A SYSTEMATIC STUDY OF THE BEHAVIOUR OF THE PMEPR IN RELATION TO OFDM DESIGN PARAMETERS

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***Abstract* -** The design of systems with enhanced quality of service (QoS) and improved power efficiency has evolved into an intensive research area in wired and wireless communications engineering. Orthogonal frequency division multiplexing (OFDM) has been proven to have the potential to achieve high data rates, adapt to severe channel conditions and exhibit spectral efficiency; this has gained its popular support in the design industry, especially for fourth generation (4G) systems. However, the high peak to mean envelope power ratio (PMEPR) exhibited by OFDM signals require linear operation of analog devices, with the associated trade-off of poor power efficiency. Several methods to reduce this PMEPR problem have been effectively researched while revealing the shortcomings. In this study we recognize the need to present the effect of OFDM system parameters on the behaviour of the PMEPR. In order to provide a basis for systematic selection of OFDM design parameters for PMEPR mitigation, we first study the reaction of the PMEPR to OFDM design parameters, we then analyse the effect of OFDM design parameters on the shortcomings of the PMEPR-limiting clipping technique.

I. INTRODUCTION

OFDM has been recognized as the candidate to meet the requirements of 4G wireless systems. The driving force behind the recognition of OFDM is the desire for effective wireless communication in high mobility conditions, and the increase in multi-media applications which require high data rates. The minimum target data rate for 4G systems is expected to be 1 Gbps and at least 100 Mbps for high mobility conditions [1]. Currently OFDM is of interest to researchers in universities and industries all over the world. The scope of industrial research in OFDM is shown in [2] to have expanded to the development of its variants which include a hybrid combination of OFDM employed as a multiple access scheme and frequency hopping. OFDM is also shown in [3] to have the capacity to surpass the performance of code division multiple access (CDMA) for 4G wireless access methods.

A major drawback of OFDM is its fluctuating envelope signals. The variation of the instantaneous envelope of an OFDM signal depends on the phase of each composite sub-carrier at any particular instant. The individual sub-carriers combine randomly in phase producing a high peak envelope equivalent to the sum of the amplitude of each sub-carrier referred to as the PMEPR. The non-linearity exhibited by the PMEPR of an OFDM signal causes power inefficiency in non-linear systems like power amplifiers (PAs), as a large back-off is needed to keep the signal correctly detectable. The fluctuating envelope is shown in [4] to limit the PA's global system efficiency to 13%.

The degrading effect of the high PMEPR can be mitigated by several PMEPR reduction techniques proposed in the Literature [5-11], these techniques however achieve PMEPR reduction while trading off other design features such as computational complexity, data rate loss, bandwidth inefficiency, BER degradation amongst others [12]. In practice, in addition to applying these PMEPR reduction techniques, designers have to carefully consider the effect of OFDM system parameters on the behaviour of the PMEPR. The systematic study conducted in this paper is based on the consideration that if we can use a statistical survey to accurately define how the PMEPR reacts to system parameters, we can significantly simplify the system design process for high PMEPR mitigation. This study reveals that with respect to the application of the system being designed, the PMEPR of an OFDM signal reacts significantly to varying OFDM design parameters.

The simplest approach to reducing the PMEPR of OFDM signals is to clip the high amplitude peaks [13]. In practice, it is desirable to establish the optimum clipping level for each system design with due cognisance of the associated trade-offs. To analyze the behaviour of the PMEPR using MATLAB simulations, we generate an OFDM signal and vary the system parameters, we then deliberately clip the signal peaks at defined clipping thresholds to study how OFDM system parameters contribute to the trade-offs of the clipping technique investigated in [14], in terms of in-band distortion (IBD) and out-of-band radiation (OBR). The study in this paper builds on the statistical analysis conducted in [15] while presenting a more detailed revelation of how system parameters affect the behaviour of the PMEPR.

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II. REPRESENTATION OF THE OFDM SIGNAL AND THE PMEPR

An investigation on the characteristics of the OFDM signal conducted in [16] denotes a set of symbols $M = \{M_k, k = 0,1,2\ldots, N-1\}$ used to modulate a set of N sub-carriers $\{f_k, k = 0,1,2\ldots, N-1\}$. Orthogonality is maintained by utilizing a carrier spacing such that $f_k = k\Delta f$ where $\Delta f = 1/NT_p$ and T_p is the pulse duration. The guard interval (GI) T_g must be large enough to eliminate the inter-symbol interference caused by the multipath delay spread T_m of the channel i.e. $T_m \le T_g$. The total symbol period T then becomes $T = T_p + T_g$. The complex envelope of M then becomes:

$$S(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} M_k \, e^{j2\pi f_k t} \qquad 0 \le t \le NT \tag{1}$$

where $j = \sqrt{-1}$.

