advanced representational levels, as well as movement to more advanced representational levels.

2.3.1. The role of the implicit transitional representational levels

Table 7.8 below shows that children most often moved from abstraction non-verbal to implicit transition, rather then from implicit to implicit transition. This transitional level may indicate that children have not yet managed to successfully consolidate the abstraction nonverbal representational level, more so than children in transition from implicit to abstraction nonverbal representational levels.

Table 7.8: the RR level children show in sets prior to showing implicit transition RR levels

Representational Level	Number of times children show a level in the set prior to showing Implicit Transitional levels
Implicit	3
Abstraction Nonverbal	8
Abstraction Verbal	1
Explicit Transition	1

Table 7.9 below shows what representational levels children moved on to in the sets following displaying implicit transitional levels. It appears that "regression" to implicit representational levels occurred as frequently as progression to Abstraction Nonverbal representational levels. This again indicates that children who are showing these transitional levels are not necessarily part of an immediate progression towards a higher representational level.

Table 7.9: the RR level children show in the set following implicit transition RR levels

Representational Level	Number of times children show a level in the set prior to showing Implicit Transitional levels
Implicit	6
Abstraction Nonverbal	7
Abstraction Verbal	3
Explicit Transition	1

2.3.2. Explicit transitional levels

Table 7.10 provides some evidence that unlike Implicit Transition, explicit transitional levels show children progressing from abstraction verbal levels, more so then "regressing" from a fully explicit E3 representational level in the previous set.

Table 7.10: the RR level children show in the set prior to showing explicit transition RR levels

Representational Level	Number of times children show a level in the set prior to showing explicit Transitional levels
Implicit Transition	1
Abstraction Nonverbal	3
Abstraction Verbal	10
E3	3

On the other hand, Table 7.11 shows that children very rarely progress from this stage on to E3 representational levels, but rather "regress" back to showing Abstraction Verbal or Abstraction Nonverbal representational levels. Again this provides evidence that these transitional levels may not be characterisable as periods of development in which children are about to show further representational progression, in the short-term.

Table 7.11: the RR level children show in the set following explicit transition RR levels

Representational Level	Number of times children show a level in the set prior to showing explicit Transitional levels
Implicit	5
Implicit Transition	1
Abstraction Nonverbal	8
Abstraction Verbal	10
E3	1

Discussion

This chapter focuses on how cognitive change occurs within the RR model. Two particular aspects, based on the description provided by Karmiloff-Smith (1992) are analysed. The RR model states that cognitive change can occur through increasing access to verbal knowledge. Representational change involves a process of representational redescription which involves knowledge which is initially encoded in a procedural format (in order to perform a specific task) being redescribed into different formats, becoming increasingly consciously accessible, and eventually verbalisable. This process is contrasted with Piagetian notions of assimilation and accommodation, which involve the introduction of new knowledge based purely on task performance conflicting with the child's current cognitions

The second aspect of importance is that representational stability is thought to precede representational change. This stability contrasts with Piaget's theories of change, which focus on the role of conflict in bringing about change – Karmiloff-Smith (1992) contrasts cognitive change following success with Piaget's cognitive change following failure on a task.

These 2 aspects are inter-linked: stability in representational levels across time is thought to provide the basis for redescription. The evidence for these 2 aspects of the RR model analysed in this experiment will now be discussed.

1. Is representational change driven by increasing access to verbal knowledge?

This experiment took a microgenetic approach to investigate cognitive change at a very high level of magnification. Children's ability to balance asymmetrical beams in the subsets immediately prior to, and following first use of new and more advanced explanations were analysed, to check whether changes in verbal explanations offered precedes changes in performance on the balance beam task. Four predictions were used to test this, based on 4 different types of verbal explanations: implicit weight, explicit weight, distance, and weight and distance. For the first 2, a downturn in performance was expected to follow first use of the more advanced verbal explanation. For the second 2 types of verbal explanations, an upturn in performance was expected to follow first use of these more advanced verbal expectations. This is in line with the U-shaped curve in performance in relation to verbal explanations which the RR model predicts. The balance beam task was chosen as a prime example of a task in which a U-shaped curve in performance in relation to the emergence of explicit verbal knowledge is expected to arise (Pine and Messer, 2003). This U-shaped curve allows for the possibility of significant results arising in terms of sharp changes in performance following the introduction of new and more complex verbal explanations. Tables 7.2-7.5 indicate that the patterns predicted by the RR model (e.g. changes in performance coming after first use of more complex explanations) did arise. The patterns are clear enough, though One-Way ANOVAs revealed significant results for only one of the verbal explanation types – explicit weight explanations. There are several reasons why the results for 3 of the 4 types of verbal explanations failed to achieve significance. First, not all children achieved the more advanced distance and weight & distance explanations. A smaller sample for these types of verbal explanation may have lead to an inability to achieve significant results. The second problem arises from the fact that comparisons were being made between performance on 2 asymmetrical beams in the subset prior to first use of more advanced explanations and performance on 2 asymmetrical beams in the subset following first use of more advanced explanations. The small number of trials being used to compare may make it more difficult for a significant difference to arise – indeed this is one of the difficulties in using microgenetic methods (see Cheshire et al, 2007). Though significant differences in performance do not always arise on subsets prior to and following first use of more advanced verbal explanations, they nonetheless follow the patterns predicted by the RR model. This provides a strong indication that changes in verbal explanations precede changes in performance – e.g. changes in representational level (which are based on children's performance on the balance beam task in relation to the most complex verbal explanations offered) can be driven by changes in verbal explanations. Though of course external experience is still needed first in order for this change to occur, it is important that the change in verbal explanations precedes the change in performance, as it provides evidence that it is not always simply the case that changes in verbal explanations follow changes in performance; there is a human capacity to use an internal process to come up with more complex, or more verbally accessible formats of knowledge. The process of "redescription" as described by Karmiloff-Smith is one way to account for how this change in representational levels occurs. The microgenetic method has proved beneficial in this respect, as it captures trends in performance in the trials immediately prior to, and following first use of more complex explanations, emphasising the utility of the microgenetic methods as a means of exploring how cognitive change occurs, in comparison with cross-sectional testing methods.

2. The Role of stability in representational change

Another important aspect of change within the RR model (Karmiloff-Smith, 1992) is that it is thought to be preceded by stability in representational levels. This original position was contrasted with the Piagetian view that instability must precede cognitive change. A third view arising from the work of Pine et al (2003) states that there may be a transitional period prior to representational change, which may indicate that children are beginning to use more advanced representations, which are not yet verbally accessible. If this is the case, then stability may still play an important role, but will not arise immediately prior to representational change. Children's representational levels per set were analysed to see if children showed stability in representational levels in the 2 sets immediately prior to representational change, or, if change is indeed more gradual, whether they show stability in representational levels at some point prior to representational change. Table 7.6 shows that children were frequently stable in maintaining representational levels prior to change. Stability occurred more often at some point prior to change rather than in the 2 sets immediately prior to change. Nonetheless, the data suggest that children do regularly show stability prior to representational change. This is as stated at odds with the Piagetian model, and is perhaps the biggest departure from the model. The importance of the period of stability is for the process of abstraction to occur. And it is important to note that this process is not merely a "process of equilibration” following a period of disequilibrium, but rather an independent, internally-driven process of redescribing information into a more verbally accessible format. The one exception to this finding of stability preceding cognitive change is for the abstraction nonverbal representational level, which may well be due to the fact that very few of the children who showed implicit representations throughout this experiment showed further progression beyond this representational level. These findings generally compliment the findings of the previous chapter: the previous chapter showed that children are generally characterisable as being stable in representational levels. This finding shows that this stability often was found to precede representational change. This supports the possibility of representational change occurring based on a period of stability in representational level, which is not part of the piagetian model.

Karmiloff-Smith (1992) stated that children showed stability in representational levels before changes between phases (e.g. from Implicit to Abstraction Nonverbal Representational levels). She did not however rule out the possibility of variability occurring “within phases” – this means a period of variability occurring between the first time children show a more advanced representational level, and when they begin to maintain this representational level on a more stable basis. To investigate this, stability in representational levels was analysed following representational change. Table 7.7 shows that children do not always show stability in maintaining the representational level immediately after representational change. Indeed for all representational levels included, with the exception of abstraction nonverbal representational levels, stability is more common at some point following representational change rather than in the set immediately following representational change. This finding supports the notion stated by Karmiloff-Smith (1992) that variability is present “within phases” – e.g. a period of variability following first showing a more advanced representational level, prior to stable use of this representational levels.

2.1 The role of transitional levels

These findings raise doubts about the role of transitional levels introduced by Pine et al (2003). It has been established that at some point prior to representational change, children display representational stability. Transitional levels (Pine et al, 2004) were thought to capture the gradual development of more advanced representational levels. The implicit transitional representational level for example was thought to capture "*a transitional level between implicit representations and the first abstraction of a centre theory*" (Pine et al, 2003, p 298) At these "transitional" points, children may be accessing multiple representations, in particular accessing knowledge which is not yet verbally accessible; they may for example show gesture speech mismatches (Pine et al, 2004, Goldin-Meadow 2001). Do the transitional levels arise prior to representational change as Pine et al (2003) suggest, or do they capture children showing variability following Karmiloff-Smith's suggestion that this may be when variability arises?

With regard to the Implicit Transitional representational level, Table 7.8 shows that children most often showed Abstraction Nonverbal representational levels in the set immediately prior to showing Implicit transitional levels. This indicates that rather than playing a role in the gradual change to a "centre-based" theory of balance, this transitional level often captures a "regression", or variability in representational levels following initial use of Abstraction Nonverbal representational levels. Interestingly, Table 7.9 also shows that in the set following use of implicit transitional levels, children showed Implicit representational levels almost as frequently as showing Abstraction nonverbal representational levels. Therefore, children seem to regress from abstraction non-verbal to implicit transition, and from implicit transition to implicit representational levels. It seems clear that the implicit transitional levels often captured children showing variability following change to abstraction nonverbal representational levels. It is equally the case however, as was described in the previous chapter, that a majority of children who displayed implicit representations for the concept of balance in this experiment maintained it throughout the course of the experiment, rather then showing further advancement. Therefore, the implicit transitional level may indeed still capture a part of a gradual process of change prior to first displaying abstraction nonverbal representational levels, but it also captures children showing "regression" following change to abstraction nonverbal representational levels.

A different story emerges for the explicit transitional level. Table 7.10 shows that explicit transitional levels are most commonly preceded by abstraction verbal representational levels. This fits in with the account of Pine et al (2003), suggesting that the transitional level captures a gradual shift towards fully explicit E3 representational levels. Table 7.11 however shows that children do not commonly move on from explicit transition to E3 representational levels, but rather "regress" to abstraction verbal representational level. Two conclusions are drawn from this – first, the change from abstraction verbal to E3 does not occur quickly, especially on a spontaneous basis (e.g. simply by getting children to repeatedly perform one task, without any interventions or instruction). Therefore, it is quite possible that explicit transitional levels do capture a gradual movement towards E3 representational levels. This may indeed be due to multiple representations being accessed. However, in the absence of children showing explicit verbal explanations referring to both weight and distance, children seem to regress to Abstraction Verbal representational levels. For the second conclusion, it is important to note that children coded at explicit transitional tend to maintain a focus on weight in the verbal explanations they offer, but balance too many asymmetrical beams in order to be classified as Abstraction Verbal. The fact that children often regress from explicit transitional to abstraction verbal shows that it is not likely that this "increase" in performance (e.g. children coded as explicit transition balance more asymmetrical beams than children coded as abstraction verbal representational levels) provides the basis for representational change to E3 – rather it would seem that the introduction of weight and distance explanations brings about movement to E3 representational levels.

The findings for Implicit and Explicit Transitional levels provide evidence both for the positions adopted by Pine et al (2003) and by Karmiloff-Smith (1992). Though change is often preceded by stability, immediately prior to change some variability can occur, which may be partially accounted for by children showing "transitional levels". These transitional levels may also arise following change – which may also be explained in terms of multiple representations being accessed, prior to children beginning to show stable use of the more advanced representational level.

Conclusions on Representational change

This chapter attempted to provide empirical support for the process of representational change described by Karmiloff-Smith (1992). By looking at children's performance on the balance beam task in the subset prior to, and following first use of more advanced verbal explanations, there were clear signs that children showed changes in performance following the introduction of new and more complex verbal explanations. Furthermore, children typically showed stability in representational levels prior to (though not necessarily always immediately prior to) representational change. These 2 findings together support the possibility that representational change occurs via a process of redescription, for which a period of stability in representational levels is required. This provides an alternative to the piagetian approach that cognitive change occurs as a result of cognitive conflict, and following failure on a task. It is interesting to note that the RR model does challenge the traditional notion of how change occurs. This is quite surprising given that Bryant (2001) states that Piaget's mechanisms for change, and the concept of equilibration in particular are not testable. Yet, modern models such as Dynamic Systems theory (Thelen and Smith, 1994) and to a lesser extent the Overlapping Waves model (Siegler, 1996) do not challenge the classical Piagetian mechanisms for change.

The addition of transitional levels to the model by Pine and Messer (2003) means that variability in relation to cognitive change can be accounted for – both through transitional levels arising prior to change, and a period of variability immediately following change. This experiment highlights the importance of the RR model as it offers a different view and a potentially different set of mechanisms through which cognitive change may occur.

