Skewed coding for suppression of pattern-dependent errors

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Abstract We present information theory analysis of the trade-off between BER improvement and the data rate loss using skewed channel coding to suppress pattern-dependent errors.

Introduction

Inter symbol interference (ISI) or the patterning effect manifests itself in digital communication as the dependence of the transmission result for an information bit on the surrounding pattern, i.e. the neighboring bits (see e.g. recent papers [1,2]). An important and actively studied example of transmission with pattern-dependent errors is optical fibre communication at high bit rates limited by intra-channel four-wave-mixing (ICFWM) [3,4] through the generation of the "ghost" pulses. Various techniques have been proposed and implemented to suppress ICFWM. In this paper, using ICWFM as a key example we consider information theory approaches to reduce pattern-dependent errors. Suppression of nonlinear intra-channel effects by channel coding has first been discussed in [5]. The use of the modulation codes to decrease ICFWM effects has been proposed in [6]. In this paper we present information theory analysis of the trade-off between BER improvement and loss in the data rate using *skewed channel coding*.

Patterning effects can be partially characterized at the receiver by analysis of error rates for eight elementary triplets corresponding to all possible combinations of the nearest neighboring bits. Probabilities of errors for the central bit in the eight elementary triplets are presented by "error vector" $\vec{Q} = (Q_0, Q_1, Q_2, ..., Q_7)$. Total BER of the long transmitted pattern is then given by the $BER = \sum_{k=0}^{7} P_k Q_k$ where P_k is the probability of the occurrence of a triplet with the index k in the input bit string, and Q_k is the error probability for the central digit in the triplet. A non-equal distribution of errors Q_k caused by ISI provides an opportunity to reduce the error rate by *reducing the probability of the triplets that affect the BER most,* by employing skewed coding. Obviously, this can only be done at the expense of the information content, which is represented by the transmitted signal entropy H (measured in bits/digit). Redundancy of the signal coding with non-uniform probabilities of different patterns is defined as $R = H_{max} - H = 1 - H$.

The skewed encoding

For illustration purposes, in the rest of the paper we focus on the example of an ICFWM-limited system, although our approach is general. As shown in [2], the major contribution to the BER in systems limited by ICFWM is from the pattern 101. It was observed in [2] in extensive numerical simulations that the probability of error for that pattern is approximately 20 times (under the stated system/signal parameters) that of any other triplet. To exemplify the error distribution between the triplets, we would like to look at the situation when $Q_{101} = Q_5 = MQ_k, k \neq 5$, with M varying from 10 to 40. Thus, the error vector here is in the form $\vec{Q} = q(1,1,1,1,1,M,1,1)$. Our goal now is to quantify the BER improvement using a pre-encoding of transmission data so that certain combinations of input digits are less frequent. Note that the two important characteristics of the transmission system performance are the BER and the channel throughput. The former is improved by pre-encoding the latter is worsened by it. Hence, the trade-off here is between BER improvement and loss in the data rate (or spectral efficiency) and increase in the complexity of an encoder/decoder.

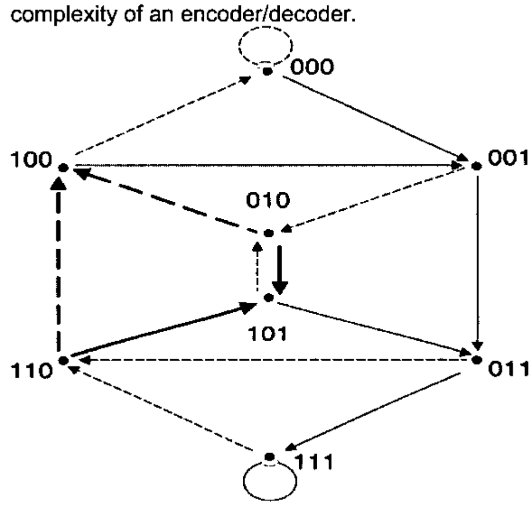

Figure 1. Transition graph.

The first issue to consider is the source information content as a function of the pre-encoding skew. We consider the encoder model of an encoder as a Markov chain shown in Fig. 1, which is a random process that attempts to avoid producing the combination of 101 found error-prone in [2]. The vertices of the graph correspond to the state of the process, which consists of the three last digits up to and including the current digit: $d_{k-2}d_{k-1}d_k$. The probability of the next digit is depicted as a transition from that state to the next one, $d_{k-1}d_kd_{k+1}$, with d_{k+1} equal to either 1 or 0, with a probability depending on the current state $d_{k-2}d_{k-1}d_k$. We use dashed arrows for $d_{k+1} = 0$ and solid ones for