The analysis of the PMEPR is performed in the time domain, it is thus necessary to present an oversampled time domain representation of $S(t)$.

$$S(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} M_k e^{j\frac{2\pi kn}{LN}}, n = 0,1,2,\ldots LN-1 \tag{2}$$

where L represents the oversampling factor, an oversampling factor $L = 1$ and $L = 3$ was used to analyze the PMEPR in the simulations. Sampling the time domain signal by a factor L yields a length LN IFFT operation achieved by inserting $(L\text{-}1)N$ zeros into $S(t)$.

The PMEPR of $S(t)$ can be defined as:

$$PMEPR\{S(t)\} = \frac{\max_{t\in[0,T]} |S(t)|^2}{P_{av}},$$
$$P_{av} \equiv E\left\{ |S(t)|^2 \right\} \tag{3}$$

where P_{av} is the average power and E denotes the expectation based on Parseval's Theorem. The resultant PMEPR of the L times oversampled time domain OFDM signal can be represented by:

$$PMEPR\{S(n)\} = \frac{\max\left|S(n)^2\right| \ , \ 0 \le n \le LN-1}{E\left\{|S(n)|^2\right\}} \tag{4}$$

III. SIMULATION MODEL - I

The simulations were performed using MATLAB m-files, MATLAB m-files were selected because it provides the convenience and flexibility required to vary system parameters.

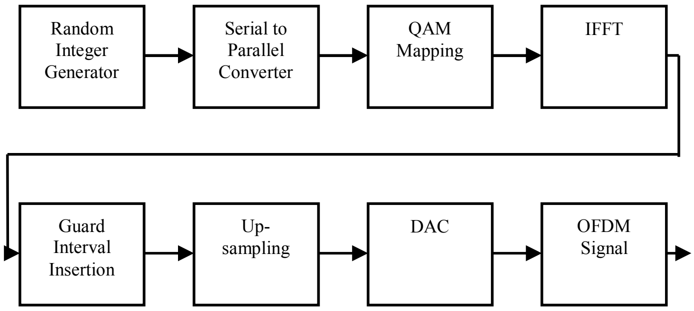

Fig. 1. OFDM Transmitter Model

The block diagram of the simulation model that statistically analyzed the behaviour of the PMEPR is as shown in Figure 1. Here, the OFDM receiver is not considered because the predominant analysis is made from the time domain representation of the OFDM spectrum before the receiver section. The detailed description of the composite blocks of the transmitter model is as follows:

- The random integer generator generates random uniformly distributed integers in the range [0, M-1], where M is the M-ary number for example for 4-QAM it generates 00, 01, 10 and 11 at its output.
- The serial random data is converted to parallel bit streams to enable parallel transmission with N sub-carriers, each bit stream then undergoes M-QAM modulation. Baseband modulation is used as it operates at reduced speed enabling effective investigation of the operations. The output is converted to a complex vector equivalent to M-QAM constellation points.
- The parallel data on the frequency axis are fed into the IFFT block where parallel to serial conversion takes place, generating a time domain OFDM signal.
- Guard interval is inserted to eliminate ISI caused by multipath properties of the channel. Zero padding aims to increase the original sampling rate, each symbol is up-sampled by a factor L, meaning L-1 zeros are inserted between any two symbols.
- The output undergoes; digital to analog conversion, up-conversion to RF, and transmission through the radio spectrum.

To effectively study the behaviour of the PMEPR over the different OFDM parameters used in practical systems, we select a range of parameters based on the upper and lower limits specified in [17] and [18]. Simulation parameters were selected using the following guidelines:

- N represents the number of sub-carriers for $1 \le N \le 2048$
- The modulation order M for M-QAM is given as $M = 2^m$ for $m \in \{2,4,6,8\}$

- The number of IFFT points $N_{IFFT} = 2^p$ where p is an integer $1 \le p \le 11$.

- Length of the guard interval G for $G \in \left\{ \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{4}{N} \right\}$
- Up-sampling factor L was varied between 1 and 4 inclusive.