Chapter 8: General Discussion and Conclusions

This discussion will begin by briefly recapping the key points from the literature review, and the main research questions addressed. Following this, the results of the experiments will be discussed and their implications for general theories of development will be highlighted.

Key Points and Research Questions arising from the Literature Review

The literature review began by discussing the importance of general models in psychology to provide a clear understanding of the human mind, and raised the possibility that the RR model (Karmiloff-Smith, 1992) could be used as a general model for cognitive development to complement and extend the Piagetian model of cognitive development. Three key features of the RR model were identified. First, the RR model operates on a domain-specific basis, rather then the universal stages envisaged by Piaget. The second key feature was the focus on development as a process of gaining increasing access to verbal knowledge, along an implicit-explicit continuum, rather than focusing on the development of "formal logical thinking". The third key feature is that development is seen to occur as a result of stability and task success, rather than failure in a task bringing about cognitive change.

Following a review of experimental research, and a comparison with other contemporary cognitive developmental models, 4 sets of issues (some of which overlap) were raised about the RR model as a general model for cognitive development. These 4 sets of issues relate to the 3 features of the RR model.

First, there is a need to look at the generalisability of knowledge. This is important as the RR model states that explicit knowledge should be generalisable, and some evidence has been found in support of this statement (Pine et al, 2003). A number of other studies have provided conflicting evidence of what type of knowledge children can generalise, and at what RR level children can start to generalise (Tolmie et al, 2005, Barlow et al, 2003, Hollis and Low, 2005). There is also evidence arising from a number of microgenetic studies stating that children are poor at generalising knowledge from one task to another (Adolph, 1997, Alibali, 1999, Siegler, 2002, Bowerman, 1982). In light of this, there was a need to define what type of knowledge children can generalise and apply to different types of task, and to what extent this adds to our knowledge of the RR model.

A second concern for the RR model was that the levels of the RR model had been applied to relatively few domains described by Karmiloff-Smith, 1992 (though see Pine et al, 1999, Critten et al, 2007, Tolmie et al 2005, Hollis and Low, 2005). There was therefore a need to show that the levels of the model apply to the domains described by Karmiloff-Smith (1992), and to multiple tasks within a domain (Shultz, 1994). Given the drive to show the RR model as a domain general model for development, there was also a need to show that this model can contribute positively to our understanding of that domain. To that end, the levels of the RR model were applied to concepts pertaining to numeracy, with the research attempting to address important issues for this domain (e.g. do children have innate concepts of number, and how should numeracy be taught to children).

The third set of issues deals with the importance of gaining increasing access to verbal knowledge within the RR model, as a major part of development. There is a need to show that the pattern of development of explicit verbal knowledge following task mastery established in the balance beam task by Pine et al (1999) applies in different domains, and has clear theoretical and practical implications for children's development.

The final issue to consider was the process of cognitive change. The aim was to test the predictions arising from the RR model that stability played an important role prior to change, and that change could be driven by increasing access to verbal knowledge.

Having summarised the aims laid out in the literature review and the specific research questions addressed in the thesis, a brief description of the experiments conducted will be given.

Summary of research

1. The generalisation of knowledge across balance tasks

The first 2 experimental chapters address the question of whether or not children could generalise knowledge across tasks, and whether, following this, the RR levels could be used as a domain-specific, rather than a task-specific description of a child's knowledge. The first step in doing this was to attempt to apply the levels of the RR model to a task other than the balance beam task which has been used by Pine and her colleagues (e.g. Pine et al, 1999), as well as by Karmiloff-Smith (e.g. Karmiloff-Smith and Inhelder, 1974). A number of balance scale tasks which have been used in other studies (see Messer et al, 2007, Surber and Gzesh, 1984, Kliman, 1987, Oshima and Okada, 1996) were assessed in terms of their compatibility with the levels of the RR model. Of the 3 balance scale tasks used, the levels of the RR model were applicable only to an unconstrained balance scale task. However, the same types of verbal explanations were being used across all three tasks, giving some indication that verbal knowledge may be thought of as generalised or at least domain-general knowledge. The task to which the RR model was applied used a different apparatus (e.g. scale) and methodology (e.g. an unconstrained task where children were given free access to the scale and the weights), showing that the levels of the RR model were not limited by only being applicable to Pine et al's (1999) balance beam task for the domain of balance. The next step in the research was to compare children's representational levels across balance beam and balance scale tasks. In a study involving 87 participants, using a simple implicit-explicit dichotomy (e.g. whether children showed implicit or explicit representations), a majority were found to show the same type of representations on both the balance scale and balance beam tasks. In spite of the differences in apparatus and tasks, children provided the same types of explanations on both tasks. The implication drawn from this is that once children achieve an explicit level of representation, they can access verbal knowledge across tasks within a domain. Therefore, it seems that verbal knowledge is generalisable across tasks within a domain, leading to the possibility that it is this type of knowledge, which is achieved earlier within the model than Karmiloff-Smith (1992) originally indicated which allows for the increasing flexibility in thinking which is one of the key features of the RR model. A chi square analysis also revealed a significant association between the RR levels on the 2 tasks. However the relation between the actual RR levels on both tasks was not as strong as when using a simple Implicit-Explicit distinction, indicating that in spite of having access to the same level of verbal knowledge, children did not show the same levels of representation on the 2 balance tasks.

2. Exploring children's representations for the principles for counting

The second set of experiments in this thesis focused on applying the levels of the RR model to children's performance on numeracy tasks. This follows on from Sophian's (1998) call for a model of development of numerical knowledge which looks beyond children's ability to perform numerical tasks. The principles laid out by Gelman et al (1976) formed the basis for this research on numeracy. The first experiment focused on the one-to-one principle for counting. The question addressed was whether or not the principles as laid out by Gelman et al (1976) were innately specified, to the extent that children could demonstrate knowledge of these principles prior to being able to count accurately themselves. The question is an important one, and has provoked much debate (Gelman et al, 1982, 1984, 1986, Briars and Siegler, 1984). By using 2 separate tasks to measure performance (a simple counting task) and verbal knowledge (an error detection task), children were assigned to representational levels. The youngest group of children study (nursery school children) were shown to have pre-implicit representations of the one to one counting principle (i.e. could not apply the one-to-one principle accurately in their own counting, and could not detect or explain errors in others' counting). This indicates that if children have some innate knowledge of the principles, they are more primitive than implicit representations. By year one, a majority of children had explicit representations for the one-to-one counting principle.

The second study on counting focused on the cardinality principle, and what forms of teaching might best help children develop explicit representations of this principle. The efficacy of two different types of teaching interventions were predicted to vary in relation to initial RR level. The findings indicated that a "procedural" intervention was most useful for children who have yet to achieve explicit verbal knowledge, and the "conceptual" intervention (derived from the work of Muldoon et al, 2003) was most effective for children who were on the verge of achieving explicit verbal knowledge, or still in the early stages of using explicit verbal explanations (e.g. abstraction verbal representational levels.). This is a tentative finding at present however, as there was not a great number of children who were initially at this level within the conceptual condition. However, the findings provide some indication that the RR levels give a good indication of what sort of teaching method is best suited for the development of explicit representations of the cardinality principle.

3. A Microgenetic Analysis of children's representations on the balance beam task

The final experiment focused again on the domain of balance, and investigated cognitive change within the RR model. Two specific aspects were focused on:
First, were children generally stable in maintaining RR levels across time, and more specifically, were they stable in maintaining representational levels prior to representational change occurring. The second aspect being analysed was the role of increasing access to verbal knowledge in relation to changes in performance on the balance task. With regard to the first issue, in light of the focus of the overlapping waves model (Siegler, 1996) on variability in thinking, there was a need to establish whether or not children were in fact stable in maintaining their representational levels across time, to see if the variability Siegler speaks about in relation to strategy use also occurs when looking at children's RR levels across time. The experiment focused on children's representations across 4 sessions, measuring representational levels at difference degrees of magnification. Most Children were found to show stability in their representational levels both across time, and were equally stable in maintaining representational levels across time at the different degrees of magnification. This is not to say that children only tended to offer only one type of explanation – indeed children routinely offered different

types of explanations, but this did not give rise to any significant changes in performance, or cause children to routinely be classified to different representational levels, even at the highest degree of magnification. From this it is concluded that children's development on this task is progressive rather than overlapping as shown in the overlapping waves model.

With regard to representational change, children were found to show stability in representational level prior to representational change. This stability was more often found at some point prior to change, rather than in the 2 sets immediately prior to representational change occurring. Therefore, variability must still play some role in the process of cognitive change. It was also demonstrated that changes in performance followed first use of new and more complex verbal explanations, rather than new and more complex verbal explanations following changes in performance on the balance beam task. Increasing verbal access to knowledge appears to play a key role in representational change, rather then changes in performance driving changes in verbal explanations offered. This allows for a view of Representational Redescription occurring through a process whereby representational stability brings about increasing verbal access to knowledge as a means of cognitive change occurring. This has very important implications for a comparison between the RR and Overlapping Waves model, as the relationship between verbal explanations and performance on a task is not considered within the Overlapping waves model.

The utility and validity of the RR model as a general model for cognitive development

Having described the experiments in this PhD, their contributions to 3 key aspects of the RR model will be discussed in greater depth, along with a focus on the tasks used to measure RR levels within this thesis. These will provide a clearer picture of whether or not there is a basis for using the RR model as a general model for cognitive development. To recap, these key aspects are: (1) The RR model describes development on a domain specific basis. (2) The importance of increasing access to verbal knowledge for cognitive development. (3) The role of stability and success in cognitive change

1. The RR model as a model for domain specific cognitive development

A number of issues arise from the fact that the RR model is a model which describes the process of development occurring within domains. First, there was a need to continue to show the domains in which the model could be applied. Second, there was a need to examine and explain phenomena such as generalisation and access to knowledge across tasks within a domain. There was a need to show that the RR model had clear utility for the domains in which it was applied, and the different types of tasks used to apply the RR model in these domains will be discussed.

1.1. Applying the levels of the RR model in new domains

If the RR is to be judged as a general model of cognitive development, it must be shown to be applicable across a wide variety of domains. Karmiloff-Smith (1992) described a number of domains in which the general principles of the RR model may be seen to apply. Similarly, a number of studies in the domains of pedestrian skills (Tolmie et al, 2005) drawing (Hollis and Low, 2005, Barlow et al, 2003) and mathematics (Chetland et al, 2007) have invoked the RR model to explain findings. Prior to this thesis however, the levels of the RR model had only been applied experimentally to the domain of balance (Pine et al, 1999), and more recently to the domain of literacy (Critten et al, 2007). The application of the levels of the model to different tasks within the domain of balance, and to the domain of numeracy goes some distance in demonstrating that the RR model as described by Karmiloff-Smith can be actively applied in the domains she described in Beyond Modularity (1992). That is to say, it provides support for the interpretations used by Karmiloff-Smith (1992) in Beyond Modularity, which are used to demonstrate the basic tenets of the RR model, but which had not been tested on an a priori basis in the research she describes (e.g. the 1974 work on the balance beam task which is reinterpreted by Karmiloff-Smith to demonstrate the levels of the RR model). The experimental application of the levels across different tasks and different domains serves to counter previous criticism of the model as being underdescribed by showing in detail how the levels of the model apply to the domains both of balance and numeracy. This begins to indicate that the RR model can be applied to a number of domains in the same way for example that Piaget described the stages of his model occurring across a variety of domains (e.g. conception of number, [Piaget, 1952], space [Piaget and Inhelder, 1956], moral judgement [Piaget, 1932], physical causality [Piaget, 1960], quantity [Piaget and Inhelder, 1974]). It is clear however that a variety of domains need to be investigated using the RR model to show that it exhaustively applicable to all domains.

1.2. Application of the RR model to the domain of numeracy

The utility of a cognitive developmental model is dependent on it having practical uses for key educational domains, such as literacy and numeracy. Muldoon (2003) notes that "*early years education would benefit from a curriculum that focuses on the conceptual underpinnings of mathematics*" (p 697). The work on the cardinality principle suggests that while there is a need to look at the concepts which underlie mathematical functions (for example the principles described by Gelman and Gallistel, 1976), there is an equally strong need to give the children the opportunity to perform simple mathematical tasks in order to achieve behavioural mastery which forms the basis for implicit representations within the RR model. There is a need to look both at children's ability to perform a task, and children's understanding of the mathematical concepts underlying these tasks. Therefore, the RR model would advocate an approach to education which would begin by giving the children an opportunity to perform simple tasks, before focusing on giving children the opportunity to verbalise the concepts underlying simple mathematical tasks such as counting. This provides a theoretical underpinning for the type of "curriculum" mentioned by Muldoon et al (2003) for early maths. It also offers a clear contrast to the open-ended model of development put forward by Rittle-Johnson et al (2001), by stating that the effect of different types of intervention will depend on children's representational level. This adds to the body of work of Pine et al (1999, 2001) which looked at different types of interventions for children's representations of balance and demonstrates that the same findings within that domain may well also hold true for children's early development of representations for mathematical principles – that at an early stage in development for these counting principles, children should be taught strategies to help them achieve mastery, before the focus on teaching switches to the conceptual basis of the counting principles.