$d_{k+1} = 1$. Those arrows are either thin, corresponding to the *non-skewed* transitions with the probability 0.5, or thick, corresponding to the *skewed* transitions. The latter are the probability $(1-\varepsilon)/2$ leading to the "bad" state 101, and the probability $(1+\varepsilon)/2$, leading to a neighboring "good" state 010. Notice that each state number, read as a binary quantity k has exactly two transitions from it, into states 2k+0 mod 8 and 2k+1 mod 8. The stationary distribution corresponding to the Markov process presented in Fig. 1 must satisfy the condition: $\hat{T}\vec{P} = \vec{P}$, where $\hat{T} = T_{k,l}$ is the process transition matrix graphically described in Fig. 1. The solution of this stationarity equation normalized by $\sum_{k=0}^{7} P_k = 1$ gives the number of the occurrences of the triplet with the index k in an infinite string of bits. The entropy of the transmitted signal reads

$$H = -\sum_{k=0}^{7} P_k (T_{k,2k} \log_2 T_{k,2k} + T_{k,2k+1} \log_2 T_{k,2k+1}) \quad (1)$$

Calculation of the redundancy

Omitting straightforward calculations we get an 8-component vector $\vec{P}$ representing the stationary distribution of the triplet's probabilities:

$$\vec{P} = \frac{1}{8+2\varepsilon}(1+\varepsilon, 1+\varepsilon, 1, 1, 1+\varepsilon, 1-\varepsilon, 1, 1)$$

Thus the frequency of occurrence is higher for the triplets 000, 001, and 100; and lower for pattern 101. Substitution of the stationary distribution $\vec{P}$ into (1) yields the redundancy $R = \dfrac{1-g(\varepsilon)}{4+\varepsilon}$, where

$$g(\varepsilon) = -\frac{1+\varepsilon}{2}\log_2\frac{1+\varepsilon}{2} - \frac{1-\varepsilon}{2}\log_2\frac{1-\varepsilon}{2}.$$

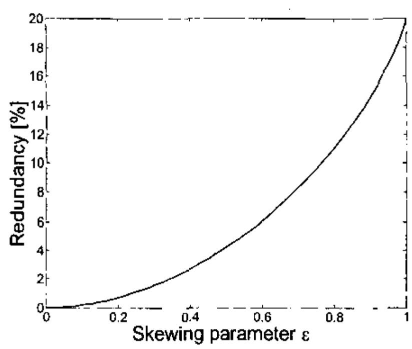

Figure 2. Redundancy versus skew parameter ε.

Figure 2 displays the result in graphic form. The largest redundancy corresponds to $\varepsilon = 1$ (extreme skew), where 20% of the throughput is lost. However, under a significant skew of 25%, the loss of throughput is at only 1%, which suggests that bit stuffing (i.e. the use of extra digits to achieve correlation behavior similar to our Markov chain) could be bandwidth-efficient when used in addition to standard forward-error correcting codes. To quantify the BER improvement due to a skewed modulation code we introduce a "code gain" factor defined as: $10\log_{10}[BER(\varepsilon=0)/BER(\varepsilon)]$. Note that here this term is used in a different context compared to the standard FEC notation (which involves energy per bit parameter) commonly used to describe linear AWGN channels.

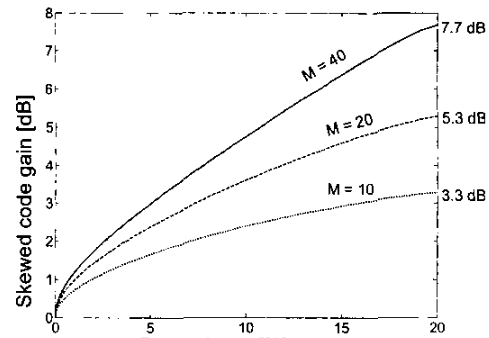

Figure 3. Improvement of the BER versus redundancy added by skewed coding.

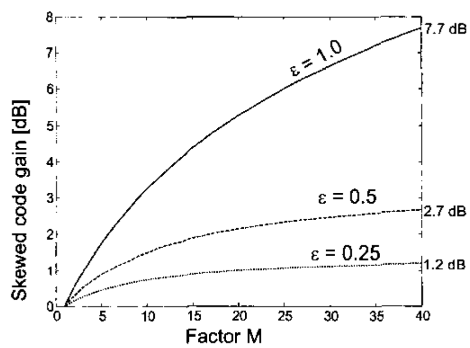

Figure 4. Code gain versus error asymmetry factor M.

Figure 3 shows the BER improvement as a function of the code redundancy. Figure 4 similarly presents "code gain" versus error asymmetry factor M for different values of the skew parameter.

Conclusions

We have examined an information theory approach to the improvement of BER in systems degraded by pattern-dependent errors. Decrease of the error rate is achieved by application of a skewed channel coding that reduces the probability of occurrence of the triplets making the major contribution to the BER. We have quantified the trade-off between BER improvement and the data rate loss using skewed coding.

References

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