- Frequency bandwidth B in MHz for $B \in \{1,2,5,10,15,20\}$

IV. SIMULATION RESULTS - I

To generate the initial OFDM signal, 64-QAM baseband modulation was used to map 1024 sub-carriers; an IFFT length of 128, a guard interval of $1/16$ and an up-sampling factor of 1 were selected. To investigate the effect of system parameters on the PMEPR of the OFDM signal, the average PMEPR E_{av} of the signal was computed while varying other system parameters, for this analysis, the length of the guard interval, G and bandwidth remain unchanged. The value of E_{av} is computed for different modulation constellations M while varying the number of sub-carriers N for each value of M, the results are shown in Table 1.

TABLE I

PMEPR VALUES E_{av} FOR VARYING SYSTEM PARAMETERS

M	PMEPR (dB)			
	$N = 128$	$N = 256$	$N = 512$	$N = 1024$
2	2.75	2.9	3.0	3.0
4	3.75	3.8	4.0	4.0
16	8.0	8.0	10.0	10.0
64	17.0	18.5	18.5	20.0

The result of the simulation reveals that PMEPR is dependent on the number of sub-carriers and the applied modulation scheme. From Table 1 it is evident that for each modulation scheme, as N increases, the PMEPR becomes higher. Also for a fixed number of sub-carriers, an increase in the modulation constellation M brings about a corresponding increase in PMEPR. The trend of this increase in PMEPR with respect to M and N is indicated with arrows in the table. The graph of PMEPR versus modulation order for different number of sub-carriers is plotted in Figure 2. Also, from the table it is noted that increase in modulation constellation has a more significant effect on the PMEPR than increase in the number of sub-carriers, this is clarified by the highlighted portions of the table. Figure 3 shows time domain illustrations of the effects of varying the modulation order, on the PMEPR. It is also observed from the simulations that; increasing the up-sampling factor from $L = 1$ to $L = 3$ yields a 2-3 dB increase in the PMEPR, further increase in L yields relatively limited increase in the PMEPR. However, change in the IFFT length brings little noticeable difference in the PMEPR.