1.3 Generalisation and access to knowledge

It is predicted by Karmiloff-Smith (1992) that as representations become more verbally accessible, they should become more generalisable, for use in other tasks in the same domain (Karmiloff-Smith, 1992). The second study in this thesis demonstrated that when children had access to explicit verbal knowledge in one balance task, they could provide the same explicit verbal knowledge in another balance task. Children were not always found to show the success on the 2 balance tasks in spite of them having access to the same knowledge for both tasks. Their performance on the balance tasks were in all likelihood effected by the characteristics of the task (Messer et al, in print), though their access to the underlying verbal knowledge was not.

This has clear implications for the generalisation of knowledge. The RR model recognises the possibility of flexibility in children's thinking, and a key aspect of representational change within Karmiloff-Smith's model is that it allows "*inter-representational flexibility and creative problem solving capacities"* (Karmiloff-Smith, 1992, p 20). Pine et al (2003) looked at the "flexibility" of their representations, by seeing if children at all representational levels could predict whether they were likely to be able to balance the beam presented them. This gave some indication that children have some form of conscious access to their knowledge, which they apply in a slightly different form of task (e.g. being able to predict whether they can balance the beam, as apart from their actual ability to balance it along a fulcrum). A majority of microgenetic studies which looked at generalisation (e.g. Adolph, 1997, Alibali, 1999, Siegler, 2002, Bowerman, 1982, Chen and Klahr, 1999) focus on the use of strategies, rather than on their actual knowledge and how it becomes available across a domain. It is not surprising therefore that children were not found to generalise in these microgenetic studies. Tolmie et al's (2005) study looks at generalisability in relation to pedestrian skills. This study focused on adult guidance and peer discussion in helping children which led to "*appropriation of E3 level representations from adult dialogue*" (Tolmie et al, 2005, p 181). The emphasis is on verbal knowledge, which is stated as emerging at the E3 representational level.

The findings of this thesis help to clarify how generalisation of knowledge may occur: once children do have explicit verbal knowledge, it is seen that they can access this knowledge for other tasks within a domain. This provides a clear explanation for the phenomena of generalisation of knowledge. It also helps to explain why the findings from microgenetic studies which focus on procedural-type strategies (i.e. Siegler, 2002), fail to show generalisation of strategy use across tasks. The development of verbal knowledge within the RR model provides the basis for the flexibility; it is the redescription of knowledge from an initially procedural basis, into generalisable verbal format. Indeed, this redescription may be thought of as a deductive process, whereby children's continued stable performance on a task brings them to deduce a general "theory" or piece of knowledge which is couched in a verbal format. The format of the knowledge is very important, given the origins of the RR model as a reaction to Fodor's (1982) work on modularity. While some types of knowledge may retain "modular" type properties and be informationally encapsulated (e.g. not available to other processes), verbal knowledge does not seem to have these properties. Verbal knowledge is therefore a cornerstone for cognitive development. Therefore, the RR model can be seen to provide a clear description and explanation for this particular phenomena, which is not described by other contemporary models of cognitive development such as the overlapping waves model (Siegler, 1996).

The overlapping waves model (Siegler, 1996), describes in great detail the development of strategies which can be used to successfully perform a task. A number of microgenetic studies have been very good in detailing the various variables which play a factor in the development and use of these strategies (see Siegler, 2006 for a comprehensive review), and a similarly intricate set of variables may be described which determine behaviour within the dynamic systems model. This model does not however attempt to describe the nature of the representation which underpinned this success. There is no sense within these models of the child clearly achieving a greater level of understanding, or certainly no desire to clearly signpost these changes when they occur. Following criticisms of Piaget's stage theory, they shy away from stating that there are clear and definable stages/levels indicating children achieving a new or more complex understanding of a domain. Siegler (2006) claims that the overlapping waves model can "*integrate qualitative and quantitative aspects of learning within a single framework*", yet there is no framework within the Overlapping waves model to indicate when children do show "qualitative advances". Chen and Siegler (2000) describe 5 processes through which strategy use develops from initial varied use through to more systematic and adaptive choice of strategies. These components however are descriptive in nature rather than explanatory, and provide no clear explanation of how this development occurs, or indeed a clear description of the framework within which the child thinks and develops – to put it strongly, the overlapping waves model is not a model in the strong sense.

One of the key points of the RR model is that it maintains a descriptive level-based approach, whilst of course moving away from the process towards the development of "formal logical" reasoning favoured by Piaget. The RR model provides a sound basis for discussing the phenomena of generalisation; and a potential general model for cognitive development should be able to provide an explanation for general phenomena such as the generalisation of knowledge, which must be a key phenomena, as what children learn in schools is supposed to be applicable to a variety of task, rather than only being applicable for the specific task in which they learnt a specific piece of knowledge.

1.4. The tasks used in each domain

A final point for this section relates to the methodologies used in this thesis. In the first set of experiments, an attempt was made to show that the levels of the RR model were applicable to more than 1 type of balance task. Having established that the levels were not limited to the balance beam task (Pine, 1999) within this domain, there was a more general need to see what types of task the levels of the model could and could not be applied to. The importance of applying the levels is that if they are not applicable within a domain, the process of redescription within the RR model is not likely to be applicable either. The two tasks to which the RR levels were applied in the domain of balance both levels of performance comparable to the U-shaped curve described by Strauss (1982). For both tasks, verbal knowledge and performance measures were elicited in one task. In the experiments on the counting task, a different approach was taken, with children's performance and verbal knowledge being measured using separate tasks.

The dip in performance for children at the abstraction level commonly seen on the balance beam task (Pine et al,1999), was less prominent for the 2 experiments on the counting principles (chapters 4 and 5). This indicates that the levels of the model were not only applicable to tasks where an appreciable u-shaped curve was apparent (as was the case in the work of Pine, 199, and Critten, 2007 on the RR model). The use of separate and straightforward tasks in the experiments on the one-to-one and counting principles demonstrates a movement towards using the RR levels as denoting knowledge for a principle, rather than simple ability to apply it on one specific task.

The use of ecologically valid, everyday type tasks fits in with the approach of the RR model which tries to give a clear and comprehensive view of children's knowledge for a domain. It is important in this respect to attempt to use multiple methods to assess children's knowledge within a domain (see Messer et al, in press). As well as ensuring the validity of the levels of the model as applying to multiple tasks within a domain, it is also important to note that certain tasks may better tap the concept underlying a domain – for example, with regard to the domain of balance, children seemed much more likely to give explicit explanations on the balance beam task, where implicit weight type explanations which seemed mainly procedural in nature were most prominent. There is a need to be aware that in spite of the claims for a model which states that the representational levels should broadly apply to any tasks within a domain, there may be some small difference in actual representational level from task to task, though as chapter 3 showed, children were highly likely to be at the same end of the implicit-explicit representational continuum on both balance tasks.

2. The importance of access to verbal knowledge

The second unique feature of the RR model is its focus on the role of access to verbal knowledge for cognitive development. Karmiloff-Smith (1992) specifically states that the process of redescription "*is a process by which implicit information in the mind becomes explicit knowledge to the mind*" (Karmiloff-Smith, 1992, p. 18). This differs from the position of Piaget, who according to Nelson (1999, p 189) "*viewed language as a* *component of the representational function, but not as an important contributor to cognition per se"*.

The microgenetic study conducted within this thesis provided evidence that increasing access to verbal knowledge (as measured by first use of new and more complex verbal explanations) may indeed play a key role in changing performance on a task, rather than the traditional Piagetian view of cognitive change being driven by processes of "accommodation, assimilation and equilibration", which come about primarily due to changing levels in performance on a particular task. On the balance beam task, children were shown to be changing in their performance in balancing asymmetrical beams following first use of new and more complex verbal explanations, rather than changes in verbal explanations being brought about by changes in performance on the balance beam task.

The view of Karmiloff-Smith's model is closer to that of Nelson (1999), in focusing on increasing access to knowledge being a key factor in cognitive development, though as Nelson notes the RR model focuses on internal analysis bringing about change, and fails to address external factors, and in particular the effect of contact with others' language use. This factor has been addressed by both Pine et al (2000) and Tolmie et al (2005), who have provided a number of clear indications of the exogenous processes which may play a role in how cognitive change occurs. The microgenetic study (chapter 6) did provide evidence that children could show what may be interpreted as endogenous change, in line with the endogenous process of redescription outlined by Karmiloff-Smith (1992).

2.1 The importance of access to verbal knowledge in the domain of numeracy

The importance of verbal access to knowledge was also seen in relation to children's knowledge of the counting principles proposed by Gelman and Gallistel (1976). The levels of the RR model were applied to the one-to-one and cardinality principles, in cross-sectional studies, providing some evidence that children do develop explicit verbal knowledge for these principles during early primary school years. This provides a stark contrast with Piaget's (1952) depiction of children's knowledge of counting. Piaget stated that children do not have a formal logical knowledge of number until they successfully perform a number conservation task. The study on the cardinality principle in this thesis showed that children could show explicit verbal knowledge of the cardinality principle (which is the key principle for the conservation task) at a younger age then this however. They may not be able to apply it to all tasks, or may be misled by other cues such as row length (see Donaldson 1976 for a critique of Piaget's experiments), but this does not mean that they have no knowledge of the concept of cardinality.

An important aspect of the RR model in relation to this is the Implicit – Explicit continuum of knowledge along which children are thought to develop, which needs to be measures to provide a clearer indicator of children's knowledge for a particular domain, given the varying performance children may show on different types of tasks within that domain. Children begin with procedural knowledge which enables them to perform tasks, which is deemed as implicit, as they cannot describe the principle underlying this procedure. At the end of development, they have access to verbal knowledge. There is a qualitative change in children's knowledge. This continuum has clearly been displayed on 2 balance tasks within this thesis, as well as with tasks relating to the one-to-one and cardinality principles for counting. The endpoint in these tasks involved children not only being able to perform the tasks successfully, but also being able to explain the concept underlying the task.

The qualitative shift from successful performance without explicit knowledge to successful performance with explicit knowledge is downplayed in Rittle-Johnson et al's (2001) studies on number. Rittle-Johnson et al (2001) used an open – ended iterative model to describe the complementary development of "conceptual" and "procedural" knowledge in relation to decimal fractions. Rittle-Johnson et al (2001) take a similar approach to the RR model in how they define their key terms conceptual (similar to explicit verbal knowledge within the RR model) and procedural knowledge (similar to behavioural mastery within the RR model). In spite of this, Rittle-Johnson et al (2001) use "novel tasks" rather than children's access to verbal knowledge to measure "conceptual knowledge". This means that they could make predictions about relative improvement in both "conceptual" and "procedural" knowledge, and use ANOVAs to easily test these predictions. The use of interval-type data for conceptual knowledge is likely to be misleading, as the key change that occurs in relation to "conceptual" knowledge is in fact qualitative rather than quantitative within their own definition of the term conceptual knowledge. That is to say, children's shift in verbal knowledge, though more gradual than a simple implicit-explicit shift is qualitative in nature, and there is a need for a model which takes this fully into account, something which the RR model, but not Rittle-Johnson et al's (2001) iterative model achieves. Indeed, the data in chapter 5 did not support the predictions of Rittle-Johnson with regard to mutual improvements in conceptual and procedural knowledge, when verbal explanations were used to measure conceptual knowledge. Rather, they highlight the need to focus on children's ability to verbalise the basic principles which are thought by Gelman and Gallistel (1976) to provide a basis for being able to count.

The main reason that the RR model was applied to the principles for counting outlined by Gelman and Gallistel (1976) was that counting provided a basis for the understanding of number, and the RR model provided an approach which looked beyond children's ability to perform tasks, as Sophian (1999) called for. The RR model is particularly relevant for numeracy given the recent focus on the importance of conceptual understanding needing to be a key part of mathematical curricula (Aubrey and Godfrey, 2003), and the increasing focus on ways to improve children's conceptual knowledge of maths (Rittle-Johnson et al, 2001, Muldoon et al, 2003). This type of knowledge is overlooked to a certain extent within the Overlapping Waves model (Siegler, 1996) – where it has for example been applied to mathematical knowledge (e.g. addition tasks), it has not provided a model to describe and explain the continued development of conceptual knowledge, but simply children's use of strategies for certain types of addition tasks.

2.2 The RR model and the overlapping waves model

This is perhaps where the RR model and the Overlapping Waves model can be reconciled. The Overlapping Waves model focuses on strategy use – e.g. procedures, and does not concern itself with the notion of children showing conscious access to their thinking. Therefore, as is the case with the development of implicit representations for cardinality, the overlapping waves model can help to depict how differing strategies to perform a task develop. It does not tell us how children develop explicit verbal knowledge of the cardinality principle, which this thesis has shown may be the element which allows for generalization of knowledge. This is where the RR model can play a role, as it shows that representational change can occur, and it does not necessarily have to be driven by changes in performance on a task.

3. Cognitive Change

Karmiloff-Smith's (1992) model emphasises that children must display stability prior to change, rather than instability or cognitive conflict as proposed by other models (e.g. the Overlapping waves model, and the Dynamic systems model). The RR model also focuses on the role of children's changing access to verbal knowledge, and its primary role in bringing about cognitive change.

3.1. The role of stability prior to change:

Siegler (1996) has promoted the notion that children are inherently variable in their thinking. This is a stark contrast to the Piagetian model, which states that children have one way of thinking at any one time, across all domains. The possibility exists that the overlapping waves model, in repudiating the Piagetian stance, goes too far the other way. That is to say, variability in strategy use may well be a feature of children's behaviour, but this may mask underlying similarity in children's level of knowledge for a domain across time. For example, a child may use a variety of different strategies to solve an addition task (Siegler and Jenkins, 1989), but this variability in strategy use does not provide any extra insight into the status of children's knowledge about addition.