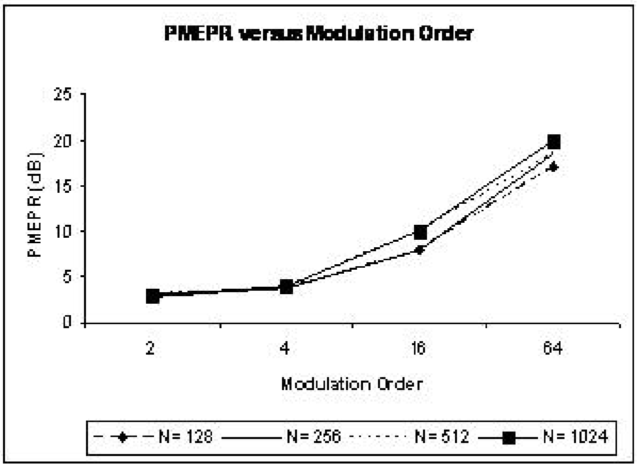

Fig. 2. Graph of PMEPR versus Modulation Order

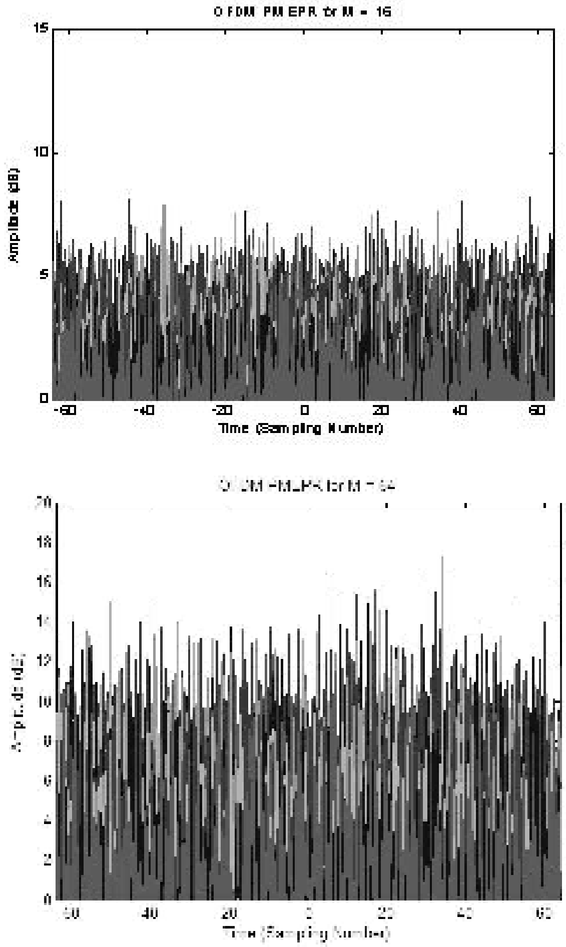

Fig. 3. Effect of Modulation Order on PMEPR

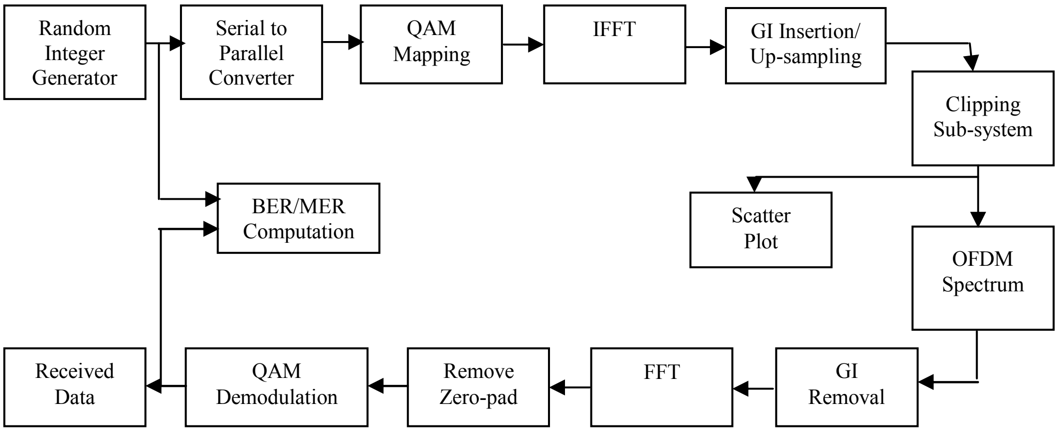

Fig. 4 OFDM Transceiver Model

V. SIMULATION MODEL - II

The second stage of the simulation involves implementation of the clipping technique for PMEPR reduction. The performance of the clipping technique is investigated; careful consideration is given to the performance in relation to the OFDM parameters, constellation noise, the bit error rate (BER), and the modulation error ratio (MER). To accomplish these analyses, the receiver section must be incorporated especially for BER and MER computations. The new block diagram of the simulation model incorporating the receiver section is shown in Figure 4. At the receiver, the guard interval is removed from the received OFDM signal, after which the data is fed into the FFT block. In the FFT block, the serial data is converted into its parallel format on the frequency axis, down-sampled, and fed into the demodulator block. Next the BER and MER are computed by comparing the transmitted and received data.

The clipping sub-system is designed to effect a non-linear transformation of the Gaussian OFDM signal; this is achieved by saturating the signal peaks at a defined threshold. We consider the clipping model as soft limiting system such that the output signal $R(n)$ is given by:

$$R(n) = \begin{cases} R(n), & for\ R(n) \leq \gamma \\ \gamma\ , & for\ R(n) > \gamma \end{cases} \qquad (5)$$

where γ is the clipping threshold. In the system, the clipping algorithm is implemented after up-sampling; this is because if the signal is clipped at the output of the IFFT, the up-sampling process causes re-growth of the clipped peaks.

VI. SIMULATION RESULTS - II

In order to evaluate the performance of the clipping sub-system, a 64-QAM OFDM signal is used to modulate 256 sub-carriers. The symbol sequence is now oversampled by a factor L = 3, guard interval of $1/16$ and a bandwidth of 20MHz are used.

Deactivation of the clipping algorithm is achieved by setting γ to an excessive 21dB. The first section of this simulation investigates the performance of the BER and MER for different clipping thresholds. In order to effect this computation, evaluation of the performance of clipping is made for N = 256 and 512, and for M = 64 and 256. The clipping threshold is adjusted, and the BER and MER computed for various clip levels as illustrated in Table 3 and Table 4.

TABLE 3
BER and MER Due to Clipping for $N = 256$

γ (dB)	64-QAM BER (10^{-3})	64-QAM MER(dB)	256-QAM BER (10^{-3})	256-QAM MER(dB)
16	0	-34.83	0	-35.01
14	0	-31.28	0.209	-31.49
12	0	-28.07	11.204	-28.13
10	0.2094	-24.86	97.906	-24.99
8	9.1099	-22.27	289.27	-22.08
6	67.016	-19.61	488.38	-19.63
4	176.07	-17.46	634.87	-17.37
2	317.07	-15.59	746.23	-15.52

TABLE 4.
BER and MER Due to Clipping for $N = 512$

γ (dB)	64-QAM BER (10^{-3})	64-QAM MER(dB)	256-QAM BER (10^{-3})	256-QAM MER(dB)
16	0	-34.79	0	-35.03
14	0	-31.41	0.261	-31.40
12	0	-28.16	11.802	-28.10
10	0.392	-24.96	101.46	-24.89
8	9.69	-22.20	287.75	-22.13
6	68.31	-19.54	490.21	-19.57
4	183.11	-17.40	638.28	-17.34
2	322.45	-15.47	743.39	-15.41

The results in the tables show that clipping causes BER and MER degradation, as the clipping threshold γ decreases the degradation is more pronounced. For high clip thresholds between 12dB and 16dB the effect of the clip is negligible for 64-QAM BER performance as highlighted in the tables, the MER degradation is however relatively more pronounced. The simulation also reveals that the number of sub-carriers N has limited effect on the BER and MER. This could be attributed to the fact that even though the clipping effect increases with N, the signal power in each sub-carrier reduces as N increases. The results also show that the BER performance due to clipping deteriorates significantly with increased constellation size. This confirms that the IBD which causes BER degradation can be minimized using lower order constellations. The effect of constellation size on the MER is however shown in the results to be limited. The graphs of BER and MER, versus γ for 64-QAM and 256-QAM with N = 256 are plotted in Figures 5 and 6.

The impact of clipping on OBR is also investigated; it is observed that spectral regrowth is negligible when the clipping sub-system is deactivated. Decreasing γ to values around 20dB indicates a slight increase in the spectral regrowth; further decrease of γ to values of about 10dB or lower increases the impact on the spectral regrowth. Figures 7 and 8 illustrate the spectral regrowth effect on the OFDM spectrum for various values of γ. The simulation also shows that the spectral distortion is more pronounced when the modulation order is increased. Effect of clipping on the constellation plot is also illustrated in Figures 9 and 10.

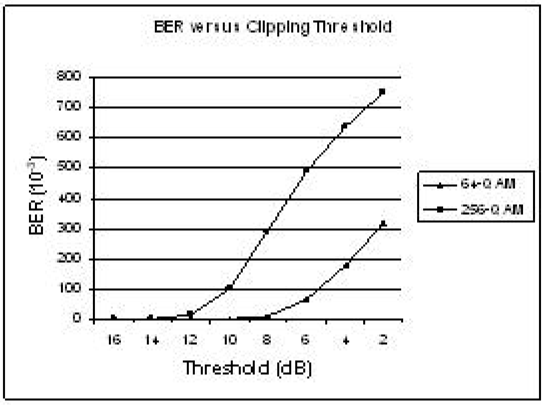

Fig. 5. BER Performance Due to Clipping

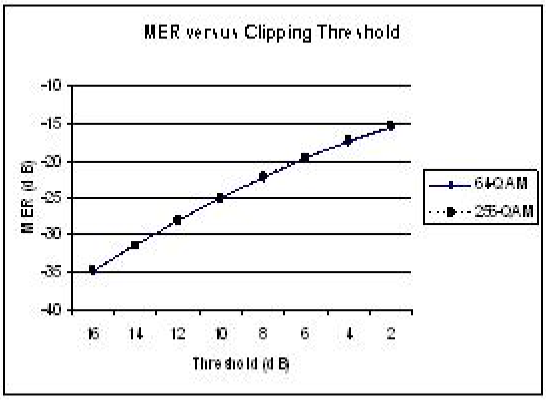

Fig. 6. MER Performance Due to Clipping

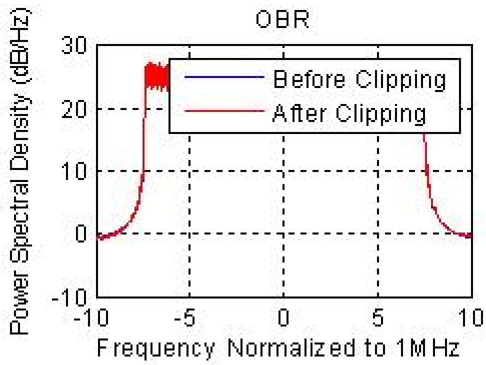

Fig. 7. Spectral Regrowth with Deactivated Clipping.