Certainly, variability in performance must be taken into account. This variability clearly signifies that the classical staircase-type models of development (Siegler, 1996, Case, 1992) are not accurate. The period of change is thought to be sudden and absolute in the case of staircase models – which give rise to the use of the staircase metaphor. The evidence from the microgenetic study showed that variability was present in representational levels prior to change. There were also notable periods of stability in representational levels at some period prior to representational change occurring. The incidence of stability prior to change occurring provides a basis for supposing that these periods of stability may play an important role in cognitive change. Indeed, some support for a similar approach has been found in other microgenetic studies involving similar cyclical patterns of stability, followed by a period of variability, followed by a return to stability thereafter once a more advanced approach has been adopted (Siegler and Svetina, 2002, Siegler and Chen, 1998, Van der Maas and Molenaar, 1992, Hosenfeld et al, 1997). The RR model provides a theoretical grounding for this cyclical pattern, and emphasises the role of the periods of stability in playing a part in the cognitive change, which were demonstrated in the microgenetic study (chapter 6).

The RR model maintains some semblance of the staircase model, insofar as it maintains a series of separate and distinguishable levels, indicating a clear progression in terms of children's access to knowledge. There is a clear educational need to maintain a perspective which emphasises these changes in thinking, even if these changes in access to knowledge are not always apparent in strategy use on tasks. The current British primary school curriculum is separated into a series of "key stages", denoting a particular pathway in the development of knowledge for key curriculum topics (e.g. literacy, numeracy, science, etc.). On a practical level, developmental models should be aiming to provide a guide for how each of the targets in each key stage should be achieved, or providing a basis for what should be achieved in each of these key stages. Given that the curriculum maintains a stage-based approach, it is pragmatic to use a stage-based model such as the RR model.

The microgenetic method (Siegler and Crowley, 1991) has been particularly important in revealing variability in children's thinking, both in general, and in relation to change. The microgenetic study included in this thesis, though finding that children were generally stable in maintaining the same representational level across time, found that periods of variability in representational levels immediately preceded, and in many cases, immediately followed representational change. What does this tell us about potential mechanisms for representational change? First, it is apparent that there is support for the overlapping waves model and dynamic systems' model, insofar as they state that variability must precede cognitive change. This has been recognised by Pine et al (2003), who introduced transitional levels into the RR model. Pine et al's (2004) work with gesture has been particularly enlightening, as it looks at the emergence of knowledge in gesture prior to speech, highlighting the notion that representational change is likely to be a much more gradual, piecemeal process, than classical stage models imply. The microgenetic experiment in this thesis provided evidence of "transitional periods" both preceding, and following achievement of a more advanced representational level, indicating that variability can both immediately precede, and follow cognitive change. Indeed, the notion of a staircase model may be somewhat misleading, as data are never likely to perfectly match a staircase, but merely be approximate so that a "staircase" metaphor may be useful – in the same way, one would not necessarily expect to find a perfect "U-shaped" curve in terms of relation to performance and verbal knowledge, merely that the metaphor best captures the idea being put across. It is important to highlight that a microgenetic method which looks in great depth at change at very high degrees of magnification may miss out on prolonged periods of stability, as they focus solely on "periods of change", or in some cases provide case studies of only a few individuals which again solely cover a period of change. This skips over the possibility of stability playing an important consolidatory role in cognitive development. This is not dismissing the microgenetic method as an approach, but merely stating that looking in close detail at periods surrounding change (which will not always occur as swiftly as the time periods of days/weeks in which microgenetic studies typically) does not provide a full picture of children's cognitive development, which does not simply encompass periods of change. The microgenetic study included in this thesis (chapters 6 and 7) provides clear evidence that the importance of periods of stability prior to change occurring should not be dismissed.

3.2. The role of increasing access to verbal knowledge in cognitive change

The RR model focuses on cognitive change being driven by increasing access to knowledge, rather than children's knowledge changing as a result of "accommodation" or "assimilation" based on children's performance on the task. The difficulties in testing how change occurs within Piaget's model has been well chronicled (Bryant, 2001). A similar criticism levelled against the RR model was that it was underdescribed, particularly in relation to the mechanisms of change (Campbell, 1994, Freeman, 1994, Scholnick, 1994). The microgenetic experiment in this thesis provided evidence that changes in performance on the balance beam task followed first use of more complex verbal explanations, rather then changes in performance leading to first use of more important verbal explanations. This acts as an indication that representational change may be an endogenously driven process (e.g. change occurring purely as a process of internal redescription of knowledge into verbally accessible formats).

Within the RR model, at certain points children may not be open to exogenous input. For the balance beam task, Pine et al (1998) have noted that children who had centre-theories for the balance task [equivalent to abstraction nonverbal or abstraction verbal representational levels] did not benefit from group collaboration and discussion of the concept of balance. The cardinality experiment reported in this thesis provided similar evidence that the effect of teaching interventions were highly likely to depend on children's initial representational levels. The intervention in this experiment, that gave children an opportunity to conceptualise the cardinality principle by explaining another person's error on a counting task, was helpful for children who had already achieved implicit representations. Relatively few children had "abstraction nonverbal" or abstraction verbal representational levels on this task, so it was difficult to assess the utility of this type of intervention for children with these levels. It is worth noting however that a U-shaped curve may not necessarily always arise, and that children's first conception within a domain will not always be as inaccurate as is the case within the domain of balance. The emergence of a "centre theory" in the domain of balance does not have an equivalent with regard to cardinality – abstraction nonverbal and abstraction verbal representational levels were codable, indicating that children may pass through these levels. The relatively small number of children coded to these levels in the experiment give a strong indication that there is not a prominent or long period during which they use an explicit but inaccurate representation for the cardinality principle. Nevertheless, in both domains relating to numeracy, and the domain of balance covered here, the importance of developing explicit conceptual knowledge is clear. Within the RR model, the joint focus on conceptual knowledge and performance, and in particular the key role played by explicit verbal knowledge playing in bringing about a cognitive change which has not been explored within other models of cognitive development. The overlapping waves model in particular fails to address this, and does not provide a clear description of how change can occur. Siegler (1996) describes the overlapping waves model, and describes 5 factors which relate to change (Path, rate, breadth, source, and variability), but gives no clear indication of how these fit with the basic phenomena of the overlapping waves model to provide a comprehensive model of how change can occur. The factors mentioned by Siegler in relation to change are no doubt important, and fit well for example with the Dynamic Systems approach, in describing multiple factors contributing to how cognitive change occurs, but there is no clear link between these factors and the fundamental phenomena described by the overlapping waves model, e.g. variability in strategy use. One problem with the overlapping waves model is the lack of depth in the description of the unit of change – the overlapping waves model has been mainly used to look at strategy change (Siegler, 2006), whereas the RR model has fairly well-specified units; e.g. Representational levels which are coded on the basis of task performance and access to verbal knowledge. Given this clear definition of the unit of analysis, and arising from this, a clearer structure of how the child is thought to be thinking (e.g. children having knowledge encoded in task-specific procedural formats which eventually becomes redescribed into a verbally accessible format).

The RR model also contributes to the discussion on how best to think of the phenomena of cognitive change. Carey (1991) discusses knowledge acquisition processes using a distinction between “enrichment” and “conceptual change”. Where does the RR model fit within this distinction? The process of redescription may be thought of as a process of "enrichment", as it involves the redescription of data already in the mind to another format. The qualitative shift in the nature of the knowledge however may be interpretable as involving "conceptual change". The microgenetic study showed changes in performance following first use of more complex verbal explanations. This sounds more in line with what Carey terms "change" rather then "enrichment". One finding from the studies on numeracy indicated that if children did have some innate basis for the counting principles laid out by Gelman and Gallistel (1876), it was more primitive than an implicit representation. Therefore, it is quite likely that "change" is required in order for children to achieve implicit representational levels.

It is also worth noting that the process of "redescription" is not necessarily sufficient to describe all the changes occurring in children's thinking. For example, the development of E3 representational levels from abstraction verbal on the balance beam task requires the introduction of explicit knowledge about the role of distance in relation to balance. This requires an adjustment away from a simple "centre theory" (which states that all objects balance at their centre). Some form of "assimilation" may in this case be needed to reconcile their original weight-based representation with the idea that the distance of a weight from the fulcrum also plays a role in whether or not an object will balance. This is demonstrated by the fact that only a small proportion of children showed spontaneous movement from abstraction verbal to E3 within the microgenetic experiment (see chapter 7). Karmiloff-Smith (1992) says that children at this level may not pay heed to external data which contradicts their own representations. Yet there was little evidence in the microgenetic study that they will spontaneously show improvement (or at least not within the 2 week period that the microgenetic experiment spanned). This begs the question of how children may overcome misconceptions, if at all. Some, but not all children may overcome misconceptions, or ever achieve more complex conceptions for a domain. Within the domain of numeracy for example, it could be hypothesised that children's difficulties with fractions arises from its incompatibility with their own conception of number (Gallistel and Gelman, 1991, for example make a similar claim that their model is supported by the fact that children have difficulties with fractions). The same may be the case for algebra, when "imaginary" and abstract numbers become more prominent, and the current thesis promotes the possibility of the RR model being used in this way to help children develop representations for these more complex forms of number.

Summary and Conclusions

This thesis has focused on addressing issues for the RR model in order to provide additional evidence that it can be employed as a general model for cognitive development. Three specific aspects of the RR model were discussed – the RR model as a domain specific model, the importance of access to verbal knowledge, and cognitive change as described by the RR model. This thesis contributed to our knowledge of each of these aspects within the RR model. With regard to the first point of the RR model as a domain specific model, the levels of the model were applied to different tasks within the domain of balance, and to two of Gelman and Gallistel's counting principles within the domain of numeracy. The issue of generalisation was also covered, with the importance of verbal knowledge being the key to generalisation being focused on. Moving on to the second key feature of the RR model, the importance of access to verbal knowledge was covered, showing that it is verbal knowledge which becomes accessible to other tasks within a domain, and provided evidence that increasing access to verbal knowledge can play a key role in cognitive change. Finally, with regard to cognitive change, the microgenetic study provided a response to the criticisms of the RR model with regard to the mechanisms of change being underdescribed were to some extent answered by providing evidence for the proposed method of redescription of the RR model (e.g. a period of stability which allows for increasing access to verbal knowledge). Together, the findings of this thesis show that the RR model can add significantly to our understanding of cognitive developmental processes.

References

Adolph, K. E. (1997). Learning in the development of infant locomotion. *Monographs of the Society for Research in Child Development, 62*(3).

Alibali, M. W. (1999). How children change their minds: Strategy change can be gradual or abrupt. *Developmental Psychology, 35*, 127-145.

Alibali, M. W., & DiRusso, A. A. (1999). The function of gesture in learning to count: More than keeping track. *Cognitive Development, 14*(1), 37-56.

Alibali, S. G.-M. a. M. W. (1994). do you have to be right to redescribe? *Behavioral and Brain Sciences, 17*(4), 718-719.

Amsel, E., Goodman, G., Savoie, D., & Clark, M. (1996). The development of reasoning about causal and noncausal influences on levers. *Child Development, 67*(4), 1624-1646.

Anderson, N. H., & Wilkening, F. (1990). Adaptive thinking in intuitive physics. In N. H. Anderson (Ed.), *Contributions to information integration theory, Vol. 1: Cognition; Vol. 2: Social; Vol. 3: Developmental* (pp. 1-42). Hillsdale, NJ, England: Lawrence Erlbaum Associates, Inc.

Antell, S. E., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child Development, 54*(3), 695-701.

Aubrey, C. G., Ray. (2003). The Development of Children's earlynumeracy through key stage 1. *British Educational Research Journal, 29*(6), 821-840.

Barlow, C. M., Jolley, R. P., White, D. G., & Galbraith, D. (2003). Rigidity in children's drawings and its relation with representational change. *Journal of Experimental Child Psychology, 86*(2), 124-152.

Baroody, A. J., Ginsburg, H. P., & Waxman, B. (1983). Children's use of mathematical structure. *Journal for Research in Mathematics Education, 14*(3), 156-168.

Bermejo, V. (1996). Cardinality development and counting. *Developmental Psychology, 32*(2).

Bermejo, V., Morales, S., & deOsuna, J. G. (2004). Supporting children's development of cardinality understanding. *Learning and Instruction, 14*(4), 381-398.

Bowerman, M. (1981). The child's expression of meaning: Expanding relationships among lexicon, syntax, and morphology. *Annals of the New York Academy of Sciences, 379*(172-189).

Briars, D., & Siegler, R. S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology, 20*(4), 607-618.

Bryant, P. (2001). Learning in Geneva: the contribution of Barbel Inhelder and her colleagues. In A. Typhon & J. Voneche (Eds.), *Working with Piaget: Essays in honour of Barbel Inhelder*. Hove, UK; Psychology Press

Campbell, R. L. (1994). what's getting redescribed? *Behavioural and Brain Sciences, 17*(4), 710-711.

Carey, S. (1985). *Conceptual change in childhood*. Cambridge, MA: MIT Press.

Carey, S. (1988). Conceptual differences between children and adults. *Mind and Language, 3*, 167-181.

Carey, S. (1991). *The epigenesis of mind - Essays on biology and cognition*. Hillsdale New Jersey: Lawrence Erlbaum Associates.

Case, R. (1992). T*he mind's staircase: Exploring the conceptual underpinnings of children's thought and knowledge*. New York: Academic Press.

Chen, Z., & Klahr, D. (1999). All other things being equal: Acquisition and transfer of the Control of Variables Strategy. *Child Development, 70*(5), 1098-1120.

Chetland, E., & Fluck, M. (2007). Children's performance on the 'give x' task: A microgenetic analysis of 'counting' and 'grabbing' behavior. *Infant and Child Development, 16*(1), 35-51.

Chi, M. T. H., de Leeuw, N., Chiu, M. H., & LaVancher, C. (1994). Eliciting self-explanations improves understanding. *Cognitive Science, 18*(3), 439-477.

Clearfield, M. W., & Mix, K. S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Psychological Science, 10*(5), 408-411.

Copley, J. V. (1999). *Mathematics in the early years*: National Council of Teachers.

Coyle, T. R., & Bjorklund, D. F. (1996). The development of strategic memory: A modified microgenetic assessment of utilization deficiencies. *Cognitive*

Development, 11(2), 295-314.

Critten, S., Pine, K.J, Messer, D.J. (2007). *A longitudinal study of the process of explicitation in children's reading and spelling representations.* Paper presented at the Society for the scientific study of reading, Prague.

Critten, S., Pine, K., & Steffler, D. (2007). Spelling Development in Young Children: A Case of Representational Redescription? *Journal of Educational Psychology, 99*(1), 207-220.

Dartnall, T. (1994). Redescribing Redescription. *Behavioral and Brain Sciences, 17*(4), 712-713.

Dehaene, S. (1997). *The Number Sense*. London: Penguin.

Dixon, J. A., & Bangert, A. S. (2005). From regularities to concepts: The development of children's understanding of a mathematical relation. *Cognitive Development, 20*(1), 65-86.

Donaldson, M. (1978). *Children's Minds*. London: Fontana.

Donlan, C. (1998). *The development of mathematical skills*: Psychology Press Ltd.

Ferretti, R. P., Butterfield, E. C., Cah, A., & Kerkman, D. (1985). The classification of children's knowledge: Development on the balance-scale and inclined-plane tasks. *Journal of Experimental Child Psychology, 39*(131-160).

Flavell, J. H. (1963). *The developmental psychology of Jean Piaget*. New York: Van Nostrand.

Fluck, M., Linnell, M., & Holgate, M. (2005). Does counting count for 3- to 4-year-olds? Parental assumptions about preschool children's understanding of counting and cardinality. *Social Development, 14*(3), 496-513.

Fodor, J. (1986). The modularity of mind. In Z. W. Pylyshyn & W. Demopoulos (Eds.), *Meaning and cognitive structure: Issues in the computational theory of mind* (pp. 3-18). Westport, CT: Ablex Publishing.

Foster-Cohen, S. H. (1994). Arguments against linguistic "modularization". *Behavioral and Brain Sciences, 17*(4), 716-717.

Freeman, N. H. (1994). Redescription of intentionality. *Behavioral and Brain Sciences, 17*(4), 717-718.

Frege, G. (1893). *Grundgesetze der Arithmetik, Band I*.

Frye, D., Braisby, N., Lowe, J., Maroudas, C., & et al. (1989). Young children's understanding of counting and cardinality. *Child Development, 60*(5), 1158-1171.

Fuson, K. C. (1988). *Children's counting and concepts of number*.

Fuson, K. C., Secada, W. G., & Hall, J. W. (1983). Matching, counting, and conservation of numerical equivalence. *Child Development, 54*(1), 91-97.

Gallistel, C. R., Brown, A. L., Carey, S., Gelman, R., & Keil, F. C. (1991). Lessons from animal learning for the study of cognitive development. In S. Carey & R. Gelman (Eds.), *The epigenesis of mind: Essays on biology and cognition* (pp. 3-36). Hillsdale, NJ, England: Lawrence Erlbaum Associates, Inc.

Gallistel, C. R., & Gelman, R. (1991). Subitizing: The preverbal counting process. In W. Kessen & A. Ortony (Eds.), *Memories, thoughts, and emotions: Essays in honor of George Mandler* (pp. 65-81). Hillsdale, NJ, England: Lawrence Erlbaum Associates, Inc.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Oxford, England: Harvard U Press.

Gelman, R., & Meck, E. (1983). Preschoolers' counting: Principles before skill. *Cognition, 13*(3), 343-359.

Gelman, R., & Meck, E. (1986). The notion of principle: The case of counting. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 29-57). Hillsdale, NJ, England: Lawrence Erlbaum Associates, Inc.

Gelman, R., Meck, E., & Merkin, S. (1986). Young children's numerical competence. *Cognitive Development, 1*(1), 1-29.

Ginsburg, H. (1975). Young children's informal knowledge of mathematics. *Journal of Children's Mathematical Behavior, 1*(3), 63-156.

Goldin Meadow, S. (2002). From thought to hand: Structured and unstructured communication outside of conventional language. In E. Amsel & J. P. Byrnes (Eds.), *Language, literacy, and cognitive development: The development and consequences of symbolic communication* (pp. 121-150). Mahwah, NJ: Lawrence Erlbaum Associates Publishers.

Goldin-Meadow, M. W. A. S. (1993). Gesture-speech mismatch and mechanisms of learning: What the hands reveal about a child's state of mind. *Cognitive Psychology, 25*, 468-523.

Goswami, U. (2002). *Blackwell handbook of Cognitive Development*: Blackwell Publishing.

Greco, P. (1962). Quantite et quotite: Nouvelle recherches sur la corrspondance terme a terme et la conversation des ensembles. In P. Greco, & Morf, A. (Ed.), *Structures mi,eroqies elementaires: etudes d'epistemologie genetique* (Vol. 13, pp. 35-52). Paris: Presses Universitaires de France.

Grupe, L. (1998). *A microgenetic study of strategy discovery in young children's addition problem solving: A thesis*.Unpublished manuscript.

Grush, R. (1994). Beyond connectionist versus classical AI: A control theoretic perspective on development and cognitive science. *Behavioral and Brain Sciences, 17*(4), 720.

Halford, G. S. (2002). Information-processing models of cognitive development. In U. Goswami (Ed.), *Blackwell handbook of childhood cognitive development* (pp. 555-574). Malden, MA: Blackwell Publishing.

Halford, G. S., Andrews, G., Dalton, C., Boag, C., & Zielinski, T. (2002). Young children's performance on the balance scale: The influence of relational complexity. *Journal of Experimental Child Psychology, 81*(4), 417-445.

Hampson, P. J. (1994). Representational redescripiton, memory, and connectionism. *Behavioral and Brain Sciences, 17*(4), 720.

Hollis, S., & Low, J. (2005). Karmiloff-Smith's RRM distinction between adjunctions and redescriptions: It's about time (and children's drawings). *British Journal of Developmental Psychology, 23*(4), 623-644.

Hosenfeld, B., van der Maas, H. L. J., & van den Boom, D. C. (1997). Indicators of discontinuous change in the development of analogical reasoning. *Journal of Experimental Child Psychology, 64*(3), 367-395.

Inhelder, B., Piaget, J., Parsons, A., & Milgram, S. (1958). The growth of logical thinking: From childhood to adolescence.

Inhelder, B. P., J. (1958). *the growth of lofical thinking from childhood to adolescence: an essay on the construction of formal operational structures*. New York: Basic Books.

Jansen, B. R. J., & van der Maas, H. L. J. (2002). The development of children's rule use on the balance scale task. *Journal of Experimental Child Psychology, 81*(4), 383-416.

Karmiloff Smith, A. (1983). A note on the concept of "metaprocedural processes" in linguistic and non-linguistic cognitive development. *Archives de psychologie, 51*(196), 35-40.

Karmiloff Smith, A. (1992). *Beyond modularity: A developmental perspective on cognitive science*. Cambridge, MA: The MIT Press.

Karmiloff Smith, A., & Inhelder, B. (1974). If you want to get ahead, get a theory. *Cognition, 3*(3), 195-212.

Kliman, M. (1987). Children's learning about the balance scale. *Instructional Science, 15*(4), 307-340.

Kuhn, D. (1994). the power of explicit knowing. *Behavioral and Brain Sciences, 17*(4), 722-723.

Kuhn, D., Amsel, E., O'Loughlin, M., Schauble, L., Leadbeater, B., & Yotive, W. (1988). *The development of scientific thinking skills*. Academic Press Inc. New York

Kuhn, D., Schauble, L., & Garcia Mila, M. (1992). Cross-domain development of scientific reasoning. *Cognition and Instruction*, 9 (4), 285-327

Kuhn, D. P., E. (1982). the development of problem-solving strategies. In H. Reese (Ed.), *Advances in child development and behaviour* (Vol. 17, pp. 1-44). New York: Academic Press.

McLaughlin, G. H. (1963). Psycho-logic: A possible alternative to Piaget's formulation. *British Journal of Educational Psychology, 33*, 61-67.

Muldoon, K. & Freeman N. (2003). Putting counting to work: preschoolers' understanding of cardinal extension. *International Journal of Educational research, 39*, 695-718.

Muldoon, K., Lewis, C, & Towse, J (2005). Because it’s there! Why some children count, rather than infer numerical relationships. *Cognitive Development*, 20 (3), 472-491

Metz, K. E. (1998). Emergent understanding and attribution of randomness: Comparative analysis of the reasoning of primary grade children and undergraduates. *Cognition and Instruction, 16*(3), 285-365.

Nelson, K. (1996). *Language in cognitive development: Emergence of the mediated mind*: Cambridge University Press.

Normandeau, S., Larivee, S., Roulin, J. L., & Longeot, F. (1989). The balance-scale dilemma: Either the subject or the experimenter muddles through. *Journal of Genetic Psychology, 150*(3), 237-250.

Olson, D. R. (1994). Where redescriptions come from. *Behavioral and Brain Sciences, 17*(4), 724.

Oshima, J., & Okada, T. (1996). Process of children's knowledge acquisition in a balance scale task. *Hiroshima Forum for Psychology, 17*, 1-12.

Pascual-Leone, J. A. (1970). A mathematical model for the transition rule in Piaget's developmental stages. *Acta Psychologica, 32*, 301-345.

Pauen, S., & Wilkening, F. (1997). Children's analogical reasoning about natural phenomena. *Journal of Experimental Child Psychology, 67*(1), 90-113.

Perry, M., & Elder, A. D. (1997). Knowledge in transition: Adults' developing understanding of a principle of physical causality. *Cognitive Development, 12*(1), 131-157.

Philips, S., & Tolmie, A. (2007). Children's performance on and understanding of the Balance Scale problem: The effects of parental support. *Infant and Child Development, 16*(1), 95-117.

Piaget, J. (1932). *The moral judgement of the child*. New York: Free Press.

Piaget, J. (1952). *The child's conception of number*. London: Routledge and Kegan Paul.

Piaget, J., Inhelder, B. (1956). *The child's conception of space*. New York: Norton.

Piaget, J. (1960). *The child's conception of physical causality*. Paterson, NJ: Littlefield, Adams.

Piaget, J., Inhelder, B. (1974). *The child's construction of quantities*. London: Routledge and Kegan Paul.

Pine, K., & Messer, D. (1999). What children do and what children know: Looking beyond success using Karmiloff-Smith's RR framework. *New Ideas in Psychology, 17*(1), 17-30.

Pine, K., & Messer, D. (2003). The development of representations as children learn about balancing. *British Journal of Developmental Psychology, 21*(2), 285-301.

Pine, K., Messer, D., & St John, K. (2001). Children's misconceptions in primary science: A survey of teachers' views. *Research in Science and Technological Education, 19*(1), 79-96.

Pine, K. J., Lufkin, N., & Messer, D. (2004). More gestures than answers: Children learning about balance. *Developmental Psychology, 40*(6), 1059-1067.

Pine, K. J., & Messer, D. J. (1998). Group collaboration effects and the explicitness of children's knowledge. *Cognitive Development, 13*(1), 109-126.

Pine, K. J., & Messer, D. J. (2000). The effect of explaining another's actions on children's implicit theories of balance. *Cognition and Instruction, 18*(1), 35-52.

Pine, K. J., Messer, D. J., & Godfrey, K. (1999). The teachability of children with naive theories: An exploration of the effects of two teaching methods. *British Journal of Educational Psychology, 69*(2), 201-211.

Rittle Johnson, B., & Siegler, R. S. (1998). The relation between conceptual and procedural knowledge in learning mathematics: A review. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 75-110). Hove, England: Psychology Press/Taylor & Francis (UK).

Rittle Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346-362.

Rutkowska, J. C. (1994). Situating representational redescrition in infants' pragmatic knowledge. *Behavioral and Brain Sciences, 17*(4), 726-727.

Schauble, L. (1990). Belief revision in children: The role of prior knowledge and strategies for generating evidence. *Journal of Experimental Child Psychology, 49*(1), 31-57.

Scholnick, E. K. (1994). Redescribing development. *Behavioral and Brain Sciences, 17*(4), 727-728.

Shultz, T. R. (1994). The challenge of representational redescription. *Behavioral and Brain Sciences, 17*(4), 728-729.

Siegler R.S & Chen, Z. (1998). Developmental differences in rule learning: A microgenetic analysis. *Cognitive Psychology, 36*, 273-310.