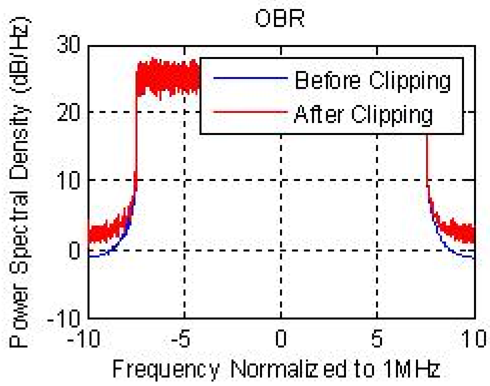

Fig. 8 Spectral Regrowth with 10dB Clipping Level

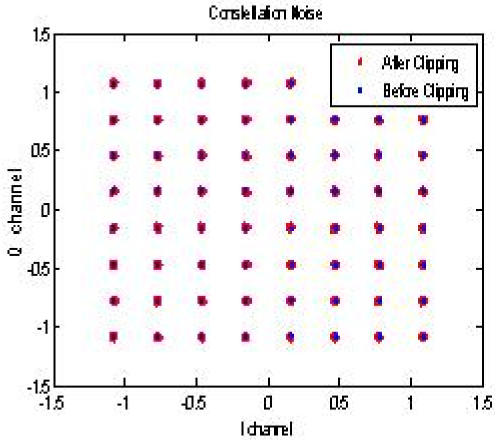

Fig. 9. 64-QAM Constellation with Deactivated Clipping.

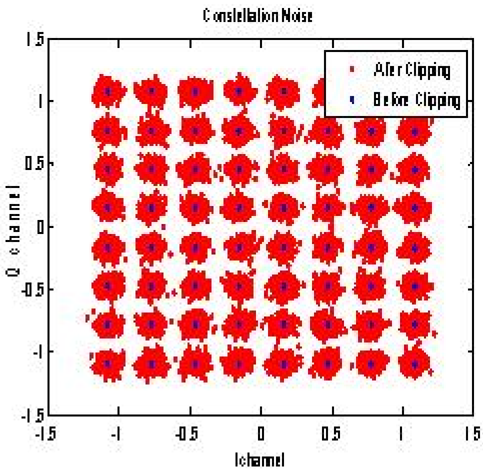

Fig. 10. Distortion Effect of 10dB Clipping

VII. CONCLUSION

The results of this simulation can conclude that the PMEPR of an OFDM signal can be effectively controlled by selectively varying OFDM design parameters. The PMEPR increases with increased number of sub-carriers N and constellation size M, the statistical survey conducted in this paper shows that the PMEPR is more sensitive to M than N. In the results, it is noted that for all values of N, 50% of the symbols possess a PMEPR higher than 17 dB for a 64-QAM modulation scheme. The simulation also reveals that varying the up-sampling factor from $L = 1$ to $L = 3$ increases the PMEPR, further increase in L has negligible effects. Also, an increase in the length of the IFFT operation results in a slight increase in the peak envelope. The effect of M and N on BER and MER degradation due to clipping is also analyzed; it is observed that while M significantly deteriorates BER performance, the effect of N on BER performance is negligible.

PMEPR reduction techniques should be selected for specific applications relative to their performance. Analysis of the spectral performance of the clipping algorithm presented in the simulations section makes it necessary to discourage its application in areas where spectral emission is tightly controlled. In other words, clipping for PMEPR reduction will not be sufficient for personal communication services (PCS) or cellular applications where OBR requirements are high. The BER degradation effect of clipping must be considered in practice relative to the application, systems with low error tolerance can compromise the OBR for good BER performance, and otherwise. In practice when the desired error probability is low, the clipping threshold can be set high enough such that clipping occurs very in-frequently.

The systematic study conducted in this paper is limited to the clipping algorithm; however it provides a basis for researchers to study the effect of OFDM system parameters on the shortcomings of the PMEPR mitigation techniques contained in the Literature, this approach is envisaged to provide designers with guidelines for optimum parameter selection.

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