Siegler R.S & Crowley, K. (1991). The microgenetic method: A direct means of studying cognitive development. *American Psychologist, 46*, 606-620.

Siegler R.S & Svetina, M. (2002). A Microgenetic/cross-sectional study of matrix completion. *Child Development, 73*, 793-809.

Siegler, R. S. (1976). Three aspects of cognitive development. *Cognitive Psychology, 8*(4), 481-520.

Siegler, R. S. (1982). The rule-assessment approach and education. *Contemporary Educational Psychology, 7*(3), 272-288.

Siegler, R. S. (1987). Strategy choices in subtraction. In J. A. Sloboda (Ed.), *(1987). Cognitive processes in mathematics. Keele cognition seminars, Vol. 1* (pp. 81-106). New York, NY: Clarendon Press/Oxford University Press.

Siegler, R. S. (1995). How does change occur: A microgenetic study of number conservation. *Cognitive Psychology, 28*(3), 225-273.

Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*: London, Oxford University Press. (1996). viii, 278 pp.

Siegler, R. S. (2006). Microgenetic Analyses of Learning. In D. Kuhn, R. S. Siegler, W. Damon & R. M. Lerner (Eds.), *Handbook of child psychology: Vol 2, Cognition, perception, and language (6th ed.)* (pp. 464-510). Hoboken, NJ: John Wiley & Sons Inc.

Siegler, R. S., & Jenkins, E. (1989). *How children discover new strategies*: Lawrence Erlbaum Associates.

Siegler, R. S., & Richards, D. D. (1979). Development of time, speed, and distance concepts. *Developmental Psychology, 15*(3), 288-298.

Siegler, R. S., & Shipley, C. (1995). Variation, selection, and cognitive change. In T. J. Simon & G. S. Halford (Eds.), *Developing cognitive competence: New approaches to process modeling* (pp. 31-76). Hillsdale, NJ, England: Lawrence Erlbaum Associates, Inc.

Siegler, R. S., & Vago, S. (1978). The development of a proportionality concept: Judging relative fullness. *Journal of Experimental Child Psychology, 25*(3), 371-395.

Smith, C., Carey, S., Wiser, M. (1985). on differentiation: a case study of the development of size, weight, and density. *Cognition, 21*, 177-237.

Smith, L. (1994). Model knowledge and transmodularity. *Behavioral and Brain Sciences, 17*(4), 729-730.

Sophian, C. (1997). Beyond competence: The significance of performance for conceptual development. *Cognitive Development, 12*(3), 281-303.

Starkey, P., & Cooper, R. G. (1980). Perception of numbers by human infants. *Science, 210*. 1033-1035

Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition, 36*(2), 97-127.

Strauss, S. (1982). *U-shaped behavioural growth*. New York: Academic Press.

Surber, C. F., & Gzesh, S. M. (1984). Reversible operations in the balance scale task. *Journal of Experimental Child Psychology, 38*(2), 254-274.

Thelen, E., & Corbetta, D. (2002). Microdevelopment and dynamic systems: Applications to infant motor development. In N. Granott & J. Parziale (Eds.), *Microdevelopment: Transition processes in development and learning* (pp. 59-79). New York, NY: Cambridge University Press.

Thelen, E., & Smith, L. B. (1994). *A dynamic systems approach to the development of cognition and action*: MIT Press.

Tolmie, A., Thomson, J. A., Foot, H. C., Whelan, K., Morrison, S., & McLaren, B. (2005). The effects of adult guidance and peer discussion on the development of children's representations: Evidence from the training of pedestrian skills. *British Journal of Psychology, 96*(2), 181-204.

Van der Maas, H. L., & Molenaar, P. C. (1992). Stagewise cognitive development: An application of catastrophe theory. *Psychological Review, 99*(3), 395-417.

Van Loosbroek, E., & Smitsman, A. W. (1990). Visual perception of numerosity in infancy. *Developmental Psychology, 26*(6), 911-922.

Wellman, -. H.-M. (2002). Understanding the psychological world: Developing a theory. In U. Goswami (Ed.), *Blackwell handbook of childhood cognitive development* (Vol. Blackwell Publishing, pp. 167-187). Malden, MA.

Wellman, H. M. (2002). Understanding the psychological world: Developing a theory of mind. In U. Goswami (Ed.), *Blackwell handbook of childhood cognitive development* (pp. 167-187). Malden, MA: Blackwell Publishing.

Wilkening, F., & Anderson, N. H. (1982). Comparison of two rule-assessment methodologies for studying cognitive development and knowledge structure. *Psychological Bulletin, 92*(1), 215-237.

Wynn, P. B. a. K. (1994). the real problem with constructivism. *Behavioral and Brain Sciences, 17*(4), 707-708.

Zelazo, P. D. (1994). from the decline of development to the ascent of consciousness. *Behavioral and Brain Sciences, 17*(4), 731-732.

Zelazo, P. D. (2004). The development of conscious control in childhood. *Trends in Cognitive Sciences, 8*(1), 12-17.

Appendices

Appendix A:

.1. Instruction sheets used for the balance scale tasks in chapter 2
.2. Instruction sheets used for the balance scale and beam tasks in chapter 3
.3. Instruction sheets and examples of the materials used for the experiment on the one-to-one counting principle in chapter 4
.4. Instruction sheets used for the cardinality experiment in chapter 5
.5. Intervention sheets used in the cardinality task in chapter 5
.6. Instruction sheets used for the microgenetic experiment in chapters 6 and 7

Appendix I

.1.1st Balance experiment:

3.1 Instruction sheets for the 3 types of balance scale task:

(1) Balance scale production task:

Production and commentary

The child is shown the balance scale, and told:

“This will be my side of the scale, and that will be your side. I will put a few rings on my side, and give you a few weights. I want you to try and make the scale balance and stay straight. Do you understand?

(2) 2 types of balance scale task here:

Type 1: the experimenter places weights on 1 side of the scale and asks the child on which peg they would put a specific number of weights in order to make the scale balance.

Type 2: The experimenter places weights on 1 side of the scale, and asks the child how many weights they should place on a specific peg in order to make the scale balance

Balance Scale Production Task 1

Production and Commentary

The child is shown the balance scale and the rings and told: "this will be my side, and that will be your side. I will put a few rings on my side, and I will give you a few rings to put on your side, to try and make the scale balance and stay straight. Do you understand?"

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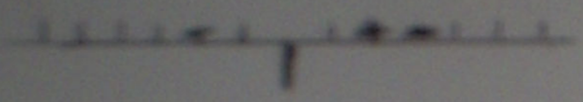

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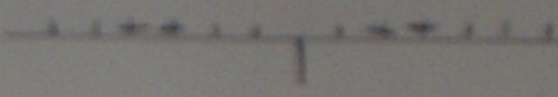

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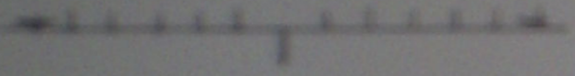

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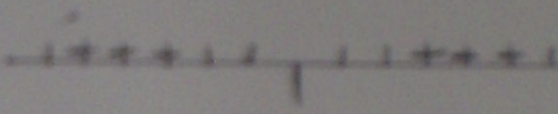

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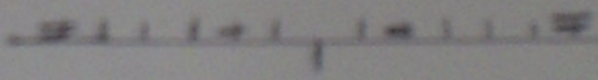

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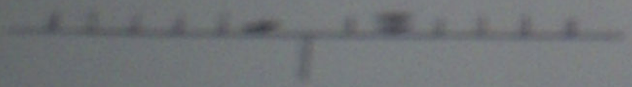

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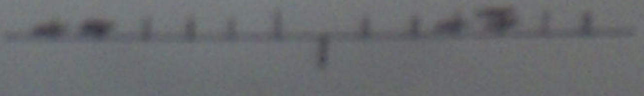

Balance Scale Production task 2: Single move condition

Type 1: The experimenter places weights on 1 side of the scale and asks the child on which peg they would put a specific number of weights in order to make the scale balance

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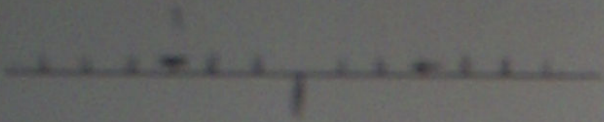

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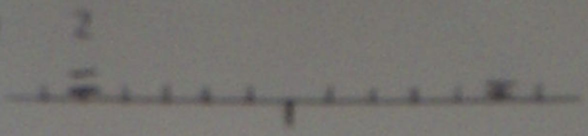

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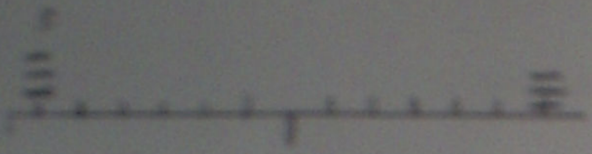

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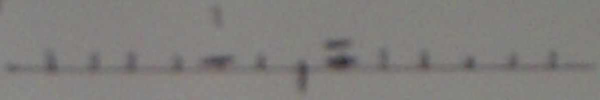

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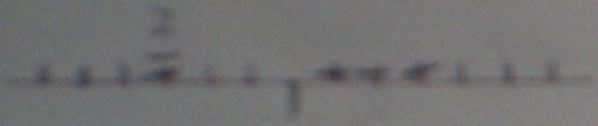

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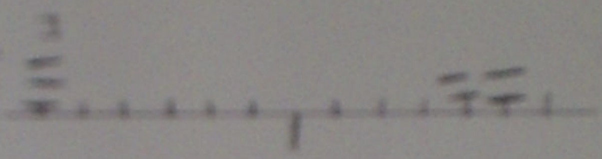

Type 2: The experimenter places weights on 1 side and asks the child how many weights they would place on a specific beam in order to make the scale balance

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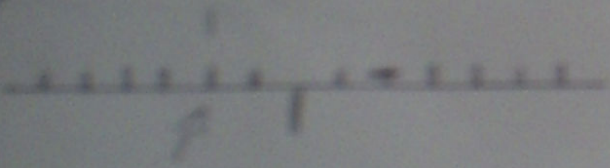

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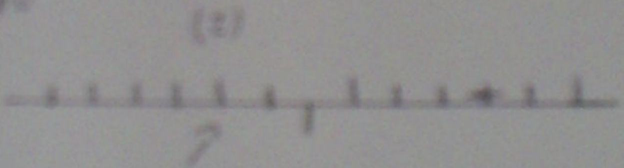

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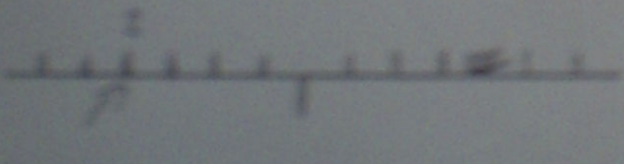

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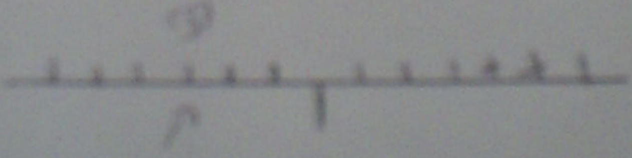

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3. Balance scale free production task:

The child is shown the balance apparatus: the scale and the weights, which they are told can be put on the pegs either side of the middle. The child is told that they are allowed play with the scale, looking at how you put on the weights so that the beam balances and stays straight. They are told they can put on as many of the weights as they want, wherever they want, in order to find out the different ways that you can make the scale balance and stay straight.

The puppet is then introduced. The child is told that the puppet does not know anything about balancing things, so they would like the child to explain what they are doing to this puppet.

When the child has said he has finished, or stops coming up with novel balancing configurations, they are again asked to explain to the puppet how the scales work. The puppet asks two questions:

(1) What if you put more weights on one side of the scale?

(2) What if the weights are put on different pegs on either side?

Once the child has answered these questions, the child is told that the game is finished, and they are thanked.

Appendix I

.2 ***Balance Scale and Balance Beam Tasks experiment***

Balance scale experiment sheet

Half the children were given the balance beam task first, and half the children were given the balance scale task first in this experiment.

Balance scale task sheet:

The child is shown the balance apparatus: the scale and the weights, which they are told can be put on the pegs either side of the middle. The child is told that they are allowed play with the scale, looking at how you put on the weights so that the beam balances and stays straight. They are told they can put on as many of the weights as they want, wherever they want, in order to find out the different ways that you can make the scale balance and stay straight.

The puppet is then introduced. The child is told that the puppet does not know anything about balancing things, so they would like the child to explain what they are doing to this puppet.

When the child has said he has finished, or stops coming up with novel balancing configurations, they are again asked to explain to the puppet how the scales work. The puppet asks two questions:

(1) What if you put more weights on one side of the scale?

(2) What if the weights are put on different pegs on either side?

Once the child has answered these questions, the child is told that the game is finished, and they are thanked.

2.2 Instruction sheet for the Balance Beam Production Task:

The child is shown the array of beams and the fulcrum. The child is told that they are going to be given the beams, and that the object of the game is to try and make the beams balance on the fulcrum – the small wooden bit. They are shown the puppet, who they are told is trying to learn about balance. The child is asked to explain to this puppet for each beam why the beam does or does not balance.

Colour	Length	Width	Weight ratio	Initial Place
Symmetrical:				
5. Wood	30cm	2 1/2cm	0:0	Left/Centre/Right
8. Red	30cm	2 cm	1:1	Left/Centre/Right
2. Green	40cm	2 ½ cm	1:1	Left/Centre/Right
Asymmetrical:				
7. Pink	40cm	3cm	3:1	Left/Centre/Right
1. Red	43cm	1 ½ cm	2:1	Left/Centre/Right
4. Mauve	30cm	2 cm	3: 1 ½	Left/Centre/Right
6. Blue	25cm	2 cm	3:2	Left/Centre/Right
3. Yellow	20 cm	2 cm	1:0	Left/Centre/Right
9. White	30cm	1 ½ cm	3:2	Left/Centre/Right

Appendix I

.3. Instruction sheets and examples of the materials used for the experiment on the one-to-one counting principle in chapter 4.

Instruction Sheet:

Half the children start with the counting task; the other half begin with the error detection task.

Children are asked to sit down at a desk in front of the camera. The camera is switched off. The child is shown a variety of the materials to be used – the counting objects, sheets, and the puppets. The child will be told they are going to count the objects, and watch the puppets trying to count the objects – the puppets are learning to count. They are told that the whole session will be recorded, for the puppets to watch again later.

Counting Task:

An array of objects is placed in front of the child. Children are asked simply to count the objects. If the child does not count all the objects, they are asked "can you count them all please?"

Counting Task Order:

4 blocks

6 dominoes

4 balls (2D)

6 dogs (2D)

8 soldiers

10 round counters

8 carrots (2D)

10 corn cobs (2D)

12 crayons

12 circular buttons (2D)

Error Detection Task:

Children are told that the 2 puppets are learning to count, and might well make mistakes. They are told to watch the puppets count, and then tell them which are right. Children are told that the puppets don't always agree on how to count. For example, show the puppets arguing over the colour of the circular counters – one thinks it's pink, the other says it's purple: which is right?

After this, they'll be told to watch the puppet carefully as they count, to see if they are counting properly. After each puppet counts, the child are asked if the puppet counted correctly – if so / if not, why? When both puppets have counted an array, it is clarified if the child thought both were right or wrong – Why did they think they were right / *wrong*?

Mistakes:

4 blocks (3D)
* puppet 1 makes gesture error (skip over object in gesture: 3)

6 dominoes (3D)
* puppet 2 makes speech error (number tagging an object twice: 4)

4 cats (2D)
* puppet 1 makes speech error (skipping over an object: 2)

6 citrus fruits (2D)
*puppet 2 makes gesture error (pointing twice to an object: 5)

8 soldiers (3D)
* puppet 1 makes speech error (number tagging an object twice: 6)

10 circular counters (3D)
*puppet 2 gesture error (skip over an object in gesture: 7)

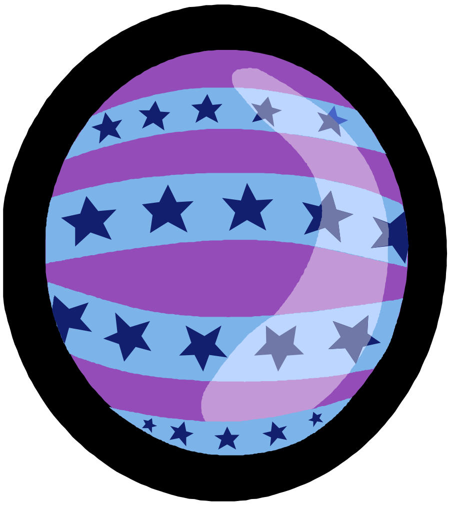
8 wildcats (2D)
* puppet 1 makes speech and gesture errors (skip over a count object, 3, count twice, 6)

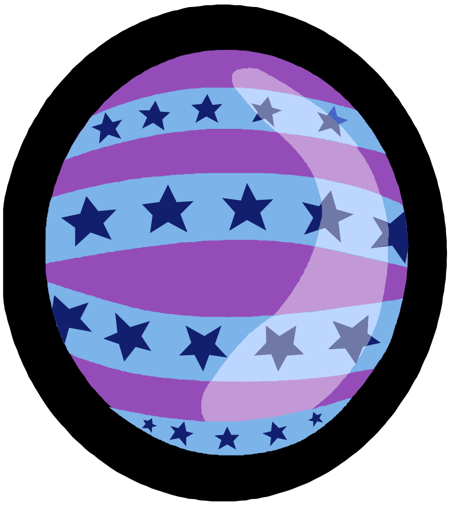
10 squares (2D)
* puppet 2 makes speech and gesture errors (gesture twice at an object, 4, skip an object in gesture, 8)

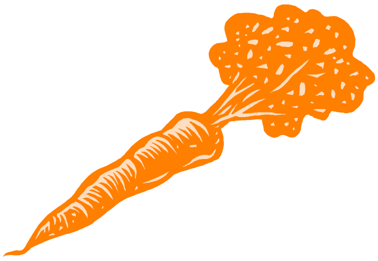
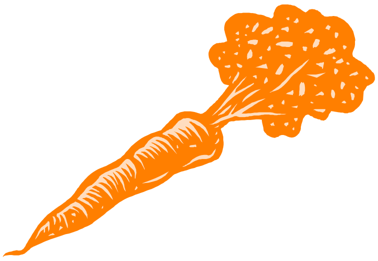
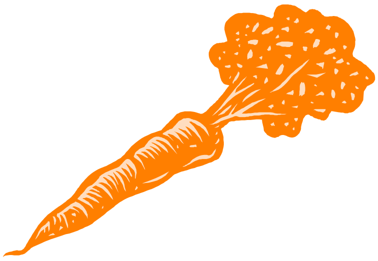
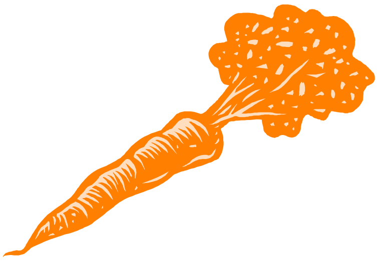
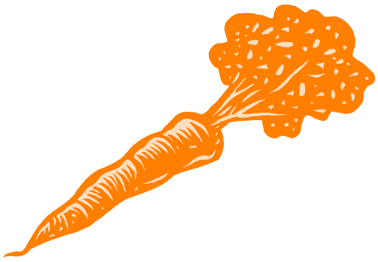
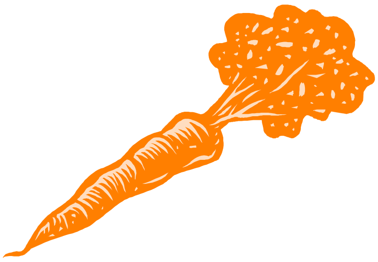
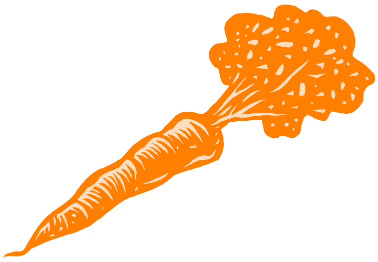
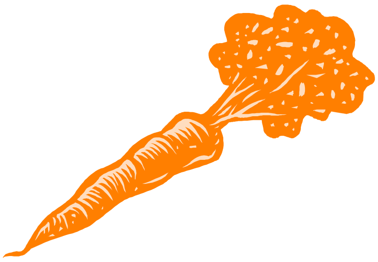
12 crayons (3D)
* puppet 1 makes gesture error (pointing twice to an object: 10)

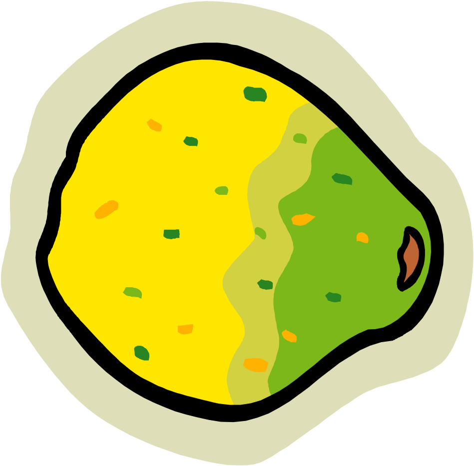
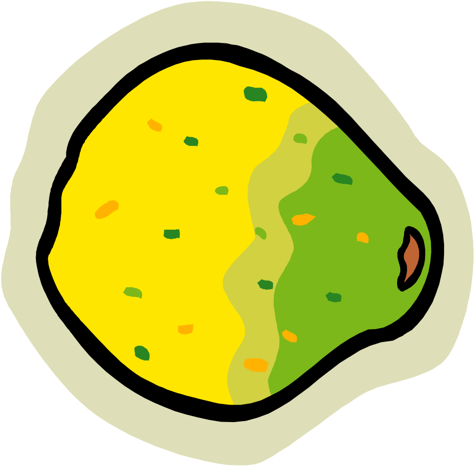
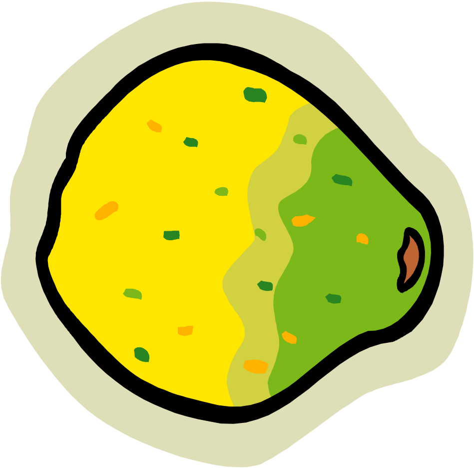
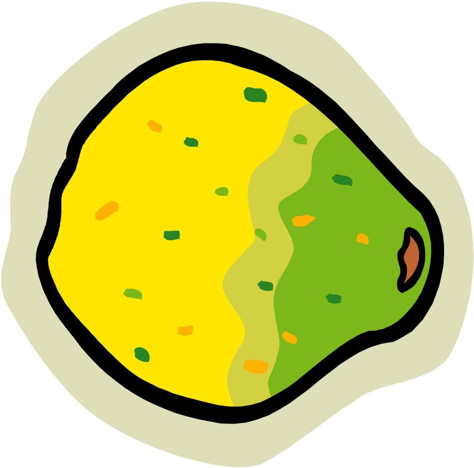
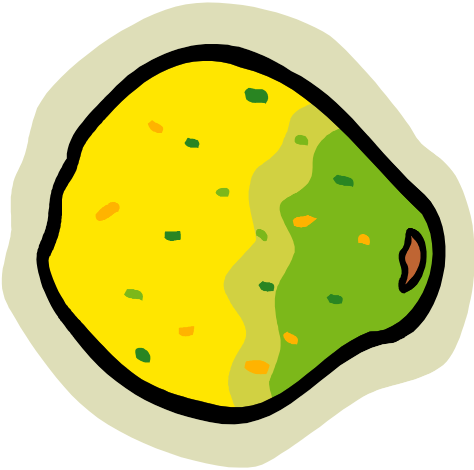
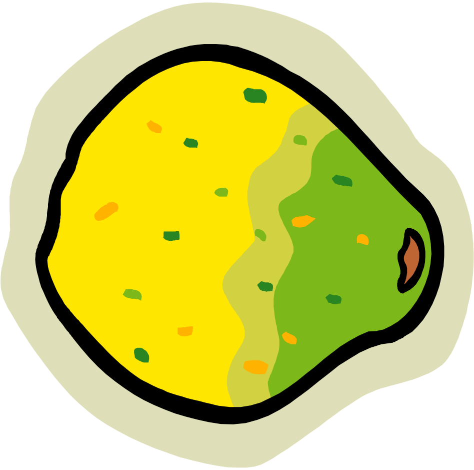
12 broccoli (2D)
* puppet 2 makes speech error (skipping over an object in count: 8)

.3.2 2D materials





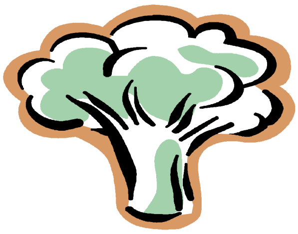
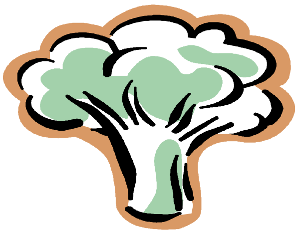
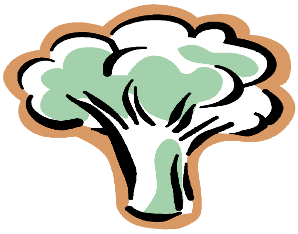
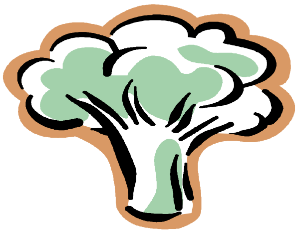
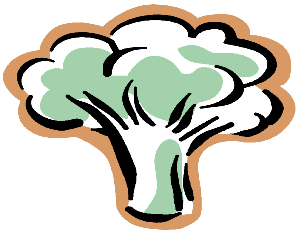
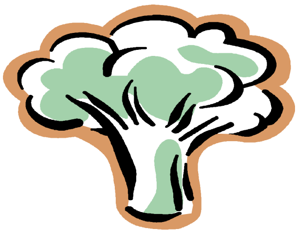
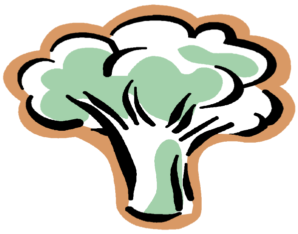
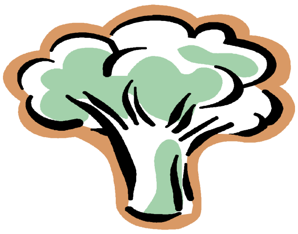
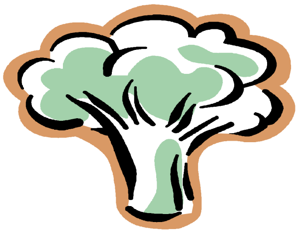
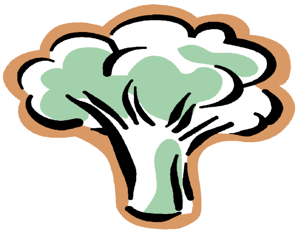
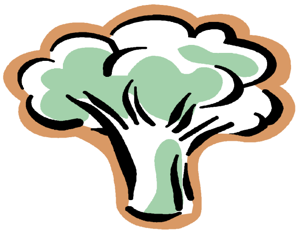
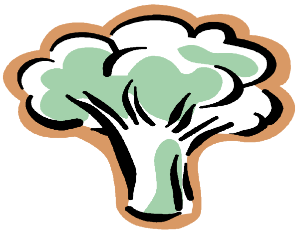







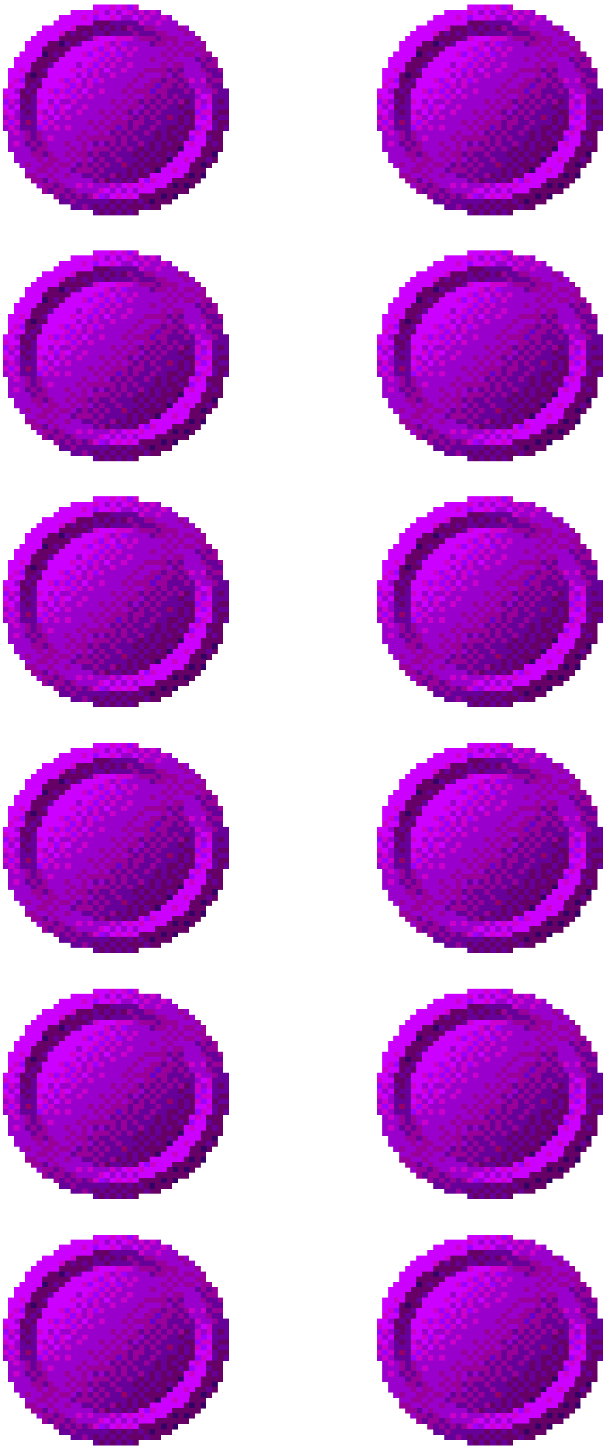


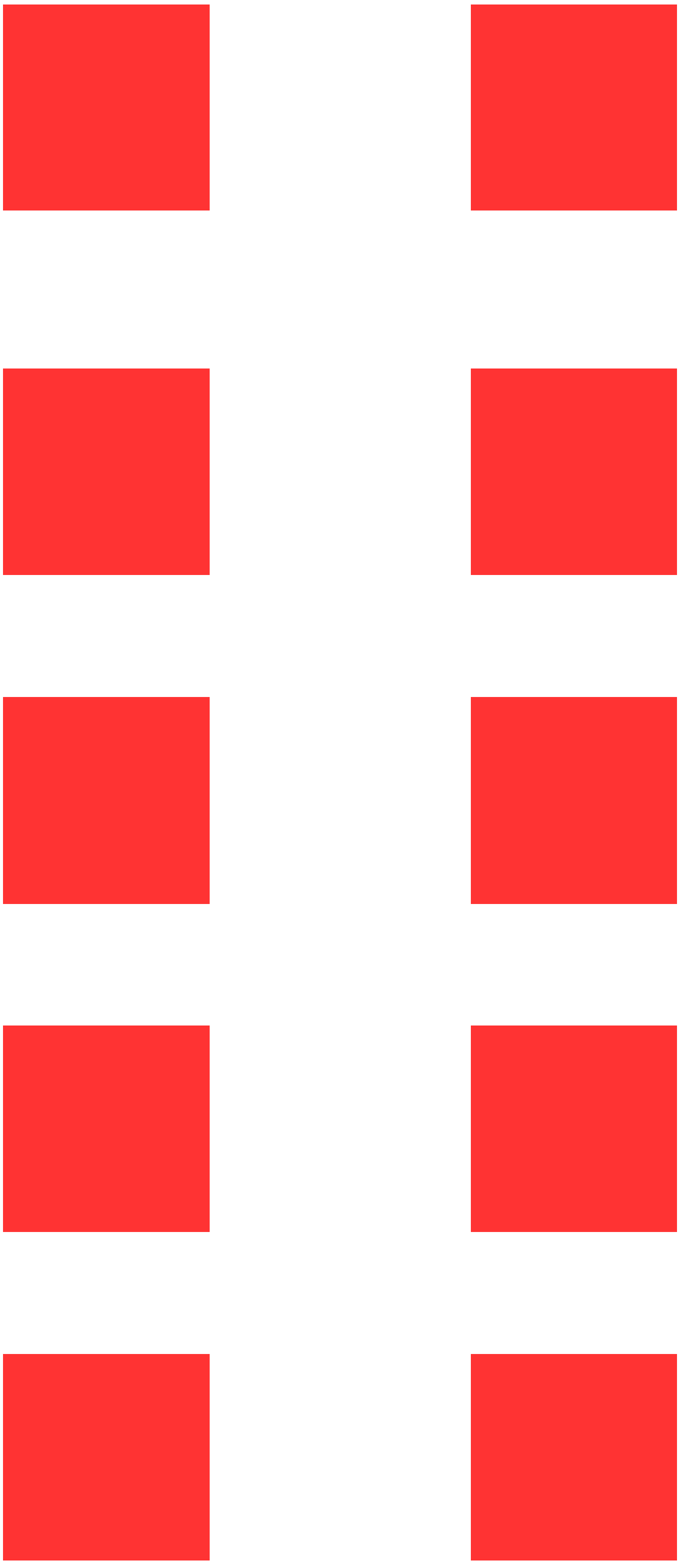










Appendix I

.4. Cardinality experiment Pre – and Post-test task Sheets:

The 2 puppets are set side by side, and a set of toys are placed in front of one of the puppets. Children are asked to make sure that the puppets both have the same amount of toys by giving the second puppet toys to play with. The child is asked how they knew that the 2 puppets now have the same number of toys to play with.

Pretest Matching Task

	Type	Organisation
2	pink counters	lines
4	dominoes	bunch
6	crayons	bunch
8	green counters	lines
10	bottle tops	bunch
12	crayons	lines

Experimental Sheets

Both puppets are set side by side, and a set of objects are placed in front of both of them. Children are asked to check and see if the 2 puppets have the same number of objects. They are asked to justify their answer – why do you think that the puppets do / don't have the same number of objects.

Pretest Comparison Task

	Type	Organisation	Match/Mismatch
3	crayons	bunch	Same
5	bottle tops	lines	1 more
7	pink counters	bunch	1 more
9	green counters	lines	2 more
11	dominoes	bunch	2 more
13	bottle tops	lines	Same

Experimental Sheet

The 2 puppets are set side by side, and a set of toys are placed in front of one of the puppets. Children are asked to make sure that the puppets both have the same amount of toys by giving the second puppet toys to play with. The child is asked how they knew that the 2 puppets now have the same number of toys to play with.

Post Test Matching Task

	Type	Organisation
3	pink counters	bunch
5	crayons	lines
7	bottle tops	bunch
9	green counters	lines
11	dominoes	bunch
13	bottle tops	lines

Experimental Sheets

Both puppets are set side by side, and a set of objects are placed in front of both of them. Children are asked to check and see if the 2 puppets have the same number of objects. They are asked to justify their answer – why do you think that the puppets do / don't have the same number of objects.

Post Test Comparison

	Type	Organisation	Match/Mismatch
2	crayons	line	1 more
4	green counters	bunch	same
6	bottle tops	bunch	2 more
8	dominoes	line	2 more
10	pink counters	bunch	1 more
12	crayons	line	same

Appendix I
.5. Cardinality experiment Intervention sheets

Children are asked to watch one puppet performing a matching task, to see how well the puppets can count. The child is to watch the puppet counting, to copy their actions, and then say whether the puppet has performed the task accurately. They are asked to justify their answer, whether the puppet was right or wrong – what did the puppet do wrong / why was it wrong?

The puppet makes a mistake by miscounting (e.g. skipping an object(s) in count on the first run through)

Conceptual Intervention

	Organisation	Match/Mismatch
3	line	Match
5	bunch	1 more
7	bunch	2 more
9	line	1 More
11	bunch	match
13	line	2 more

Experimental Sheet

Children watch a puppet using a count to compare strategy on a matching task, and were then asked to copy the type of performance of the puppet to ensure that it was correct.

Procedural Intervention

	Organisation
3	line
5	bunch
7	bunch
9	line
11	bunch
13	line

Experimental Sheet

Control Group

Children are asked to complete a series of simple pen-and-paper mazes.

Appendix I

.6. Instruction sheets for the microgenetic experiment in chapters 6 and 7

2 sets of 9 beams were used. Children were presented both sets in each session across 4 sessions. The order of the beams and of the sets was randomised (so that half the sessions they were given set 1 first, and in the other half they were given set 2 first), to minimise repetition. For each set, there were 4 different sets of order; beams were sorted in such a way that one for every triad of beams (e.g. first 3, second 3, last 3), there was one symmetrical beam. This allowed for coding R-R level per subset of 3 beams. Also, for each set of beams, children were never given them in the exact same order twice.

5.1 Instruction sheets for the Balance Beam Production Task: SET 1

The child is shown the array of beams and the fulcrum. The child is told that they are going to be given the beams, and that the object of the game is to try and make the beams balance on the fulcrum – the small wooden bit. They are shown the puppet, who they are told is trying to learn about balance. The child is asked to explain to this puppet for each beam why the beam does or does not balance.

Colour	Length	Width	Weight ratio	Initial Place
Symmetrical:				
2. Wood	30cm	2 ½ cm	0:0	Left/Centre/Right
8. Red	30cm	2cm	1:1	Left/Centre/Right
5. Green	40cm	2 ½ cm	1:1	Left/Centre/Right
Asymmetrical:				
1. Pink	40cm	3cm	3:1	Left/Centre/Right
9. Red	43cm	1 ½ cm	2:1	Left/Centre/Right
6. Mauve	30cm	2cm	3: 1 ½	Left/Centre/Right
4. Blue	25cm	2cm	3:2	Left/Centre/Right
7. Yellow	20 ½ cm	2cm	1:0	Left/Centre/Right
3. White	30cm	1 ½cm	3:2	Left/Centre/Right

Balance Beam Production Task: SET 2

The child is shown the array of beams and the fulcrum. The child is told that they are going to be given the beams, and that the object of the game is to try and make the beams balance on the fulcrum – the small wooden bit. They are shown the puppet, who they are told is trying to learn about balance. The child is asked to explain to this puppet for each beam why the beam does or does not balance.

Colour	Length	Width	Weight ratio	Initial Place
Symmetrical:				
1. Navy	30cm	2 cm	0:0	Left/Centre/Right
8. Green	30cm	1 ½ cm	1:1	Left/Centre/Right
4. Orange	40cm	2 cm	1:1	Left/Centre/Right
Asymmetrical:				
6. Blue	30cm	2cm	2:0	Left/Centre/Right
2. Green	43cm	1 ½ cm	4:1	Left/Centre/Right
7. Purple	45cm	1 ½ cm	3: 1	Left/Centre/Right
3. Wood	30cm	1 ½ cm	2:1	Left/Centre/Right
9. Wood	30 cm	1 ½ cm	3:2	Left/Centre/Right
5. Wood	30cm	2 cm	1:0	Left/Centre/Right